BEHAVIOR OF TEMPERATURE GRADIENTS AND VELOCITY FIELDS IN FUEL OIL EXPORTATION THROUGH A 12 INCHES TOROIDAL STRETCH REPRESENTED BY TWO CYLINDRICAL SURFACES

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Abstract. *The proposal of this work is to predict temperature gradients and velocity fields of a flow in an insulated stretch of a curved tube, through two cylindrical surfaces. Particularly, it was chosen the vertical central plane and the horizontal central surface of the curve. The curvature radius is considered to be the same length of the pipe diameter. Fields of velocity achieved as well as temperature gradients developed are built from initial conditions caught from the final values of the previous flow through the straight stretch. Three conditions of flow rate and initial temperature are tested, all of them at a very low Reynolds and Dean numbers and very high Prandtl number. Simulations are performed through finite difference method and a commercial code accomplishes the same simulation to be compared with. The practical case represented in this study is the transportation of fuel oil between a refinery and a distribution terminal in Brazilian Northeastern, where the initial stretch is aerial and buries into the ground with a vertical curve of 90 degrees. Results obtained through the cylindrical surfaces show good coincidence with results of the commercial code and literature and could represent an alternative way to treat the classical issue of the flow in a toroidal stretch.*

Keywords: *Gradients of Temperature, Velocity Fields, Numerical Simulation*

1 – INTRODUCTION

Studies of the flowing in curved and coiled tubes of circular section have been exhaustively carried out. It was Dean (1927 and 1928) that conducted the first analytical investigation of fully developed laminar flow in a curved pipe. He was also the first researcher to examine the pressure drop of the fully developed flow in a torus by assuming uniform stream for secondary flow. In the eighties, extensive reviews were made by Berger S. A. et al (1983), Ito (1987), Kakaç et al (1987) and Brum (1988), where it was posted the general approaches to the problem until those articles' moment. At the current time, Ghobadi and Muzychka (2016) in "A Review of Heat Transfer and Pressure Drop Correlations for Laminar Flow in Curved Circular Ducts" present the state-of-the-art of the issue. According to these authors, in pretty much one century, approximately 5,000 U.S. patents were registered and more than 10,000 research articles were performed on curved tube geometries and their applications.

In terms of coordinates to be applied in the treatment of the problem, it is important pointing that Sankarish., and Rao (1973) were the first to model equations in toroidal coordinates and this is the most employed coordinate system for the case. Patankar et al (1974 and 1975) approached it in curvilinear cylindrical ones and Tyagi and Sharma (1975) citing Kreith (1955) utilized both systems. The first, for the Navier-Stokes equations and the second, for the energy equation. Unlike the ordinary, Masliyah and Nandakumar (1984) applied an alternate development of a nonorthogonal coordinate system that is useful in studying fluid flow in helicaltubes in the limit of large pitch.

Following this path, the proposal of this article is to employ two cylindrical surfaces, to discretize equations described in regular cylindrical coordinates, to find out velocity fields and temperature gradients in a flow through a curved pipe. Simulations are performed through finite difference method and a commercial code accomplishes the same simulations to be compared with.

The case to be verified is a fuel oil transportation between a refinery and a distribution terminal through a piping 35.1 Km long in Brazilian Northeast region. A first work of flowing simulation in a straight stretch was developed by Ciambeli et al (2014), to predict temperature gradients in a section not insulated and with a reduction on the original thickness, where there is a loss of heat. On the present work, it will be held the simulation of the flow in the curve after the straight stretch.

When the straight pipe buries into the ground, is does through a vertical curve of 90 degrees. As stated above, curves, helicoids, serpentines, spirals and twisters of circular section are most commonly simulated as a torus. In this sense, to better understand the mathematical proposal of the present work, take a look at the red and blue surfaces in Fig. 1.

The vertical plan of the axial direction in a straight pipe will prolong ahead through a curve as the red plan shown in Fig. 1. This plan represents a quarter of the circular section in a cylinder. In the same way, the horizontal plan of the axial direction will prolong ahead through the curve as the blue surface. This body depicts a quarter of the hull in a cylinder. On the majority of the cases, these bodies will be elements of thermal and hydrodynamical symmetry for a flowing through a curve and Navier-Stokes and Energy equations can be described in cylindrical coordinates inside them.

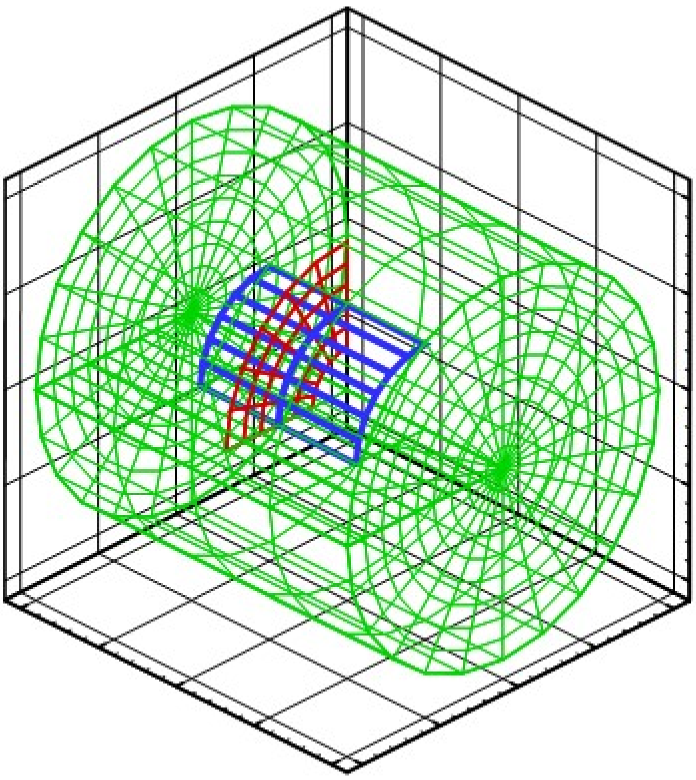

Fig. 1 Surfaces colored in red and blue are surfaces of thermal and hydrodynamical symmetry inside a vertical curve of a tube. Navier-Stokes and Energy equations can be expressed in cylindrical coordinates inside them.

2. MATERIALS AND METHODS

2.1 Equations governing the problem and assumptions

Equations of mass conservation, momentum and energy for red and blue surfaces as posted in Fig. 1 can be expressed in cylindrical coordinates as evinced below. In this mathematical proposal, r is the radial direction, θ is the axial direction and z is the transversal one. Assumptions for momentum equations are: the fluid was assumed to be incompressible and Newtonian with constant physical properties and fully developed flow. For energy equation, assumptions are: constant physical properties, ρ and k, and no energy generation. For the blue shell in Fig. 1, R is the curvature radius of the piping's curve.

Equations for a quarter of a plate, as shown in Fig. 1, for mass conservation:

$$\frac{\partial v_r}{\partial r} + \frac{v_r}{r} + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} = 0 \tag{1}$$

r Momentum:

$$\rho\left(\frac{\partial v_r}{\partial t} + v_r\frac{\partial v_r}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta} - \frac{v_\theta^2}{r}\right) = \rho g_r - \frac{\partial P}{\partial r} + \mu\left(\frac{\partial^2 v_r}{\partial r^2} + \frac{1}{r}\frac{\partial v_r}{\partial r} - \frac{v_r}{r^2} + \frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2} - \frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}\right) \tag{2}$$

θ Momentum:

$$\rho\left(\frac{\partial v_\theta}{\partial t} + v_r\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r v_\theta}{r}\right) = \rho g_\theta - \frac{1}{R}\frac{\partial P}{\partial \theta} + \mu\left(\frac{\partial^2 v_\theta}{\partial r^2} + \frac{1}{r}\frac{\partial v_\theta}{\partial r} - \frac{v_\theta}{r^2} + \frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{2}{r^2}\frac{\partial v_r}{\partial \theta}\right) \tag{3}$$

Energy:

$$\rho c_p \left(\frac{\partial T}{\partial t} + v_r \frac{\partial T}{\partial r} + \frac{v_\theta}{r}\frac{\partial T}{\partial \theta}\right) = k_{fluid}\left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}\right) + \mu\Phi \tag{4}$$

$$where, \Phi = 2\left[\left(\frac{\partial v_r}{\partial r}\right)^2 + \left(\frac{1}{r}\frac{\partial v_\theta}{\partial \theta} + \frac{v_r}{r}\right)^2\right] + \left(\frac{\partial v_\theta}{\partial r} - \frac{v_\theta}{r} + \frac{1}{r}\frac{\partial v_r}{\partial \theta}\right)^2 \tag{5}$$

Equations for a quarter of a shell, as shown in Fig. 1, for mass conservation:

$$\frac{1}{R}\frac{\partial v_\theta}{\partial \theta} + \frac{\partial v_z}{\partial h} = 0 \tag{6}$$

θ Momentum:

$$\rho\left(\frac{\partial v_\theta}{\partial t} + \frac{v_\theta}{R}\frac{\partial v_\theta}{\partial \theta} + v_z\frac{\partial v_\theta}{\partial z}\right) = \rho g_\theta - \frac{1}{R}\frac{\partial P}{\partial \theta} + \mu\left(-\frac{v_\theta}{R^2} + \frac{1}{R^2}\frac{\partial^2 v_\theta}{\partial \theta^2} + \frac{\partial^2 v_\theta}{\partial z^2}\right) \tag{7}$$

z Momentum:

$$\rho\left(\frac{\partial v_z}{\partial t} + \frac{v_\theta}{R}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}\right) = \mu\left(\frac{1}{R^2}\frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right) \tag{8}$$

Energy:

$$\rho c_p\left(\frac{\partial T}{\partial t} + \frac{v_\theta}{R}\frac{\partial T}{\partial \theta} + v_z\frac{\partial T}{\partial z}\right) = k_{fluid}\left(\frac{1}{R^2}\frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2}\right) + \mu\Phi \tag{9}$$

$$where, \Phi = 2\left[\left(\frac{1}{R}\frac{\partial v_\theta}{\partial \theta}\right)^2 + \left(\frac{\partial v_z}{\partial z}\right)^2\right] - \left(\frac{v_\theta}{R}\right)^2 + \left(\frac{1}{R}\frac{\partial v_z}{\partial \theta} + \frac{\partial v_z}{\partial z}\right)^2 \tag{10}$$

2.2 Assumptions for initial condition of velocity.

As a premise for the initial components of the velocity, v_{axial} will be the initial value of v_θ and the initial conditions of v_z and v_r will be zero. v_{axial} is given by Eq. (11):

$$v_{axial} = \frac{2Q}{\pi r_i^4}(r_i^2 - r^2) \tag{11}$$

Where Q is the flow rate and r_i is the inner radius of the pipe.

2.3 Pressure drop assumptions

It is known that for a horizontal straight pipe, where a fluid flows in laminar and stationary regime, the variation of the pressure in axial direction is constant and described by Eq. (12):

$$\frac{\partial P}{\partial z} = -\frac{8Q}{\pi r^4} \tag{12}$$

In the simulation herein proposed, horizontal stretch straight direction z will prolong inside the subsequent curve as direction θ, according to the red plan in Fig. 1. Thus, the corrected equation will be:

$$\frac{1}{R}\frac{\partial P}{\partial \theta} = -\frac{8Q}{\pi r^4} cos\Delta\theta \tag{13}$$

Similarly, Eq. (12) will be expressed in radial direction on the red plan as:

$$\frac{\partial P}{\partial r} = -\frac{8Q}{\pi r^4} sen\Delta\theta \tag{14}$$

In the blue plan, the projection of the Eq. (12) in axial direction will be the same as Eq. (13), but there will be no projection in transversal direction.

For the calculation of the value of ρg through the various directions, the analysis shall be similar to the value dP. As g is directed downward, the expression shall be $\rho g cos\theta$ for axial direction and $\rho g\ sen\theta$ for radial direction. Over the direction z there is no projection of the gravity and there will be no value of ρg.

2.4 Initial conditions of temperature and flow rate

Initial values of temperature at the entrance of the curve will be the same as those after completed the initial 1080 meters. It means, initial conditions of (A) 348 K and 150 m^3/h, (B) 353 K and 150 m^3/h and (C) 353K and 250 m^3/h on the exit of the refinery, which results to achieve the curve with the gradients evinced in Tab. 3.

Table 3 – Initial conditions of temperature at the entrance of the curve

Initial Temperature Condition	(A)	(B)	(C)
Inner wall Temperature (K)	341.5454	345.7387	348.6889
First layer Temperature nearby the wall (K)	346.1753	350.9472	352.2207
	347.6453	352.6009	352.9066
	347.9480	352.9414	352.9916
	347.9939	352,9930	352.9994
	348	353	353
	348	353	353
	348	353	353
	348	353	353
	348	353	353
Temperature at the center of the flowing (K)	348	353	353

As thermal boundary conditions, it was considered peripherally constant heat flux at the wall and constant temperature wall. According to Ghobadi and Muzychka (2016), these are conditions H1.

2.5 Physical properties of materials

Ciambeli, et al (2014) informed all physical properties of the fuel oil and materials employed on the infrastructure of transportation that this work demands to perform the simulation.

2.6 Dimensionless values

Taking into account all properties and characteristics of the fluid and process, as an additional information for researchers, Reynold Number *Re* resulted to be around 0.32, Dean Number *De* gave around 0.23, with very high Pradtl, Peclet and Graetz Numbers.

Nusselt Number *Nu* for the curve can be calculated according to Akiyama and Cheng's (1971) indication for the same approach of boundary condition herein employed, i.e., constant heat flux in the wall with peripherally constant wall temperature. So, Eq.(15) is applicable:

$$N_u = \frac{48}{11}0.181E(1 - 0.839E^{-1} + 35.4E^{-2} - 207E^{-3} + 417E^{-4}) \quad (15)$$

Where

$$E = De^{1/2}Pr^{1/4} \quad (16)$$

Thus, calculating through these expressions, it can be found *Nu* around 20.

Considering the low values of *Re* and *De* numbers, Fanning friction factor shall be predicted through Dean (1927) equation, written in terms of friction factor ratio:

$$\frac{f_c}{f_s} = \left(1 - 0.03058\left(\frac{De^2}{288}\right)^2 + 0.00725\left(\frac{De^2}{288}\right)^4\right) \quad (20)$$

where f_c is the Fanning friction factor for the curved tube, and f_s is the Fanning friction factor for the straight tube as given by Poiseulle's law:

$$f_s = \frac{16}{Re} \quad (21)$$

This resulted in *fc* around 50.

3. RESULTS AND DISCUSSION

From left to right, Fig. 2 shows the shape for axial velocity and vortexes in cross section at the exit of the curve of 90 degrees, given by the commercial code. Values decrease from dark red to dark blue colors. The line at the top corresponds to condition (A) pointed in Tab. 3. It can be seen condition (B) at the middle and so on.

Figure 3 shows the shape of axial velocities and temperature gradients, viewed through the red and blue surfaces evinced in Fig. 1, inside the herein proposed tool.

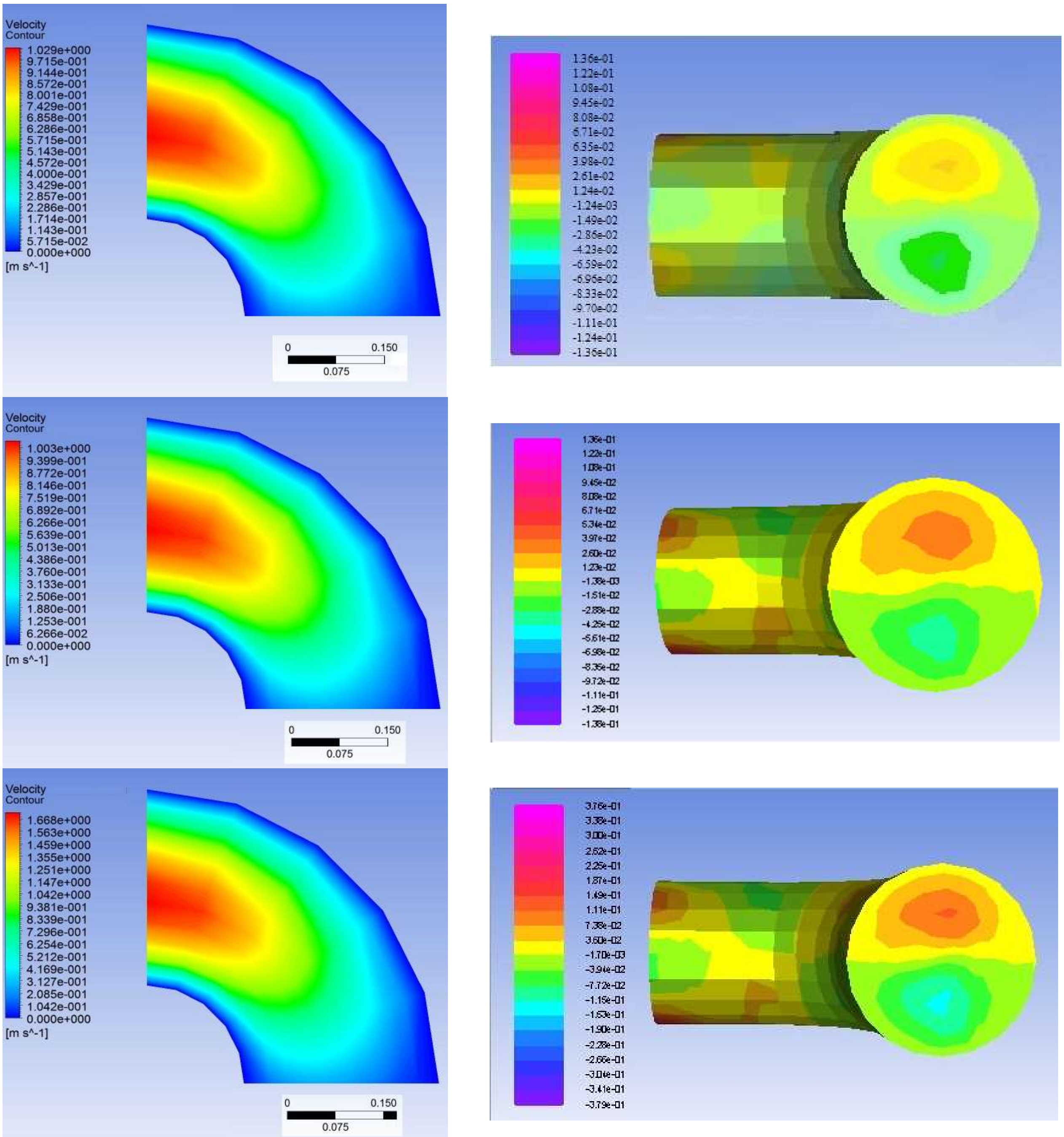

Figure 2 – From left to right, it is shown axial velocity and vortexes in the cross section predicted by the commercial code, for each condition of initial temperature gradient and flow rate. In accordance with Table 3, at the top, condition (A), at the middle, (B) and at the bottom, (C).

3.1 Comparing results of the herein proposed method and results of the commercial code

Comparing shapes of axial velocity given by the first column in Fig. 2 with those ones from the first and second columns in Fig. 3 indicates that predictions through two central surfaces are pretty viable. Small differences can be attributed to some floating of each numerical method and some discrepancies between initial velocities of each method. Responses from the commercial code, as classical shapes, endorse the proposed tool. However, it is not possible predicting the cross section profile only though these two surfaces. This way, to view vortexes and the secondary flow, it is necessary the aid of the commercial code as evinced on the second column of the Fig. 2. Another possibility is to describe the cross section's geometry in toroidal coordinates, to build a mathematical model, discretizing it in some numerical method to implement in computational language, in order to simulate the flow, which in fact represents a conventional method of simulation

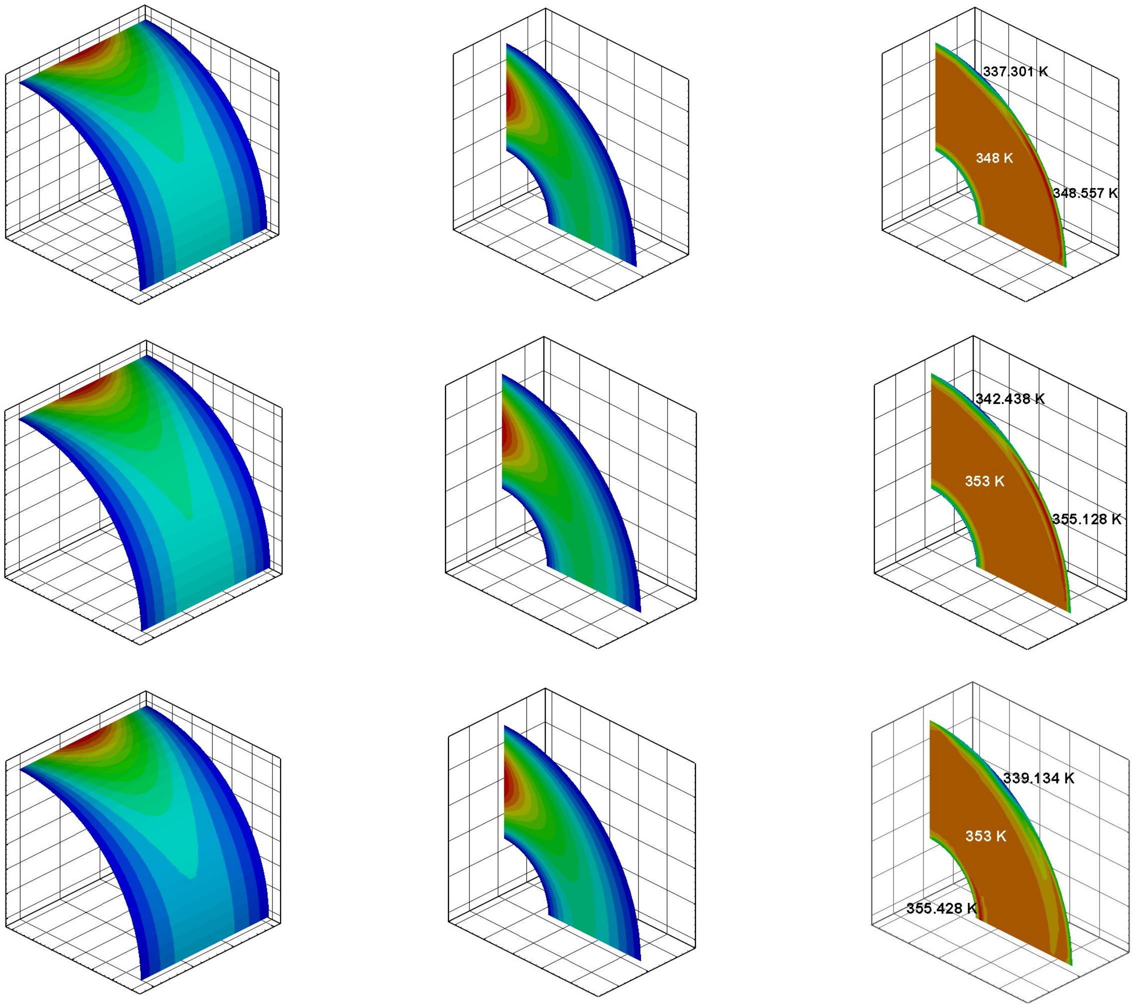

Figure 3 – From left to right, it is shown axial velocity shapes and temperature profiles, for each condition of initial temperature gradient and flow rate. At the topline, condition (A), at the middle, (B) and at the bottom, (C), in accordance with Tab. 3.

3.2 The shape of temperature gradients

Upraising Fig. 3 third column, temperature profiles show a big inner region where the initial central temperature at the entrance remains stable until the end of the curve. It agrees with results found in related works. According to Liu and Masliyah (1993) the temperature distribution in curved tubes split into two profiles for low Prandtl number fluids (similar to two vorticities), and had only one profile for high Prandtl number, as the present case.

Localization of the temperature's higher value evinced in the peripheral region of the curve, as viewed in Fig. 3 also find endorsement in literature. In curved tubes, as with the velocity fields, the secondary flow distorts temperature profiles, pushing the temperature peak toward the tube outer wall. Consequently, a higher heat transfer rate at the coil outer wall than at the inner wall is expected. Cioncolini and Santini (2006) showed that the secondary flow initiated inside the coiled tubes increases both hydraulic resistance and the heat transfer effectiveness in comparison to straight tubes. Fig. 3 at the third column is an evidence of that. It is possible to see that both for the first and second combinations of initial temperature and flow rates. However, on the third condition, with a higher flow rate, result has shown the opposite behavior, which might be an indication that flow rate influenced the heat transfer and the location of the temperature peak for this fuel oil.

This behavior might be classical for any conventional insulated cylindrical geometry and mainly for insulated electrical wires if Eq. (22) is satisfied, as literature acknowledge (Bejan, 1993). In this case, the rate of heat transfer increases even with the augmentation of the insulation thickness.

$$\frac{k}{hr_i} > 1 \qquad (22)$$

Although this equation is not true for the present case, it is possible that the insulation of the tube conjugated with the influence of the higher flow rate could be changing the behavior of the temperature gradient, shifting the peak of temperature to the opposite side of the inner wall, according to that shown in the last condition. A more accurate study shall be performed in order to confirm this postulate.

4. CONCLUSIONS

A simulation of a flow in a curved tube through two cylindrical surfaces, as representative of a whole flowing process with very high Prandtl Number fluid and very low Dean Number curve was presented. It was performed via finite difference method. Additionally, the same process was carried out in a commercial code. The real case studied was the transference of fuel oil between a refinery and a distribution terminal in Brazilian Northeast region.

The behavior of velocity fields and temperature gradients resulted to be that expected for laminar regime. Predictions of velocity shapes performed through the cylindrical surfaces according to the herein proposed simulation found good agreement with those got through the commercial code. The cross section with the classical vortexes is not possible to be seen through this proposed tool.

Larger amplitude of temperature gradients achieved was a remark and were endorsed by the literature, which is explained because the secondary flow initiated inside the curved tubes increases heat transfer effectiveness in comparison to straight tubes. A higher flow rate seems to have influenced peak temperature location for the fuel oil employed on this study.

5. ACKNOWLEDGEMENTS

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