

OPTIMIZATION OF THERMAL EFFICIENCY OF FLAT PLATE SOLAR COLLECTORS USING MATLAB

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Abstract. Solar energy is becoming an alternative in front of the scarcity of fossil fuel reserves. Furthermore, the solar energy is a cleaner way to harness energy, when compared with fossil fuel, since it does not emit CO₂ to the environment. Thus, when using this energy, we contribute to a sustainable development of the planet. One of the simplest and most direct applications of this energy is the conversion of solar radiation into heat, which can be used in systems that aim to heat water or any kind of fluid; so as in air-drying systems. A solar heating system widely used is the flat plate solar collector. It consists of an absorber plate, to absorb the solar radiation; tubes fixed on the plate, through where the water flows; an insulation on the back and on the edges; and at least one glass cover, to reduce the top losses. Many studies have been executed in order to analyze the operation of a flat plate solar collector and increase their thermal efficiency. The aim of this research project was to use a MATLAB's function, *fmincon*, to perform a multivariable optimization of a flat plate solar collector thermal efficiency function. So, it was possible to simultaneously find the optimal values of six dimensions (thickness of insulation at the base, the lateral insulation thickness, distance between tubes, the tube inner diameter, the tube outer diameter and thickness of the absorber plate), which maximized the efficiency of the collector. After that, the individual influences, of some of these parameters on the thermal efficiency, were evaluated. The study results can be used to design more efficient flat plate solar collectors, which means to capture the required energy using a smaller area and having a lower production cost.

Keywords: flat plate solar collector, solar energy, MATLAB, optimization, thermal efficiency.

1. NOMENCLATURE

T	temperature [K]	q	heat transfer rate [W]
A	area [m ²]	l	collector length [m]
U	global heat transfer coefficient [W/(m ² .K)]	m	empirical constant [dimensionless]
c_p	specific heat [J/(kg.K)]	\dot{m}	mass flow rate [kg/s]
C	constants [dimensionless]	N	tube number
D_i	internal diameter [m]	R	thermal resistance [m ² .K/W]
D_o	external diameter [m]	t	time [s]
V	velocity [m/s]	t	fin thickness [m]
de/dt	energy variation rate [W]	N	number of covers
F_R	heat removal factor [dimensionless]	<i>Greek symbols</i>	
F'	efficiency factor [dimensionless]	τ	transmissivity [dimensionless]
G	mass flow rate per area [kg/(s.m ²)]	α	absorptivity [dimensionless]
h	heat transfer coefficient [W/(m ² .K)]	σ	Stefan Boltzmann constant [W/(m ² .K ⁴)]
h_r	radiation heat coefficient [W/(m ² .K)]	β	collector angle inclination [°]
I	solar irradiation [W/m ²]	ε	emissivity [dimensionless]
k	thermal conductivity [W/(m.K)]	η	thermal efficiency [dimensionless]
L	duct length [m]	ϕ	empirical constant [dimensionless]

2. INTRODUCTION

Solar energy is gaining great attention these days in front of the scarcity of fossil fuel reserve and the necessity of renewable energy sources. Besides, it is a cleaner way to harness energy, which contributes to the sustainable development of the planet. One of the simplest and direct way to use solar energy is to convert solar radiation in thermal energy that could be used to heat water, air or any kind of fluid. Flat plate solar collector are more common for residential water heat systems and space heating. Figure 1 shows a typical liquid-type flat plate solar collector:

Solar radiation and environmental conditions are different around the world, so each collector must have the proper design for its specific localization. It is commonly necessary to create models for the designs, but models require assumptions of initial values, which generate errors and inaccuracies. Wojcicki (2015) affirmed that Typical Day Concept, when applied, using balance equations calculates a unique correct solution for the out data every 24h. It helps

to save time, money and difficulties. Akhtar and Mullick (2007) created a semi-analytical model to compute the glazing temperatures and the top loss coefficient. It is a function of the collector's parameters and the atmosphere variables. This method eliminates the necessity of numerical solutions for a following energy balance. After six years, Subiantoro and Tiow (2013) found an optimum space between the absorber plate and the glazing increasing the fluid maximum temperature in 14 % because of the reduction of the top losses. In a recent work (Sandhu *et al.*, 2014) an experimental investigation proved that inserters devices increase the Nusselt number and, consequently, the thermal performance. Aware that the distribution of the fluid flow through the tubes has a strong influence on the thermal performance, Facão (2015) concluded that the out header must have a higher diameter than the inner header to achieve a good fluid flow. According to him, commercial collectors have equal diameters. In the same year, Cerón *et al.* (2015) made possible to evaluate the impact of distinct operational conditions and design, in the flat plate solar collector thermal performance, with a 3D numerical model, which reduces prototypes expenses. Jiandong *et al.* (2015) conducted a numerical simulation with the structural parameters of the collector and concluded that the efficiency increases when expanding the absorber plate width, contracting the tubes distance, decreasing the tubes lengths or enlarging its diameter.

The aim of this research project is to use MATLAB to perform a multivariable optimization of thermal efficiency function of a flat plate solar collector.

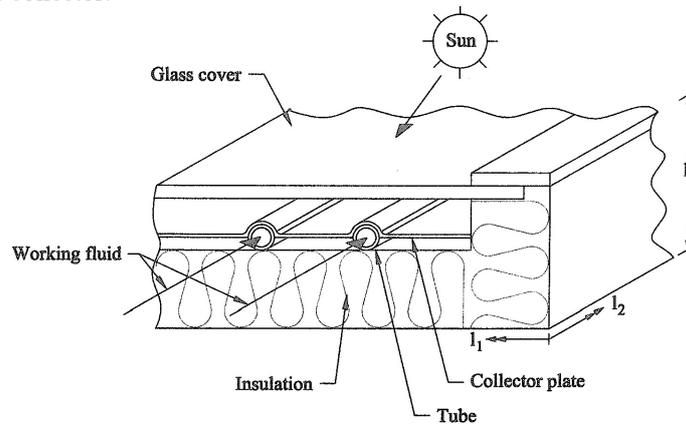


Figure 1. Flat plate solar collector scheme. (Goswami *et al.*, 1999).

3. MATHEMATICAL MODELING

An energy balance can estimate the thermal performance of a solar collector. Equation (1) represents the energy balance among the incident radiation heat, the useful energy, the heat losses and the generated energy:

$$I A_c \tau \alpha = q_u + q_{loss} + \frac{de_c}{dt} \quad (1)$$

The collector instantaneous efficiency η_c is simply the ratio between the useful energy that the fluid gains and the incident total solar energy rate. This efficiency is shown in Eq. (2):

$$\eta_c = \frac{q_u}{A_c I} \quad (2)$$

3.1 Global heat loss coefficient of a collector (U_c)

The global heat loss coefficient of a collector serves to calculate the heat loss from the collector to the ambient when the mean absorber plate temperature (T_c) and the ambient temperature (T_a) are known. Equation (3) shows how this calculation is:

$$q_{loss} = U_c A_c (T_c - T_a) \quad (3)$$

Even though the Eq. (3) is apparently simple, U_c cannot be set directly without a detailed analysis of all the heat losses. To formulate the mathematical model, some simplifying assumptions have been taken into account such as permanent regime, uniform temperature between the top and the bottom of the absorber plate, among others. Figure 2 shows the flat plate solar collector thermal circuit.

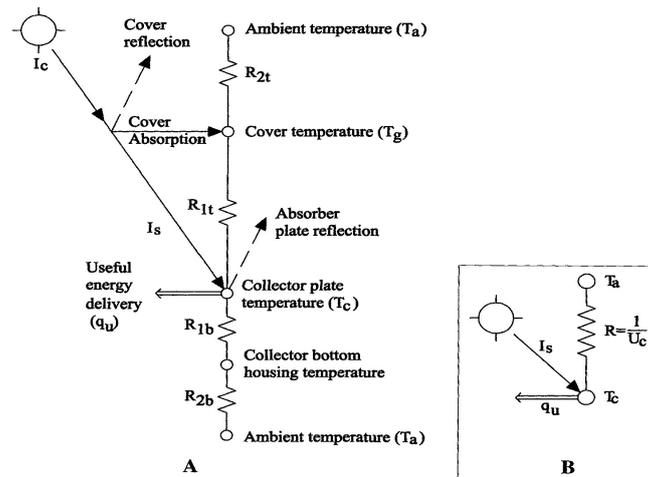


Figure 2. The flat plate solar collector thermal circuit: (a) detailed circuit; (b) equivalent circuit (Goswami *et al.*, 2000).

So, the useful energy can be described as the difference between the absorbed incident solar radiation energy and the heat losses – that can be calculated by the circuit present in the Fig. 2. Equation (4) is the useful heat and Eq. (5) is the heat losses:

$$q_u = IA_c \tau \alpha - q_{loss} \quad (4)$$

$$q_{loss} = U_c A_c (T_c - T_a) = \frac{A_c (T_c - T_a)}{R} \quad (5)$$

Three parallel heat losses occur from the plate to ambient, as Eq. (6) shows. Top, bottom and edge heat losses:

$$U_c = U_{top} + U_{bottom} + U_{edge} \quad (6)$$

where the bottom thermal resistance are shown, respectively, in Eq. (7):

$$R_{1b} = \frac{l_i}{k_i} \quad \text{and} \quad R_{2b} = \frac{1}{h_{c,bottom}} \quad (7)$$

R_{2b} is usually neglected for being much smaller than R_{1b} , so U_b can be expressed as in Eq. (8):

$$U_b = \left(\frac{k}{L} \right)_{insulation} \quad (8)$$

Equation (9) shows the edge coefficient:

$$U_e = \frac{(UA)_{edge}}{A_c} = \frac{K_{insulation}}{L_e} \frac{A_e}{A_c} \quad (9)$$

The top loss is a little bit complicated. There are convection and radiation losses from the plate to the glass cover and from the glass cover to the ambient. Equation (10) and Eq. (11) show, respectively, the resistances between the absorber plate and the glass cover and the resistance between the glass cover and the ambient.

$$\frac{1}{R_{1t}} = \frac{1}{R_{r,1}} + \frac{1}{R_{c,1}} = h_{r,1} + h_{c,1} \quad (10)$$

$$\frac{1}{R_{2t}} = \frac{1}{R_{r,\infty}} + \frac{1}{R_{c,\infty}} = h_{r,\infty} + h_{c,\infty} \quad (11)$$

where the radiation losses coefficients can be calculated and are expressed in Eq. (12) and Eq. (13):

$$h_{r,1} = \frac{\sigma(T_c + T_g)(T_c^2 + T_g^2)}{(1/\varepsilon_{p,i} + 1/\varepsilon_{g,i} - 1)} \quad (12)$$

$$h_{r,\infty} = \frac{\varepsilon_{g,i} \sigma(T_g^4 - T_{ceu}^4)}{(T_g - T_a)} \quad (13)$$

Hottel and Woertz (1942) and Klein (1975) suggested a simplified procedure for calculating the top heat loss coefficient when all the covers have the same materials. Their equation is expressed in Eq. (14). This method agreed closely with the values obtained before. (Duffie and Backman, 2013):

$$U_t = \frac{1}{N/(C/T_c)[(T_c - T_a)/(N + f)]^e + 1/h_{c,\infty}} + \frac{\sigma(T_c^2 + T_a^2)(T_c + T_a)}{1/[\varepsilon_p + 0.00591N h_{c,\infty}] + (2N + f - 1 + 0.133\varepsilon_p)/\varepsilon_{g,i} - N} \quad (14)$$

where:

$$f = (1 + 0,089h_{c,\infty} - 0,1166h_{c,\infty}\varepsilon_p)(1 + 0,07866N); \quad N = \text{Number of covers};$$

$$C = 520(1 - 0,000051\beta^2) \quad \text{for } 0^\circ < \beta < 70^\circ; \quad \text{if } \beta > 70^\circ, \quad \text{use } \beta = 70^\circ;$$

$$e = 0.430(1 - 100/T_c); \quad \varepsilon_{g,i} = \text{emittance of glass (0.88)}; \quad \varepsilon_p = \text{emittance plate};$$

$$h_{c,\infty} = 5,7 + 3,8V \quad \text{and } V = \text{the wind speed in m/sec.}$$

3.2 Useful energy

To determinate the thermal efficiency, firstly we need to define the useful heat transfer to the working fluid. As we know, the incident radiation on the absorber plate arrives to the fluid in two different ways. Some of that energy is transferred directly to the top surface of the flow channels, which is represented in Eq. (15):

$$q_{duct}(x_0) = D\{\alpha I - U_c[T_b(x_0) - T_a]\} \quad (15)$$

While the other part focuses on the region of the plates connecting two adjacent channels of flow and can be observed in Eq. (16). This energy is conducted in the transverse direction to the flow in the channels and the flat plate, in this case, can be considered as a fin:

$$q_{total}(x_0) = w\eta_f\{\alpha I - U_c[T_b(x_0) - T_a]\} \quad (16)$$

where $\eta_f = \tanh(mw/2)/(mw/2)$ is the fin efficiency, $m^2 = U_c/kt$ and w is the distance between tubes. Thus, the useful energy rate per unit length in the direction of flow is expressed bellow, in Eq. (17):

$$q_u(x_0) = (D + 2w\eta)\{\alpha_s I_s - U_c[T_b(x_0) - T_a]\} \quad (17)$$

There is a problem with the equation above. It is easier to know the working fluid temperature than collector's temperature. In order to obtain the useful energy in function of the known parameters, we can use the concept of collector's efficiency factor F' . The useful heat and the collector's efficiency factor is shown, respectively, in Eq. (18) and Eq. (19):

$$q_u(x_0) = l'F'\{\alpha_s I_s - U_c[T_f(x_0) - T_a]\} \quad (18)$$

where:

$$F' = \frac{1/U_c}{w[1/(U_c(D_0 + (w - D_0)\eta_f)) + 1/C_b + 1/(\bar{h}_{c,i} \pi D_i)]} \quad (19)$$

The collector's heat removal factor, F_R , is the relation between the real heat transfer to the working fluid, with the heat transfer rate at the minimum temperature difference between the absorber and the environment and is represented in Eq. (20). Using this concept, we can let the useful energy in function of the inlet fluid temperature (T_{fe}):

$$F_R = \frac{Gc_p(T_{f,out} - T_{fe})}{\alpha I - U_c(T_{fe} - T_a)} \quad (20)$$

where $G = \dot{m}/A_c$ and the useful energy calculation with F_R is in the Eq. (21):

$$q_u = A_c F_R [\tau \alpha I - U_c(T_{fe} - T_a)] \quad (21)$$

The Hottel-Whillier-Bliss equation, Eq. (22), is a combination of Eq. (2), Eq. (18), Eq. (20), Eq. (21) and an energy balance for a dx duct section flow. This is the equation to be optimized, where F' has been expressed in Eq. (19):

$$\eta_c = \frac{\dot{m}c_p}{U_c A_c} \left[1 - \exp\left(-\frac{U_c F'}{Gc_p}\right) \right] \left[\tau \alpha - \frac{U_c(T_{fe} - T_a)}{I} \right] \quad (22)$$

5. OPTIMIZATION

This research project had the objective to optimize, simultaneously, six geometric parameters aiming the collector thermal efficiency maximization. In this purpose, the efficiency equation was algebraically modeled being in function of these parameters. Next, throughout a computational routine developed in MATLAB, the optimal value of the collector dimensions have been found within a pre-established range. The other parameters were adopted from commercial or local values. Table 1 and Tab. 2 shows, respectively, the variable parameters and the adopted parameters.

Table 1. Variable parameters.

$L_B \in [2, 10]$ mm	Base insulation thickness	$D_i \in [32.5, 57]$ mm	Tube internal diameter
$L_e \in [2, 10]$ mm	Edge insulation thickness	$D_o \in [35, 60]$ mm	Tube external diameter
$W \in [87, 98]$ mm	Distance between tubes	$t \in [0.2, 2]$ mm	Absorber plate thickness

Table 2. Adopted parameters.

$\dot{m} = 70$ kg/s	$N = 1$	$1/C_b = 0$	$\tau_s \alpha_s = 0.96$	$\alpha_s = 0.95$
$T_i = 323$ K	$T_a = 300$ K	$T_p = 373$ K	$I_c = 700$ W/m ² *	$c_p = 4200$ J/(kg.K)
$\beta = 45^\circ$	$V = 2.5$ m/s	$\varepsilon_p = 0.95$	$k_{ins} = 0.04$ W/(m.K)	$\sigma = 5.67 \times 10^{-8}$ W/(m ² .K ⁴)
$A_e = 0.3$ m ²	$A_c = 2.53$ m ²	$\varepsilon_g = 0.94$	$k_{copper} = 1250$ W/(m.K)	$h_f = 1000$ W/(m ² .K)

* I_c was captured in a cloudy day in Bauru – SP, Brazil

The specifically chosen MATLAB's function is the *fmincon*. It attempts to find a constrained minimum of a scalar function of several variables starting at an initial estimate. In other words, it returns to the user the values of each desirable parameter, which gives the minimum value of the evaluating function. The syntax used in this project can be observed bellow, in Eq. (23):

$$[x, fval] = \text{fmincon}(fun, x0, A, b, Aeq, beq, lb, ub) \quad (23)$$

This structure minimizes the function *fun* by finding and returning the values of the variables, *x*. Those results in a minimum function value, *fval*. The user has to provide an initial guess, *x0*. He can also furnish the lower and upper boundary, *lb* and *ub*, in which the parameters can vary. If it is wanted to restrict some of the variables, the matrix, *A*, and the vector, *b*, should be used to impose linear inequalities while *Aeq* and *beq* should be used to establish linear equalities.

In this case, *fun* would be the thermal efficiency, Eq. (22). However, as it was wanted to obtain the maximum value of the efficiency, the function was the inverse of the thermal efficiency in the program. The range of the parameters values is limited by the lower and upper boundary vectors (*lb* and *ub*) and the middle values were chosen as an initial guess, *x0*. Some constrains had to be made to avoid impossible values of the parameters. The boundary of the parameters

variation can be observed in the Tab. 3 and the constrains established were: The distance between adjacent tubes is at least two and a half of the external diameter value ($W \geq 2.5D_o$); The tube thickness is at least one millimeter ($D_o \geq D_i + 2 \text{ mm}$).

Table 3. Parameters variation boundaries.

	L_e	L_b	D_o	D_i	W	t
<i>lb</i>	0.002	0.002	0	0	0.007	0.0005
<i>ub</i>	0.020	0.020	0.070	0.065	0.105	0.0050

In order to find the optimum values of the geometrical parameters, which would furnish the maximum thermal efficiency, it was needed that the efficiency was function of the desirable parameters. With this purpose, the Eq. (22) was manipulated with the Eq. (19) and Eq. (6). Then, it was made an optimization routine in MATLAB. Therefore, the wanted values could be found. After the optimization, an evaluation of the individual influence that some parameters have in the thermal efficiency has been made. The individually analyzed parameters were ambient temperature (T_a), solar irradiation (I), inlet fluid temperature (T_{fe}) and absorber plate area (A_c).

6. RESULTS AND DISCUSSIONS

6.1 Optimization

After the geometrical parameters optimization, it has been found that the maximum flat plate solar collector thermal efficiency, in those conditions, is $\eta = 0,633$ where the parameters values are: $L_e = 0.02 \text{ m}$, $L_b = 0.02 \text{ m}$, $D_o = 0.42 \text{ m}$, $D_i = 0.42 \text{ m}$, $W = 0.10 \text{ m}$ and $t = 0.0005 \text{ m}$.

It have been observed, as expected, that the insulations thickness were the highest values, inside the limits pre-established, once the bigger is the thickness, the smaller is the heat losses through the bottom and the edges.

To have the thermal efficiency maximization, the internal diameter (D_i) trended to be as big as it is possible, since the convection coefficient was held constant, because higher is the heat transfer area between the duct and the fluid. In addition to that, the efficiency also increases with the reduction of the distance between tubes since it decrease the thermal resistance and the heat losses once the incident heat between the tubes has to travel a smaller distance.

Curiously, the distance between tubes was as big as it is possible while the diameters established in intermediary values. When it was analyzed, a conclusion have been made: the inner diameter is the most significant among these three parameters. Because of the constrains, D_i had the higher value possible, making D_o assume 0.042 mm (2 mm bigger than D_i , the minimum that it would assume after the internal diameter value assumed) while W just had the higher value possible because of the constrain $W \geq 2.5D_o$.

If the distance between tubes were the most relevant parameter, it would achieve its smaller value, 0.07 mm and D_o and D_i would be, respectively, 0.035 mm and 0.033 mm . In addition to that, if the external diameter were the most relevant parameter, it would have the smallest value leading W and D_i down.

In the other hand, when the internal diameter is increased, the fluid velocity, the Reynolds number, the heat transfer coefficient and, consequently, absorbed heat by the fluid are decreased. With a higher complexity of the mathematical modelling, which consider the variation of the internal heat transfer coefficient, an optimum value of D_i would appear.

Considering that the adopted ambient conditions are the mean conditions of a determined region, it is possible to affirm that those are the best parameters for a local project.

6.2 Individual influences

Aiming to analyze the thermal efficiency variation, fluid inlet temperature, ambient temperature, solar irradiation and superficial area were varied, one at a time, maintaining constant the rest of the parameters.

As it can be observed in the Fig. 3.a, the thermal efficiency decreases linearly when T_{fe} rises up. When the inlet temperature has the same value of the ambient temperature, the efficiency approaches to 100 %. As T_{fe} gets smaller than T_a , the thermal efficiency gets bigger than 100 %. This means that the useful heat is bigger than the heat received from the solar irradiation because the fluid is also receiving energy from the ambient. On the other hand, when T_{fe} reaches approximately 363 K, the thermal efficiency is null, which means that all the heat received from the sun is been lost. After that value, the fluid loses more energy than the provided solar energy making the efficiency to become negative.

When the ambient temperature decreases, the heat losses increases and, consequently, the useful heat is smaller dropping the thermal efficiency. Figure 3.b shows the graph T_a versus η :

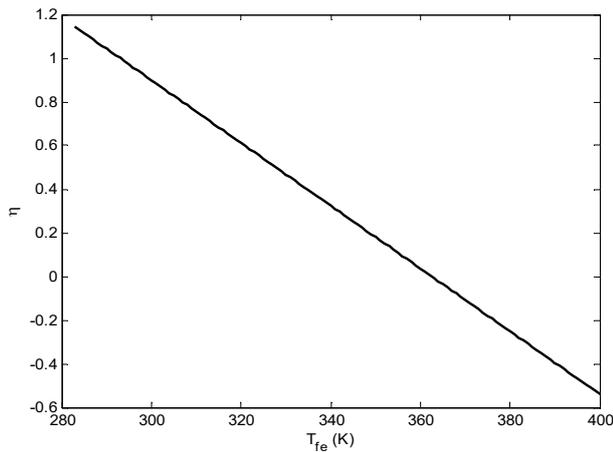


Figure 3.a – Thermal efficiency in function of the inlet fluid temperature ($\eta \times T_{fe}$).

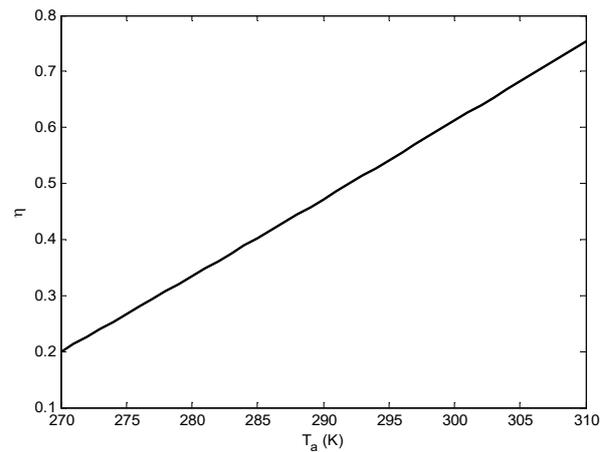


Figure 3.b – Thermal efficiency in function of the ambient temperature ($\eta \times T_a$).

In the Fig. 4.a, it can be perceived, as expected, that the thermal efficiency is as big as the solar irradiation is. However, the curve on the graph has a logarithmical behavior where the efficiency trends to a maximum when the irradiation is grown. It also can be observed that when I is smaller than 180 W/m^2 , η is negative because the heat losses are bigger than the useful energy. Knowing that the thermal irradiation varies during the day, it can be affirmed that the timetables that I is smaller than 180 W/m^2 , it is not viable to operate the solar collector.

Figure 4.b shows that the curve $A_c \times \eta$ also has a logarithmical behavior, where the efficiency increases with the increase of the collector superficial area but trends to a maximum while the area is grown. Curiously, the variation of the area had no significant impact on the thermal efficiency, when compared with the others analyzed parameters. That is because with a bigger A_c both the incident thermal irradiation and the heat losses are bigger too.

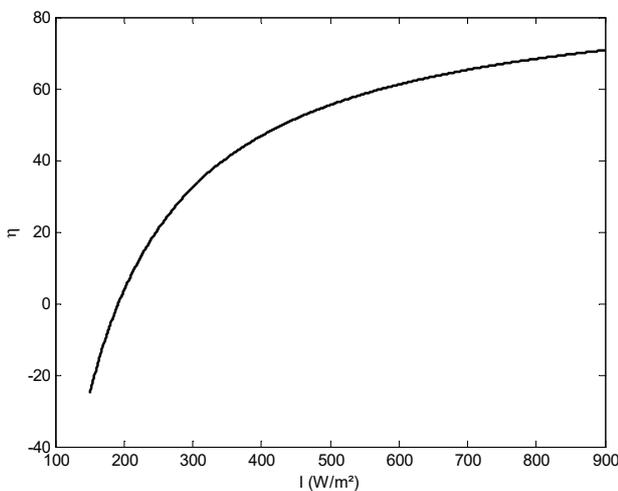


Figure 4.a – Thermal efficiency in function of the solar irradiation ($\eta \times I$).

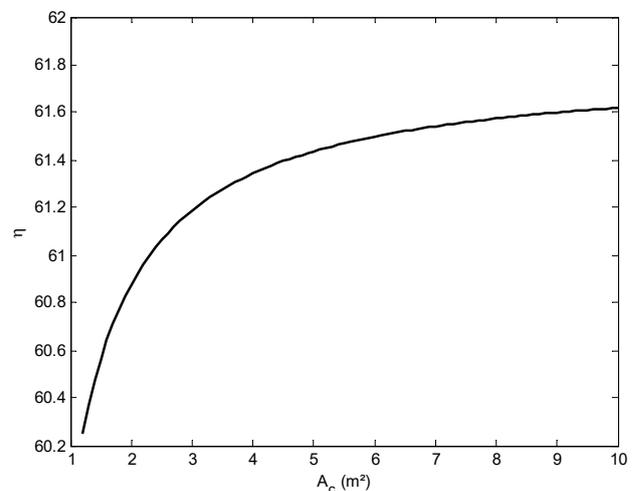


Figure 4.b – Thermal efficiency in function of the collector top area ($\eta \times A_c$).

7. CONCLUSIONS

In this project, it was made a mathematical modeling by an energy balance in the components of the collector. Then, an MATLAB computational routine was developed in order to obtain the optimum values of the design geometrical parameters of the flat plate solar collector and the respectively maximum thermal efficiency. When some parameters were varied individually, it was possible to evaluate the efficiency behavior.

The results could generate some rich discussions and were shown in two ways: numerically and graphically. Therefore, it was concluded that the thermal efficiency is very sensitive to operational variables of the solar collector. This way, to achieve the maximum efficiency, in function of many parameters, is a hard work, once its behavior differs too much by the analyzed operational parameter.

Finally, a good technique is to organize the thermal efficiency in a reduced number of variables trying to classify behavior in separated analyzes. The analyses made here, if it had been made in larger comprehensiveness, could be a useful tool to optimize a flat plate solar collector leading to a higher efficiency and lower cost.

8. ACKNOWLEDGEMENTS

This research was supported by FAPESP (São Paulo Research Foundation): grant 2015/08440-5.

I thank Santiago del Rio Oliveira, my professor advisor of this project, by all his teachings, guiding and insights that I had the pleasure to receive.

We also thank UNESP, our University, by making this research possible.

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