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Truss optimization including the effects of geometric nonlinearity in structural analysis

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Abstract. The consideration of nonlinearities is extremely relevant in the structural analysis of trusses, since the optimization of the structure requires cautious analysis regarding the effects of buckling. Therefore, this study investigates the perturbations in the topology of the objective function due to nonlinearities, potentializing the use of heuristic methods, being one of them the PSO, used in this work. An algorithm was developed to predict global stability by detecting singularities of the global stiffness matrix to overcome the difficulties of load control method. The largest deflections in the structures analyzed in relation to the linear analysis were replicated to the objective function of the optimization problem, causing fluctuations and the formation of local minima. The stability criterion adopted allowed the satisfactory use of the line control method to the optimization problems, making feasible the analysis of the highly nonlinear behavior of the structures.
Keywords: truss optimization, structural analysis, geometrical nonlinearity, finite elements method, PSO.

INTRODUCTION

Trusses have wide application in structural engineering, whose main objectives are to obtain lighter, efficient and reliable structures. To reduce the mass of the structure, fewer elements are usually used, which become slender, which requires that the analyzes of these structures be more cautious with respect to the effects of buckling. (Pereira et al., 2007; Leite and Lavall, 2000). Therefore, the consideration of nonlinearities (physical, geometric and boundaries) are extremely relevant to obtain the best approximation of the real behavior of these structures when applied in optimization of actual problems. In literature, there are several researches available for optimum structural design, on the other hand, there are a few researches that explores the effects of geometric nonlinearities in objective function. (Medeiros, 2005; Leite, 2000). So, this study investigates structures studied previously in literature, which are regarding the effects caused by constraints in topology of the objective function. Nonlinear effects cause disturbances at topology of the function being observed, potentiating the use of heuristic methods.

STRUCTURAL ANALYSIS

According to Geschwindner (2000), first order elastic analysis is the most common structural analysis, in which deformations are considered small so that the equilibrium equations can be written based on the configuration without deformations of the structure. Any inelastic behavior of the material is disregarded and the principle of effect superposition is valid. As a result, the load-displacement curves by linear analysis cannot describe the deformation of the structure in detail. To obtain a more reliable and accurate analysis, in this work was used the finite element method considering the geometric nonlinearity.

Nonlinear Finite Element

The behavior of a typical bar element can be obtained by stiffness equations formulated in terms of the linear variation of the translational displacement (Hill et al., 1989). However, for large displacements, the elastic and geometric linear stiffness equations deduced from the Green–Lagrangian strain (Eq. (1)) based on Total Lagrangian (LT) description are more adequate.

Considering a bar with initial length L_0 and final length L, relating the kinematic equations, the generalized three-dimensional deformation in matrix form is presented in Eq. (1).

$$\varepsilon = \frac{L^2 - L_0^2}{2L_0} = \mathbf{B_0 u} + \frac{1}{2}\mathbf{Bu} \tag{1}$$

where:

$$\mathbf{B_0} = \frac{1}{4\alpha_0^2}\left[\mathrm{x_{ij}}, \mathrm{y_{ij}}, \mathrm{z_{ij}}, -\mathrm{x_{ij}}, -\mathrm{y_{ij}}, -\mathrm{z_{ij}}\right] \tag{2}$$

$$\mathbf{B} = \frac{1}{4\alpha_0^2}\left[\mathrm{u_{ij}}, \mathrm{v_{ij}}, \mathrm{w_{ij}}, -\mathrm{u_{ij}}, -\mathrm{v_{ij}}, -\mathrm{w_{ij}}\right] \tag{3}$$

$$\mathbf{u} = \left[\mathrm{u_i}, \mathrm{v_i}, \mathrm{w_i}, \mathrm{u_j}, \mathrm{v_j}, \mathrm{w_j}\right]^{\mathrm{T}} \tag{4}$$

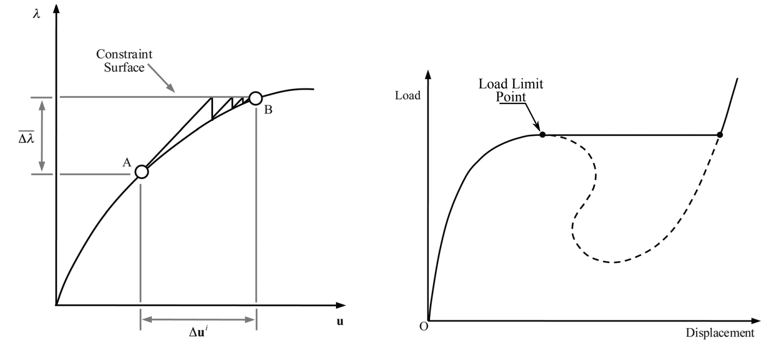
being x_{ij}, y_{ij} and z_{ij} the directional cosine in the longitudinal direction of the bar in the reference configuration to the global axis x, y and z, respectively. For the actual configuration, u_{ij}, v_{ij} and w_{ij} are the corresponding directional cosine for the actual configuration. The nodal displacements associated to nodes *i* and *j* make the vector **u**. The internal forces vector q_{i} cam be find using the principle of Virtual Work, resulting:

$$\mathbf{q_i} = 2\alpha_0 A_0 \sigma \left(\mathbf{B_0} + \mathbf{B}\right) \tag{5}$$

Deriving Eq. (5) with respect to **u**, the tangent stiffness matrix $K_{\mathbf{t}}$ is obtained:

$$\mathbf{K_t} = 2E\alpha_0 A_0 \mathbf{B_0}\mathbf{B_0}^{\mathrm{T}} + \left(2E\alpha_0 A_0 \mathbf{B_0}\mathbf{B}^{\mathrm{T}} + 2E\alpha_0 A_0 \mathbf{B}\mathbf{B_0}^{\mathrm{T}} + 2E\alpha_0 A_0 \mathbf{B}\mathbf{B}^{\mathrm{T}}\right) + \frac{2}{\alpha_0} A_0 \sigma \mathbf{A} \tag{6}$$

A complete investigation of the large deflection inelastic behavior of structures requires tracing the equilibrium path. Several methods and techniques to achieve the equilibrium path have been presented in the literature, such as load control, displacement control, generalized displacement control and arc-length methods. Each of the methods has its own advantages and disadvantages. Traditionally, load control methods (LCM) type methods have been the most popular for solving nonlinear system of equations. This method, the external loads are computed at the first iteration of each incremental step and held constant throughout the remaining iterations in the step (see Fig. 1a).



(a) Load Control Method - LCM. (b) Snap through behavior.

Figure 1 – Load control method and nonlinear equilibrium paths.

where $\Delta\bar{\lambda}$ is a prescribed initial load factor and $\Delta\bar{u}$ is the correspond displacement. The load control method is easy to implemented and extremely robust, however, since the externally applied loads are kept constant, this method fails near load limit point (i.e. when the tangent matrix is singular (Riks, 1979) – Fig. 1b)[1]. So, to overcome the difficulties of the load control method at the limit points, we developed a computer program to predict the global stability by singularity detection of the global stiffness matrix, called stability criterion. Therefore, the load control method (LCM) is employed to solve the nonlinear equilibrium equations.

The stability criterion is to verify the overall stability of the structure. This condition was defined for the purpose of identifying which elements violated some constraint (e.g. bending stress or buckling), which would no longer be responsive to the stresses as in their original state. Thus, elements that have suffered some kind of failure are removed from the system by a mathematical artifice - bars that have violated some final constraint of the loading step receive zero stiffness within the global tangent stiffness matrix of the structure ([KtG]) and then a check of the stability of the structure is carried out. This stability check aims to identify if the absence of the slash that has violated the constraint renders the structure unstable - if so, the algorithm is terminated in the load step that is breached and otherwise a new load increase is applied to the structure, continuing the analysis.

[1] For further discussion on nonlinear systems, seer Bathe (2006) – Crisfield (1997) in Refs.

A greater degree of confidence in the results is guaranteed due to this stability criterion, besides allowing a reduction of computational cost during the optimization, since the condition of completion of the structural analysis will be conditioned to the instability of the structure, and not to the applied load, in which convergence is reached only when the entire load is applied during the incremental and iterative process.

Structural Optimization

Usually, in structural optimization problems, the objective function (function that is desired to find the minimum) consists on the mathematical formulation of the optimum design problem. So, in terms of truss optimization the problem can be mathematically stated as:

$$
\begin{aligned}
\text{find} : \ & \mathbf{x^*} \\
\text{which maximizes} : \ & W = \sum_{i}^{\mathrm{N_m}} A_i L_i \rho_i \quad i = 1, \cdots, \mathrm{N_m} \text{ for all members} \\
\text{subject to the constraints} : \ & C_k^{\sigma} \leq 0 \quad k = 1, \cdots n_c \\
& C_l^{\delta} \leq 0 \quad l = 1, \cdots n_d \\
\text{where} : \ & A_{min} \leq A_i \leq A_{max} \quad i = 1, \cdots n_g
\end{aligned}
\tag{7}
$$

where W is the weight of the truss, A_i, L_i and ρ_i are the cross-section area, length and material density of member i, respectively. $\mathrm{N_m}$ is the total number of members. n_c is the total members subjected to both axial compression (buckling) and stress constraint, according C_k^{σ}. C_l^{δ} is the lateral displacement constraint of the lth degree of freedom for all restricted displacements n_d. The solution to Eq. (7) consists in finding optimal design variables $\mathbf{x}^*$ that minimize the weight of the truss. In the literature, there are many studies available for the optimum design of structures using metaheuristic techniques or based on mathematical programming. This work, we use the Particle Swarm Optimization (PSO) algorithm, proposed by Kennedy and Eberhart (1995), to deal with the optimization problem. In the PSO optimization method, each particle corresponds to a point in the search space and represents a possible solution to the optimization problem - these particles have a fitness function that individually evaluates their suitability for the solution. It is also intrinsic to these particles velocity values that define the direction of motion of the particle by the search space, this velocity is modified taking into account the best position of each particle (lbest) and the best position of the group / swarm (gbest), so, over time, the whole group ends up getting the optimal solution. (Oliveira, 2008; Medeiros, 2005).

NUMERICAL RESULTS

In this section, numerical examples are presented to verify the accuracy of the computer program developed. For the verification purpose, the results obtained are compared with the results reported in the literature. In structural analysis, the purpose of the examples is to demonstrate the capability of the proposed procedure in tracing the equilibrium path of highly nonlinear trusses. Then, the effects caused by constraints in the topology of the objective function, as well as the optimum results obtained with the optimization, are shown in structural optimization test cases.

Structural analysis

The first problem analyzed consist of a 12-bar truss structure as illustrated in Fig. 2. The material properties are assigned as E = 21 $\times 10^5$ kg/cm^2, area of 0.2 cm^2. The maximum load is 100 kN. The computational results are shown in Fig. 3, where it is observed that the LCM captured the full equilibrium path.

The second problem consists in two-dimensional truss, illustrated in Fig. 4. This structure is composed of two identical members with area of 96.77 cm^2 and moment of inertia of 745.18 cm^4. The elastic modulus of the material is E = 7.03 $\times 10^5$ kg/cm^2. Fig. 5 shows the load-displacement curves obtained by the present study for linear and geometric nonlinear (LGN) analysis in comparison with Posada et al. (2007). It is observed that the results of the present study for geometric nonlinear analysis are in good agreement with those reported by Posada et al. (2007). However, can be seen that the first snap-through occurs at the load factors of 0.5 and 0.58 (top of the load-displacement curve), where the LCM failed at the fist load limit point. This behavior is expected because the LCM can only capture the full equilibrium in stable branch, where incremental stiffness matrix is positively defined. The alternative found to deal with this problem is one of the main contributions of this work.

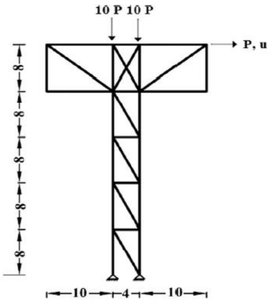


Figure 2 – 29 bar structure. Adapted from Rezaiee-Pajand et al. (2015)

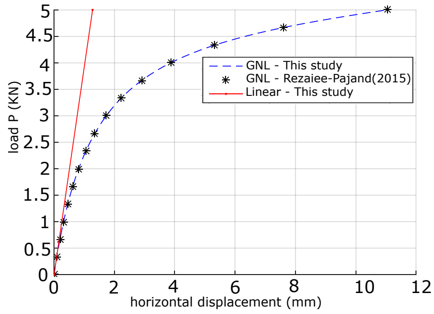


Figure 3 – Load-displacement curve.

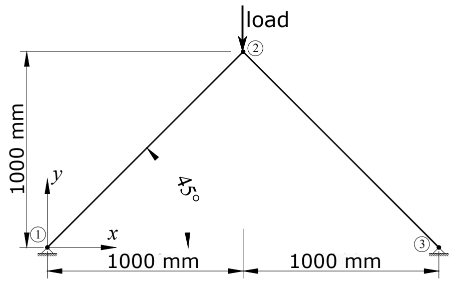


Figure 4 – Two bar symmetric structure. Adapted from Posada et a. (2007).

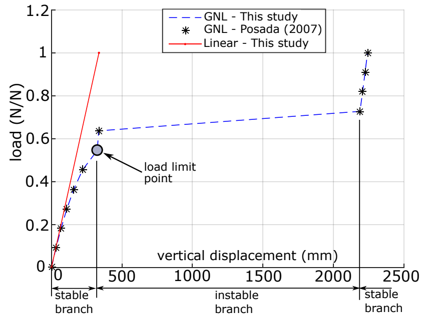
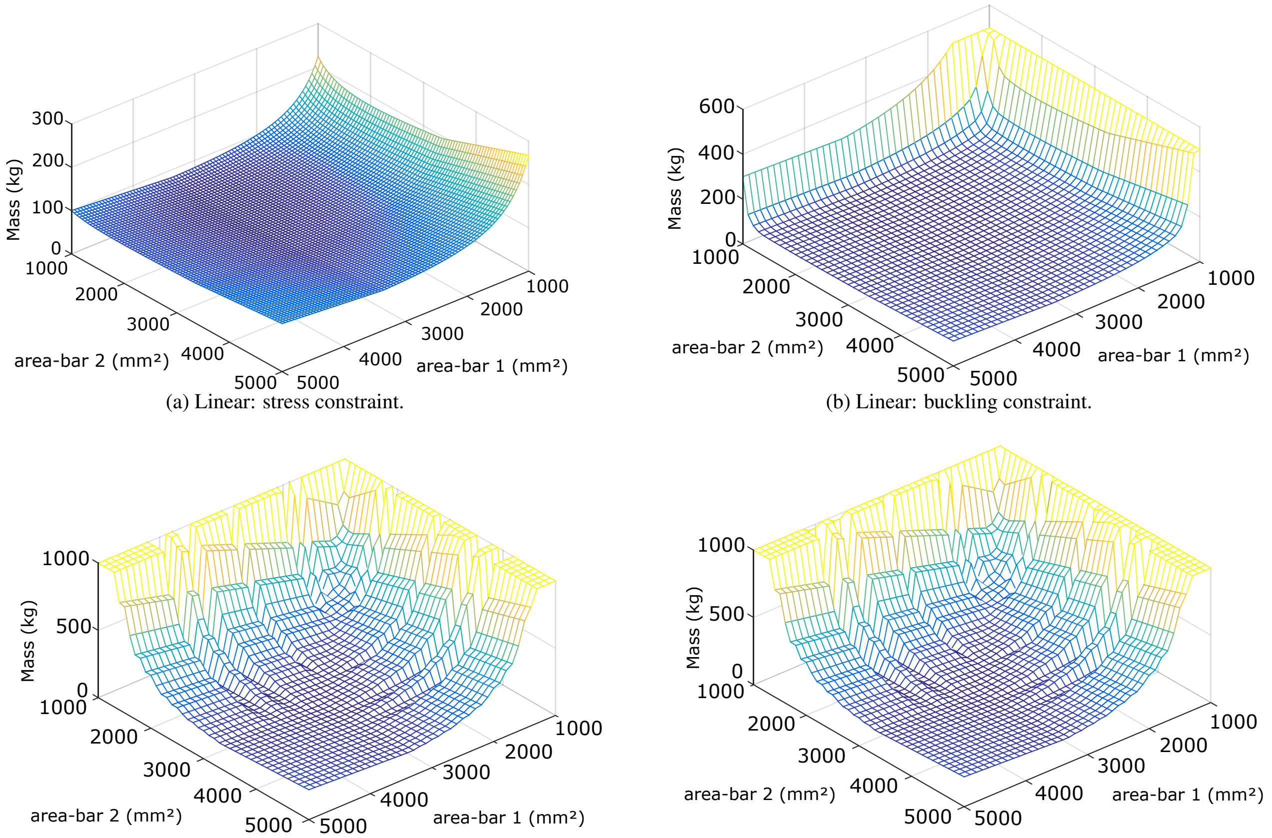


Figure 5 – Load-displacement curve.

Structural Optimization

The previous two-bar truss illustrated in Fig. 4 has been analyzed previously using the LCM. It is observed that the LCM cannot capture the equilibrium path beyond load limit point. With the coupling based on global stability prediction in LCM, it had been realized the optimization two-bar truss subjected to axial compression (critical Euler buckling stress) and stress constraints, according Eq. (7). Figure 6a and 6b shown the objective function find by linear analysis whit stress and buckling constraints. It can be observed a soft behavior in full domain function, as stress and buckling constraints penalize the objective function smoothly and proportionally to the loads applied to the bars. Figure 6c and 6d show the objective function by geometric nonlinear analysis considering stress and buckling constraint, respectively. It is observed that the objective function has several local minima. This behavior is result of the redistribution of strains and oscillation of deformation, which can be caused by geometric nonlinear analysis, potentiating the application of heuristic methods, such as PSO, for this type of problem because they not depend on the function gradient, they can find the optimal solution without getting stagnant in the local minimums.

The second case consist in the optimization of the twelve bar truss. This problem consists of a 12-bar truss structure as illustrated in Fig. 7. This example features highly nonlinear behavior (see Fig. 8), studied by Yang and Leu (1991), Yang et al. (2007), Krenk and Hededal (1995), and Leon and Paulino (2012). As stability criterion developed in this work allows a use of the load control method for the nonlinear geometric analysis of this problem, we perform the parametric optimization to verify the effects of the constraints on the optimal solution. Like the two-bar truss example, the Eq. (7) are used to include axial compression and stress constraints. The material properties as design variable ranges of the twelve bar truss are assigned as $E = 200$ GPa, $\rho = 7850$ kg/m^3, design variable lower bound $A_{min} = 4 \times 10^{-5}$ m^2, and the external load $F = 150$ kN. So this is a truss optimization on size with two stress constraints and 12 design variables. The same truss adopting pipe sections for all members, was also designed by the algorithm considering the linear and geometric nonlinear analysis behaviour. Two optimization were performed. The first considering linear analysis (see column 2 - Tab. 1) and other considering geometric nonlinear analysis (see column 3 - Tab. 1). Note that linear analysis had a lighter structure than did geometric nonlinear analysis. This was due to the fact that while the buckling constraints were critical in the case of linear and nonlinear behaviour, buckling constraints become more active in the case of nonlinear behaviour due to the redistribution of compensation loads. In addition, it was also reported a large difference between the values of member forces in both analyzes. Thus, geometric nonlinearity results in large sections to accommodate member forces. Furthermore, there a possibility of having a change in the sign of the member force. That is, while the member was in tension according to linear analysis, it can be found in compression after the non-linear analysis.



(a) Linear: stress constraint.
(b) Linear: buckling constraint.
(c) GNL: stress constraint.
(d) GNL: buckling constraint.

Figure 6 – Objective function by optimization of the two-bar truss.

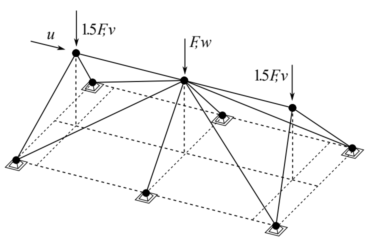


Figure 7 – Twelve-bar truss schematic,3D view. Adapted from Leon and Paulino (2012).

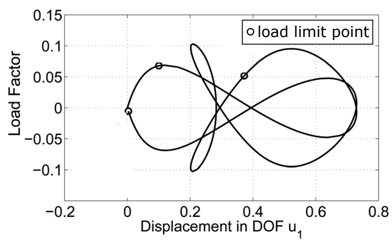


Figure 8 – Load-displacement curve.

Table 1 – Optimal solution.

Bar	Area [mm^2]	
	Linear	GNL
1	40.00	273.55
2	315.73	590.64
3	40.00	273.55
4	40.00	273.55
5	315.72	590.64
6	40.03	273.55
7	515.09	515.09
8	515.09	515.09
9	40.00	40.00
10	40.00	40.00
11	515.09	515.09
12	515.09	515.09
Mass [kg]	33.999	56.303

Summary

The brief results presented in this work demonstrate relevant characteristics regarding the behavior of the structural systems analyzed. It’s observed that the geometric nonlinearity resulted in larger deflections in the analyzed structures if compared to the linear analysis. In addition, it was observed that this behavior was replicated to the objective function of the optimization problem, causing fluctuations and formation of local minima. It is worth mentioning that the stability criteria adopted in this work allowed the satisfactory use of the LCM to optimize problems addressed in this work, making possible the nonlinear geometric analysis and optimization of slender structures with highly nonlinear behavior. These results intensify the analyzes performed in this work searching for the best numerical representation of the real behavior of these structural systems.

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REFERENCES

Bathe, K.J., 2006, "Finite element procedures", Prentice-Hall, Upper Saddle River.

Crisfield, M.A., 1997, "Non-Linear Finite Element Analysis of Solids and Structures", Vol 2: Advanced Topics, John Wiley & Sons.

Geschwindner, L.F., 2000, "Unified Design of Steel Structures", Ed. Wiley, University of California, USA.

Hill, C.D., Blandford, G.E. and Wang, S.T., 1989. "Post-buckling analysis of steel space trusses". Journal of Structural Engineering, Vol. 115, No. 4. doi:10.1061/(ASCE)0733-9445(1989)115:4(900).

Kennedy, J., Eberhart, R.C., 1995, "Particles Swarm Optimization", IEEE International Conference on Neural Networks.

Krenk, S., and Hededal, O., 1995, "A Dual Orthogonality Procedure for NonLinear Finite Element Equations", Comput. Methods Appl. Mech. Eng.

Leite, F. N., Lavall, A.C.C., 2000, "Uma Formulação Teórica Consistente para Análise Não Linear de Estruturas Treliçadas Espaciais", masters dissertation, UFMG.

Leon S.E., Paulino G.H., Pereira A., Menezes I.F.M., and Lages E.N., 2012, "A Unified Library of Nonlinear Solution Schemes"

Medeiros, J.A.C.C., 2005, "Enxame de Partículas como Ferramenta de Otimização em Problemas Complexos de Engenharia Nuclear", doctoral thesis, UFRJ.

Pereira, A., Martha, L.F., Vaz, L.E., 2007, "Otimização baseada em confiabilidade: aplicação a treliças espaciais", doctoral thesis, PUC-Rio.

Posada, L.M., 2007, "Stability Analysis of Two-dimensional Truss Structures", masters dissertation, Universidade de Estugarda, Alemanha.

Rezaiee-Pajand, M., Naserian, R., 2015, "Using residual areas for geometrically nonlinear structural analysis", Ocean Engineering Vol. 105.

Riks E. (1979), 'An incremental approach to the solution to the solution of buckling ans snapping problems', Int. J. Solids Struct.

Yang, Y.-B., and Leu, L.-J., 1991, "Constitutive Laws and Force Recovery Procedures in Nonlinear Analysis of Trusses", Comput. Methods Appl. Mech.Eng.

Yang, Y.-B., Leu, L.-J., and Yang, J. P., 2007, "Key Considerations in Tracing the Postbuckling Response of Structures With Multi Winding Loops", Mech. Adv. Mater. Struct.

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