NUMERICAL ASSESSMENT OF LAMINAR FLOW AND CONJUGATE HEAT TRANSFER IN PARALLEL-PLATE CHANNELS WITH INCLINED FINS AT LOW REYNOLDS NUMBER

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Abstract. *The conjugate heat transfer of the laminar flow of a Newtonian fluid in a two-dimensional horizontal finned channel is numerically evaluated. The channel comprises heat conducting thick walls with prescribed temperature in the external surfaces. Transversal, equally spaced fins are placed in the internal surface of the channel along the flow direction. Fluid flow enters the channel at constant velocity and temperature. Heat transfer takes place in the channel, at the fluid-solid interface and in the thick channel walls by heat conduction. Furthermore, the angle between fins and the channel walls, β, was assumed to vary from 0° (smooth channel without fins) to 90° (classical case) in the clockwise direction. The maximum blockage ratio of the geometry is 0.4 whereas aspect ratio is 2 for all cases. The numerical simulations were obtained using the commercial code CFX®, that solves the transport equations using unstructured meshes. The influence of different fin configurations on heat transfer is compared and discussed.*

Keywords*: CFD, laminar flow, conjugate heat transfer, parallel-plate, fins*

1. INTRODUCTION

Heat transfer is very important to many industries, thus instigating a great number of researchers to continuously improve the design and operation efficiency of heat transfer systems. Some classes of compact heat exchangers, such as electronic equipment, may present fluid flow under laminar regime. In addition to the fluid characteristics and material used to manufacture the equipment, the heat exchange geometry can also directly affect the heating or cooling performance of the heat transfer process. Therefore, this work aims to investigate the efficiency of different finned surface configurations with respect to heat exchange. In this case, heat conduction in the channel walls are also accounted for so that the heat transfer process is said to be conjugated. Interestingly, the literature on similar studies is focused on the fluid and investigates heat exchange enhancement disregarding of the heat conduction in the solid wall.

The introduction of fins or baffles in the fluid flow path periodically interrupts the growth of the hydrodynamic and the thermal boundary layers and creates zones of recirculation close to the fins positions. The overlapping flow is likely to impinge the channel wall at the reattachment point downstream the fins positions. It is expected that this fin disposition would increase the heat transfer characteristics by washing of the channel walls (Mousavi and Hooman, 2006). However, since it causes flow obstruction, higher streamwise pressure drop is also predictable.

Yang et al. (2010) conducted a numerical study of forced convective cooling of a single fin in a channel. The authors concluded that the optimum aspect ratio for the finned channel corresponding to maximum heat transfer increases with the Reynolds number and decreases with the ratio between the solid and fluid thermal conductivity. Webb and Ramadhyani (1985), computed a laminar conjugate heat transfer problem in a channel with staggered plain ribs. It was observed that the increase in heat transfer for gases is not substantial enough to offset the added friction factor penalty. Strapattanapipat and Promvonge (2009) investigated laminar heat transfer in a channel with diamond-shaped baffles and reported an increase in thermal performance compared to flat baffles. Therefore, a deeper understanding of reattaching flows is very important, mainly due to the large variations of the local heat transfer coefficient this flow pattern can cause.

Complementary, researchers have been studying different fin positioning and construction materials. Santos and Lemos (2006), presented numerical simulations for laminar flow in a channel containing solid and porous baffles and reported no advantages on the use of porous baffles. Furthermore, many parameters that interrelates the channel and fin geometries are worthy of attention because of the heat transfer augmentation they can promote. Different types of aspect ratios are possible to be studied, e.g., fin height per channel height, fin pitch per channel height, etc. Cheng and Huang (1991) performed a numerical study of laminar forced convection in parallel-plate channels with transversal fins. The authors indicated that the relative position of the fin arrays causes larger effects for increasing fin height. They also reported a general ineffective behavior of the in-line arrangement due to flow recirculation between consecutive fins.

A great number of works have been reported considering the fluid flow as *fully periodically developed*, making possible to simulate only a geometric module with periodic dimensional characteristics. Patankar et al. (1980) first presented this concept, as mentioned by Mousavi and Hooman (2006), who investigated numerically the entrance region of a two dimensional channel with isothermal walls and staggered baffles for laminar fluid flow. Furthermore, the authors concluded for the Prandtl number ranging from 0.35 to 10 that it also affects the location of the periodically fully developed region. In the same work, it was observed that larger blockage ratios and Reynolds number cause the Nusselt number to increase. Similarly, Bazdidi-Tehrani and Naderi-Abadi (2004) carried out a numerical investigation of the heat transfer problem under laminar fluid flow for the entrance region of a two-dimensional channel with in-line ribs. It was found that a moderate decrease in the local Nusselt number with increasing blockage ratio. On the other hand, larger rib pitch caused a marginal increase in the local Nusselt number and a decrease in the friction factor. When addressing fluid flow in a backward-facing step, Vogel and Eaton (1985) reported that the fluctuating skin friction controls the heat transfer rate near the reattachment region.

In the present work, the analysis of the laminar fluid flow and conjugate heat transfer for parallel-plate channels with inclined fins was conducted for a single Reynolds and Prandtl numbers. The same blockage ratio was assumed for each different angle, so that fins with the in-line arrangement have different length when compared to the staggered configuration. The fin pitch is assumed identical for all cases. In addition to the flow topology, the local Nusselt number and the total heat transfer energy rate were evaluated against the presence of the fins. The effect of the fins on the temperature distribution in the solid domain (wall channels) was also investigated.

2. GEOMETRY AND THEORETICAL FORMULATION

Figure 1 shows the two geometric configurations studied in this work. Both fins and walls have the same thermal conductivity so that thickness of the latter is parametrized with relation to channel height. Prescribed temperature is assumed for the wall outer surfaces. For both arrangements, it was studied the following cases: $\beta = 0°$ (smooth channel without fins), $\beta = 23°$, $\beta = 45°$, $\beta = 68°$ and $\beta = 90°$ (classical configuration), where β is the angle between the fins and channel walls, as depicted in Fig. 1. The channel length downstream the last fin is 20 times the fin height in order to avoid outlet fluid flow effects on the numerical solution (Zdanski et al., 2015). The channel geometry was designed based on the following ratios: e/a = 1/2, l/a = 4/10, t/l = 1/10 and s/a = 2, where a is the channel height.

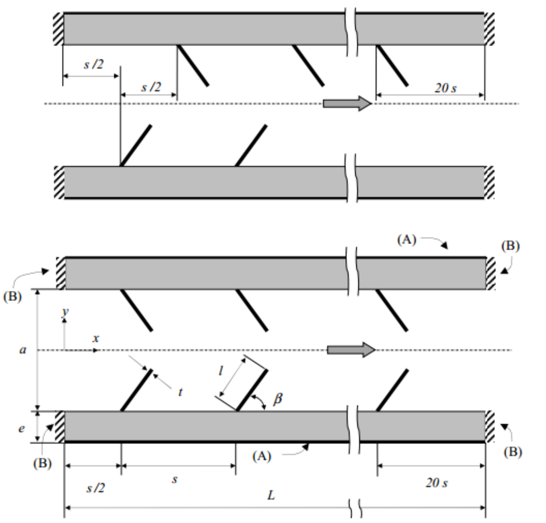

Figure 1. Problem geometries. (a) staggered fins and (b) in-line fins.

The mathematical modeling was carried out including the following considerations: two-dimensional steady fluid flow with conjugate heat transfer; laminar incompressible flow; constant physical properties; negligible radiation heat transfer; no viscous dissipation and body forces effect; fins are perfectly fixed to their basis (no thermal contact resistance). Consequently, the two-dimensional formulation of the mass, energy and momentum governing equations are as follows:

Fluid domain:

$$\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0 , \tag{1}$$

$$\left(u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} \right) = -\frac{\partial p^*}{\partial x^*} + \frac{1}{\mathrm{Re}_a} \left(\frac{\partial^2 u^*}{\partial x^{*2}} + \frac{\partial^2 u^*}{\partial y^{*2}} \right), \tag{2}$$

$$\left(u^* \frac{\partial v^*}{\partial x^*} + v^* \frac{\partial v^*}{\partial y^*} \right) = -\frac{\partial p^*}{\partial y^*} + \frac{1}{\mathrm{Re}_a} \left(\frac{\partial^2 v^*}{\partial x^{*2}} + \frac{\partial^2 v^*}{\partial y^{*2}} \right), \tag{3}$$

$$\left(u^* \frac{\partial T^*}{\partial x^*} + v^* \frac{\partial T^*}{\partial y^*} \right) = \frac{1}{\mathrm{Re}_a \, \mathrm{Pr}} \left(\frac{\partial^2 T^*}{\partial x^{*2}} + \frac{\partial^2 T^*}{\partial y^{*2}} \right) , \tag{4}$$

Solid domain:

$$\frac{k_w}{k_f} \left(\frac{\partial^2 T^*}{\partial x^{*2}} + \frac{\partial^2 T^*}{\partial y^{*2}} \right) = 0 \ , \tag{5}$$

where $x^* = x/a$, $y^* = y/a$ are dimensionless coordinates, $u^* = u/u_0$ and $v^* = v/u_0$ are the velocity components along x and y directions, respectively, $p^* = p/\rho_f u_0^2$ is the pressure, $T^* = (T - T_0)/(T_w - T_0)$ is temperature, $Re_a = \rho_f u_0 a / \mu$, and $Pr = \mu c_f / k_f$ stand for Reynolds and Prandtl numbers, respectively and k_w/k_f is the ratio between the solid and fluid thermal conductivity. It is noteworthy that the dimensionless equations were obtained by considering the channel height, a, the fluid inlet velocity and temperature, u_0 and T_0, and the outer surface temperature, T_w.

The solution of the governing equations was accomplished by using the commercial code CFX®, which uses the so-called element-based finite volume method (Ferziger and Peric, 2002). The numerical scheme adopts a co-located grid layout, but with the alternative discretization proposed by (Rhie and Chow, 1983) for the mass flows in order to avoid the pressure field decoupling (i.e., the undesirable oscillatory pressure field). The convergence criterion of 1×10^{-6} for the residual was able to keep the global imbalances well lower than 1% for the quantities evaluated by the governing equations throughout the domains.

In the solution procedure, uniform profiles for temperature and for velocity were considered at the channel inlet. The Re and Pr numbers were assumed 35 and 0.7, respectively. No reverse flow was observed at the channel outlet. At the vertical portion of the walls (see letter B in the Fig.1) it was assumed an adiabatic condition. Additionally, the ratio of thermal conductivities of the solid to the fluid was considered $k_w/k_f = 25$. Finally, the solid-fluid interface considers heat exchange without contact resistance.

3. RESULTS AND DISCUSSION

3.1. Solution verification

Verification of the numerical model was performed by comparing the skin friction coefficient (Fanning coefficient) and Nusselt number for a parallel-plate channel against the reference values. In addition, a possible dependence of the solution on the mesh grid was also investigated by assessing the solution obtained using different meshes with increasing refinement ratio. The solid domain was discretized with hexahedral elements, while the fluid domain used tetrahedral and pyramidal elements (with proper boundary conditions in the z direction to emulate 2D fluid flow).

The Fanning coefficient and Nusselt number for fully developed flow in a parallel-plate channel without fins (β=0°) are

$$f_0 = \frac{\tau_{w_0}}{\frac{1}{2}\rho_f u_0^2} = \frac{4}{\mathrm{Re}_{D_h}} \left. \frac{\partial u^*}{\partial y^*} \right|_{y^*=1/2} \quad \text{and} \quad Nu_{Dh,0} = \frac{h_0 D_h}{k_f} = 2 \frac{\left. \partial T^* / \partial y^* \right|_{y^*=1/2}}{\left(T_b^* - T_{wi}^* \right)} \tag{6}$$

where τ_{w0} is the wall shear stress, Re_{Dh} is the Reynolds number based on hydraulic diameter, $D_h = 2a$, h_0 is the convective heat transfer coefficient for the fully developed flow, T_{wi}^* is the temperature of the internal surface of the wall (in this case $k_w / k_f \to \infty$ and $T_{wi}^* = T_w^* = 1$) and T_b^* is the bulk temperature at a given section of the channel. The reference values for the friction coefficient and Nusselt number are $f_0 = 96 / Re_{Dh} = 1.371$ (White, 2010) and $Nu_{Dh,0} = 7.541$ (Bergman et al, 2006). Table 1 presents the results for the three tested meshes. It is observed that accurate results were obtained even for the coarser mesh.

Table 1. Verification results for the smooth channel without fins.

Case	Elements	f_0	$Nu_{Dh,0}$
A	96,172	1.378	7.647
B	249,285	1.377	7.555
C	888,898	1.367	7.542
Reference	-	1.371	7.541

3.2. Fluid flow topology

The Figure 2 shows the streamlines for the finned portion of the channel for all cases. Scale factors were applied to the channel height (1:1.1) and length (1:5.5) to improve visibility of the flow features. It is noticeable the differences on the behavior that dictates the interaction between fluid flow and the solid domain. The in-line configurations clearly results in a lower level of fluid-solid interaction. The fluid is directed to the channel center due to the symmetric contraction created by the alignment of the fins. Therefore, fluid flows preferably along the longitudinal direction, i.e., the horizontal components of velocity are more pronounced in this arrangement. Larger values of the angle β cause differences in velocity to increase.

For staggered fins, the differences in the fluid flow topology are more expressive. Thus, the fluid-solid interaction for the staggered configuration is comparatively more substantial to the in-line cases. Due to the non-symmetrical pattern of fin positioning, a zigzag-type of fluid flow is created inside the finned portion of the channel. By increasing the angle β, the deviation from a pure horizontal fluid flow also increases. The vertical velocity component is quite high for larger angle β. As a consequence, this arrangement leads to higher local Nusselt number and Fanning coefficient.

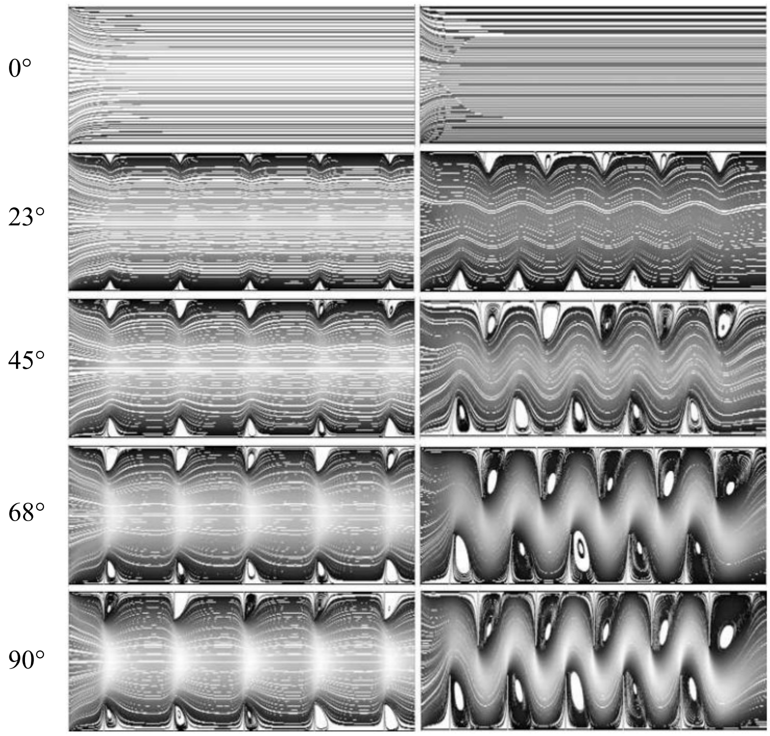

Figure 2. Streamlines for the in-line (left) and staggered (right) arrangements.

3.3. Heat transfer

In order to investigate how the fins affect the heat exchange between the solid and the fluid domains, Figure 3 shows both the fluid temperature contours and temperature at the channel center T_c^*, whereas Figure 4 presents both the solid temperature contours and temperature at the solid-fluid interface T_{wi}^*. Due to the symmetry of the in-line arrangement, it is presented only the upper wall temperature contours. In Figure 3, it is possible to notice for the in-line arrangement that the dimensionless temperature is symmetrical with respect to the channel center irrespective of the angle between fins and channel walls. On the other hand, there is no flow symmetry for the staggered arrangement, so that temperature for both walls is plotted in Figure 4. Unlike the in-line arrangement, there is a noticeable influence of the fin inclination on the temperature distribution for both fluid and solid domains. Larger angles, β, increase proportionally the deviation from to the horizontal flow, i.e., larger blockage ratio causes the vertical reflection of the flow to increase. Therefore, the portion of the fluid with lower dimensionless temperature is displaced to near-wall regions causing the temperature gradients to increase. Both Figure 3 and Figure 4 are presented with the same scaling factor of the Figure 2. As inferred by Iaccarino et al (2002), some differences between the experimental and numerical data can be eliminated if conduction heat transfer in fins is taken into account. Hence, the conjugate heat transfer implies also a more realistic solution of these configurations.

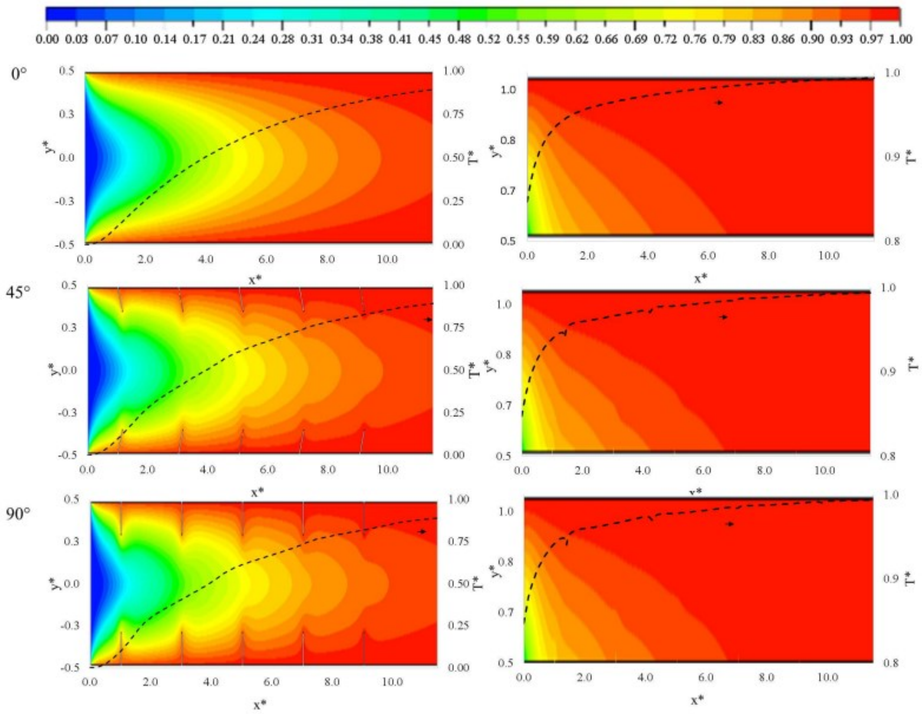

Figure 3. Fluid temperature contours and temperature profile at the channel center (left) and upper wall temperature contours and temperature profile at the solid-fluid interface (right).

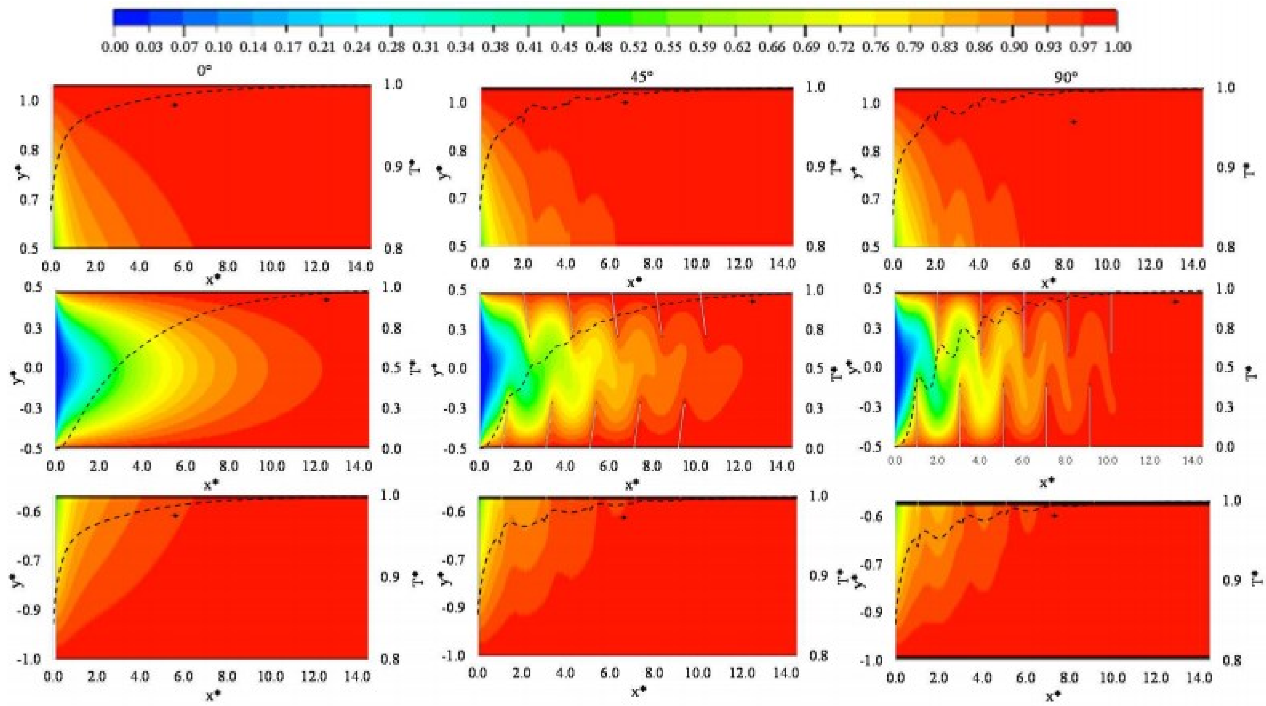

Figure 4. Fluid temperature contours and temperature profile at the channel center (horizontal middle) and wall temperature contours and solid-fluid interface temperature profile (upper and lower walls are represented in the top and bottom set of pictures).

Evaluation of the heat transfer at the channel wall is made by the local Nusselt number, $Nu_{Dh,x}$. Due to the recirculation of the fluid near the fins, the inlet temperature is adopted as reference, so that

$$Nu_{Dh,x} = \frac{h_x D_h}{k_f} = (\mp)\frac{2}{T^*_{wi}}\left.\frac{\partial T^*}{\partial y^*}\right|_{y^*=\pm 1/2}, \tag{7}$$

where h_x is the local convective heat transfer coefficient. It must be emphasized that this definition of the Nusselt number reflects the decreasing in heat exchange at the channel internal surface with as the thermal equilibrium is reached along the flow, $\partial T^*/\partial y^* \rightarrow 0$.

Figure 5 shows the behavior of the local Nusselt number for both fin arrangements. For cases with the in-line arrangement, the local Nusselt number decrease for larger blockage ratios (i.e., the increase of the angle between fins and channel walls, β), as shows Figure 5 (a). For the staggered arrangement, there is an effect of the flow impingement at the channel walls, leading to higher local heat transfer. On the other hand, the fluid recirculation causes the heat transfer near the fins to decrease.

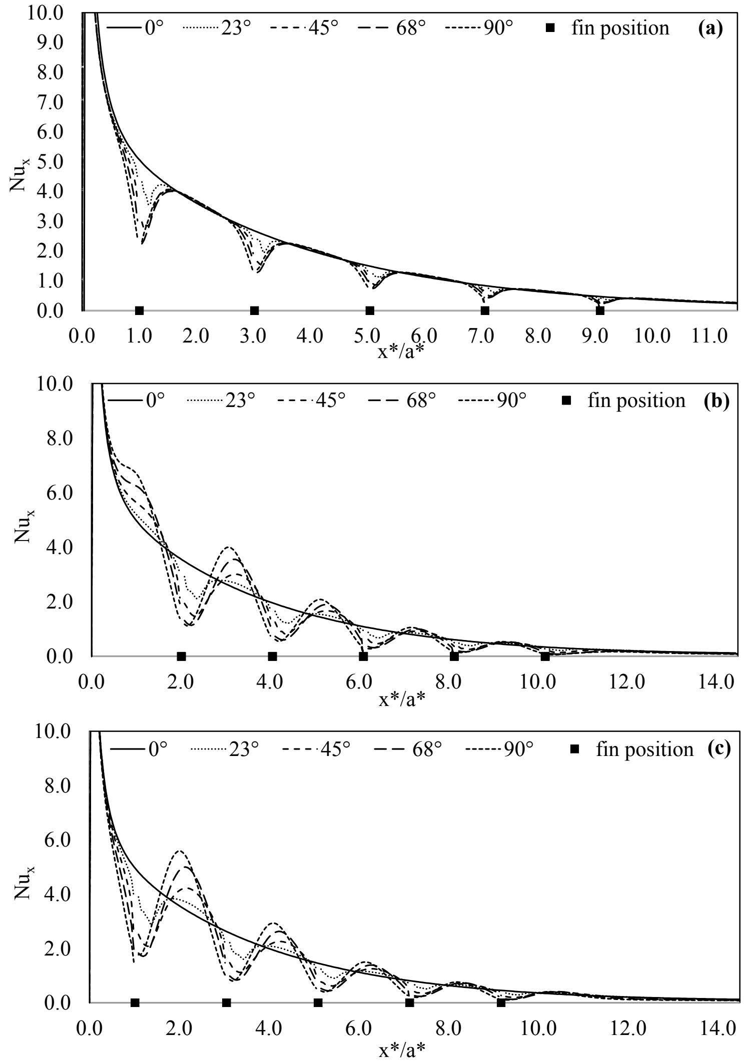

Figure 5. Local Nusselt number along the channel for the in-line arrangement (a) and for the upper wall (b) and lower wall (c) for the staggered arrangement.

The optimum design of a heat exchanger must find the best configuration to maximize the total heat transfer, q_t (with the lowest possible amount of pressure loss). Thus, it is important to point out that the total thermal energy transferred to the fluid, q_t, comprises two components: (i) heat exchanged through channel walls, q_w, and (ii) heat exchanged by the fins surfaces, q_f,. The Table **2** presents the ratio of the total heat flux to the heat flux of a channel without fins, q_0 (β=0°). In laminar flows, the heat transfer from/to the fluid at interfaces is exclusively diffusive. Near the fins, fluid recirculation causes the temperature gradient to decrease (i.e., local Nusselt number diminishes). In the in-line arrangement, this is not compensated by the increase in heat transfer surface due to the fins. Therefore, no increase in heat exchange is observed with relation to the channel without fins (actually, a little decrease takes place). In turn, for the staggered arrangement of fins, the increase in local heat transfer on the surfaces caused by the flow impingement leads to a little increase in the total heat flux, reaching a maximum of 1%.

Table 2. Total heat transfer ratio with respect to the channel without fins.

	β	0°	23°	45°	68°	90°
q_t/q_0	in-line	1.000	1.000	0.998	0.998	0.997
	staggered	1.000	0.999	1.001	1.007	1.011

4. CONCLUSIONS

The conjugate heat transfer of a Newtonian fluid in a horizontal, two-dimensional finned channel is numerically evaluated. The flow is considered laminar and under steady-state conditions. The fins and the channel walls are heat conducting and interact with the fluid flow through convective heat transfer. The problem is modeled as conjugate, i.e., the solution of the conductive heat transfer problem for the solid domain is integrated with the solution of the convective heat transfer problem for the fluid domain. Transversal, equally spaced fins are placed in the internal surface of the channel along the flow direction according to a staggered or in-line arrangement. Additionally, the angle between the fins and the channel walls are known and lies in the range from 0° (smooth channel without fins) to 90° (classical configuration). The inlet flow velocity and temperature are uniform. Due to the fins, there is a blockage ratio of the fluid flow from 0 to 0.4. Furthermore, the fin pitch and channel height present an aspect ratio equal to 2. The numerical simulations were performed using the commercial code CFX® using unstructured meshes. The influence of different fin configurations on heat transfer is compared and discussed. Generally, it was found that the local Nusselt number is very low in the regions close to fins due to fluid recirculation. On the other hand, the local Nusselt number in the regions corresponding to the fin pitch presented dependency on the arrangement and also on the inclination angle. Contrary to the staggered arrangement, the local Nusselt number for the in-line fins diminishes with increasing in the blockage ratio. For the simulated geometrical configurations and flow conditions, the increase of the heat exchange surface due to fins were barely sufficient to compensate the heat transfer reduction imposed by the recirculating zones.

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