Frequency-Domain Fatigue Analysis by Strain-Life Approach with Mean Stress Correction

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Abstract: The mean stress effect in frequency-domain fatigue analysis by strain-life approach is presented in this paper, whose focus is set on an algorithm to correct the mean effect on the fatigue intensity damage by using the zero mean strain-life spectral method proposed by Nieslony and Macha (2007). The calculations have been performed for the 2024-T351 aluminum alloy under stationary random Gaussian strain response with positive mean value for both narrow and wide-band spectrum. Frequency-domain fatigue results were based on Manson-Coffin Strain-Life Curve and Palmgren-Miner Rule. Elastic Morrow and SWT Mean Stress Correction Methods have been used with Ramberg-Osgood Relationship. The results for frequency-domain approach present good agreement against the traditional time domain method and have become a great alternative to Traditional Rainflow Counting Method for both types of spectrum mentioned before.

Keywords: frequency domain, fatigue intensity damage, Manson-Coffin Strain-Life Curve, Spectral Method, Mean Stress

INTRODUCTION

The literature shows, basically, two methods of analyzing fatigue of random loads: time domain and frequency domain methods (Dowling, 2013; Bishop, 1988). There are two methodologies to study fatigue both in time and frequency domains depending on the type of load history: stress-life and strain-life methodologies (Nieslony and Macha, 2007).

As we can see in literature, there are some semi-empirical methods for fatigue analyses in frequency domain, both for narrow-band and wide-band signals. Mršnik et al. (2013) have revised some of them, such as Rayleigh method (Miles, 1956), Wirsching-Light method (1980), Dirlik method (1985), Zhao-Baker method (1992), Benausciutti-Tovo methods (2002, 2005) and Lalanne method (2013).

Nieslony and Macha (2007) have proposed an elasto-plastic frequency-based method to estimate fatigue life for zero mean stress using spectral methods as a function of the Probability Density Function (PDF) of Strain Amplitude and following the Manson-Coffin equation both for narrow-band and wide-band spectrum. Rognon et al. (2011) have presented techniques for fatigue damage evaluation using spectral methods and a model taking into account elasto-plastic behavior for zero mean stress. Nieslony and Böhm (2015) have made a strain-based multiaxial fatigue life evaluation using spectral method.

In time domain, there are some proposed methods to include the nonzero mean stress effect, of which Elastic Morrow and SWT approaches are the most commonly used for engineering applications involving the strain-life approach (Dowling, 2013).

As we go through the literature there is no solution regarding the problem of the mean stress correction by using the strain-based fatigue law in frequency domain. Nieslony and Böhm (2015) affirms that there are few applications of the mean stress correction in frequency domain method. Kihl and Sarkani (1999) presented the mean stress contribution on fatigue life of welded steel joints. The authors applied the mean value correction to random load following a Rayleigh distribution based on the stress-life curve. Nieslony and Böhm (2012, 2015) proposed a mean value correction procedure based on stress-life approach that directly operates on the power spectral density of the signal.

In this paper, the authors have presented an application of Elastic Morrow and SWT mean stress correction procedures (widely applied in time domain) in damage intensity calculation for frequency domain methods by using the strain-life spectral methodology proposed by Nieslony and Macha (2007), both for narrow-band and wide-band process. The results are compared with the traditional time domain results. The calculations have been performed for the 2024-T3 aluminum alloy under stationary random Gaussian load with positive mean value.

FREQUENCY DOMAIN FATIGUE

The frequency domain method is presented in terms of the Power Spectral Density (PSD). The statistical properties can be described by the PSD moments. Others important statistical parameters for frequency domains are the expected number of passes through the zero level $E[0^+]$ and expected number of peaks per second $E[P]$. The irregularity factor γ is an important property that differs a process between narrow-band and wide-band. When this factor is close to 1.0, the process is considered narrow-band (Bishop, 1999; Lalanne, 2013; Lee et al., 2012; Newland, 1993).

Fatigue Damage Accumulation with Mean Stress Effect

Initially, the spectral methods were described as a function of stress amplitude. However, the approaches are defined based on PDF of amplitudes, so a definition in terms of strain amplitude is valid. It is possible to apply the strain-life method proposed by Nieslony and Macha (2007) using the PDF for Rayleigh, Dirlik, Benausciutti-Tovo and Lalanne methods written for strain amplitude - $PDF(\varepsilon_a)$. In this paper, Rayleigh is tested for narrow-band analysis and the other ones for wide-band. So, as described in literature for stress amplitude, the frequency domain formula in terms of strain amplitude ε_{a_j} and bandwidth $\Delta\varepsilon_a$ to count cycles n_j for each j-strain amplitude is showed in Eq. (1):

$$n_j = E[P]\ T\ PDF\left(\varepsilon_{a_j}\right)\Delta\varepsilon_a \tag{1}$$

Then, the intensity damage (per second) d_i is presented in Eq. (2), where N_{f_j} is the number of cycles obtained with Manson-Coffin law corrected by Elastic Morrow or SWT methods:

$$d_i = \sum_{j=1}^{k} \frac{E[P]\ PDF(\varepsilon_{a_j})\ \Delta\varepsilon_a}{N_{f_j}} \tag{2}$$

The Elastic Morrow mean stress correction method is described in Eq. (3), where σ_f' is the endurance limit, σ_m, the mean stress, E, the Young's modulus, $\widetilde{\sigma}_{fB}$, the true rupture stress, ε_f', the fatigue ductility exponent, c, the Coffin-Manson exponent and b, the exponent of endurance limit. This method reduces the mean stress effect for short lives, where the plastic strain is dominant (Ince and Glinka, 2011).

$$\varepsilon_{a_j} = \frac{\sigma_f'}{E}\left(1 - \frac{\sigma_m}{\widetilde{\sigma}_{fB}}\right)(2N_{f_j})^b + \varepsilon_f'\ (2N_{f_j})^c \tag{3}$$

The Smith, Watson e Topper (SWT) mean stress correction method is described in Eq. (4), where $\sigma_{máx_j}$ is the maximum stress. The SWT method is preferred in most cases with good results for a wide range of materials and excellent estimates for aluminum alloys (Dowling, 2013).

$$\begin{cases} \sigma_{máx_j}\varepsilon_{a_j} = \dfrac{(\sigma_f')^2}{E}\left(2N_{f_j}\right)^{2b} + \sigma_f'\ \varepsilon_f'\left(2N_{f_j}\right)^{b+c},\ \sigma_{máx_j} > 0 \\[2mm] N_{f_j} = \infty,\quad \sigma_{máx_j} \leq 0 \end{cases} \tag{4}$$

The Ramberg-Osgood cyclic stress-strain curve is presented in Eq. (5), where H' is the cyclic strength coefficient and n', the cyclic strain hardening exponent. The cyclic stress–strain curve is the relationship between stress amplitude σ_{a_j} and strain amplitude ε_{a_j} for cyclic loading. The Ramberg-Osgood relation can also be used as a monotonic path (Dowling, 2013).

$$\varepsilon_{a_j} = \frac{\sigma_{a_j}}{E} + \left(\frac{\sigma_{a_j}}{H'}\right)^{1/n'} \tag{5}$$

Material

The 2024-T351 aluminum alloy was the material used for the calculations. Its properties are shown in Tab. 1, where σ_0 is the yield strength and σ_u, the ultimate stress.

Table 1 – Cyclic Stress–Strain and Strain–Life Constants for 2024-T351 Al (Dowling, 2013)

Material	Tensile Properties			Cyclic $\sigma - \varepsilon$ Curve			Strain-Life Curve			
	σ_0 (MPa)	σ_u (MPa)	$\widetilde{\sigma}_{fB}$ (MPa)	E (MPa)	H' (MPa)	n'	σ_f' (MPa)	b	ε_f'	c
2024-T351 Al	379	469	558	73,100	662	0.070	927	-0.113	0.409	-0.713

Calculation Algorithm

MATLAB® software has been used to implement the algorithm. The Elastic Morrow and SWT methods can be used for mean stress correction in N_f. The algorithm proposed here has the same structure of zero mean stress one. The modification is applied in the intensity damage equation in the end of the algorithm. A global stress mean value σ_m^G is calculated from the iterative application of the Ramberg-Osgood cyclic stress-strain curve (assumed as monotonic stress-strain path) on the global strain mean value ε_m^G obtained from the strain random signal response $\varepsilon_{(t)}$. Figure 1 shows the algorithm scheme.
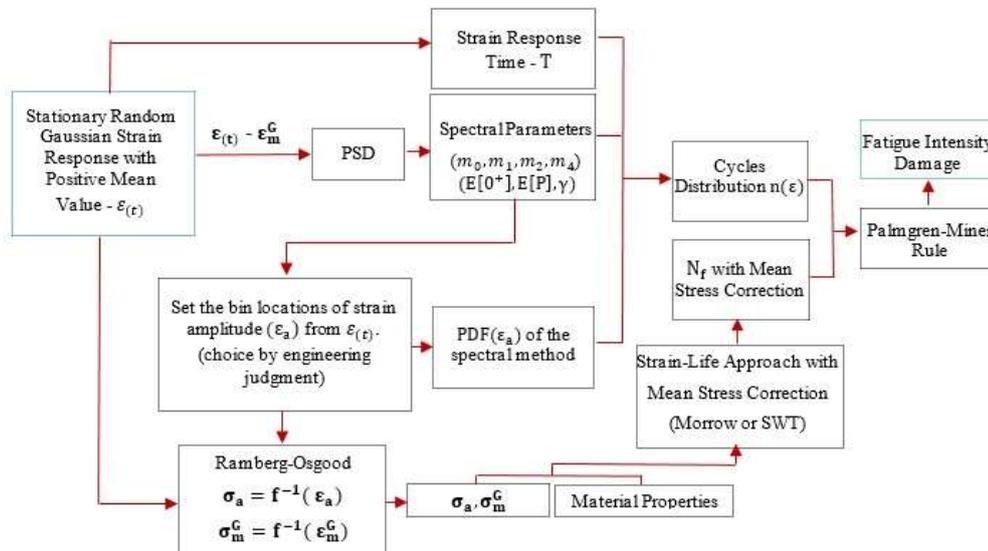

Figure 1 – Algorithm scheme for mean stress correction in frequency domain.

The algorithm follows these steps:

    A.   Initially, a Gaussian random stationary response signal $\varepsilon_{(t)}$ with positive mean value ε_m^G is generated,

    B.   From $\varepsilon_{(t)}$ is possible to use the Ramberg-Osgood cyclic relation to obtain iteratively the global mean stress σ_m^G and the stress amplitude σ_a from ε_m^G and ε_a, respectively,

    C.   The PSD and PDF calculations are performed considering a transformed strain $\varepsilon_T = \varepsilon_a - \varepsilon_m^G$, with zero mean load, and thus, the effect of the mean load is applied only in the calculation of the damage intensity in fatigue through the Elastic Morrow and SWT methods,

    D.   With the PDF values, spectral parameters, loading analysis time, global stress and stress amplitude, the mean stress correction is applied together with the calculation of the cycle distribution,

    E.   Then, using the Palmgren-Miner linear damage accumulation, the fatigue damage intensity is obtained.

RESULTS AND DISCUSSION

A stationary random Gaussian strain response with positive mean value $\varepsilon_m^G = 1.5e\text{-}03$ was generated as input. The time of random history T = 327.68s (30,000 points under sampling frequency of 100Hz) is fixed numerically. Bandpass Chebyshev Type I filter is used to define the spectral width of strain signal. (MathWorks Signal Processing Toolbox, 2018). The narrow-band signal has a dominating frequency of 13.7Hz. The wide-band has spectrum from 8Hz to 21Hz. A section of the history with narrow-band and wide-band frequency spectrums is presented in Fig 2.
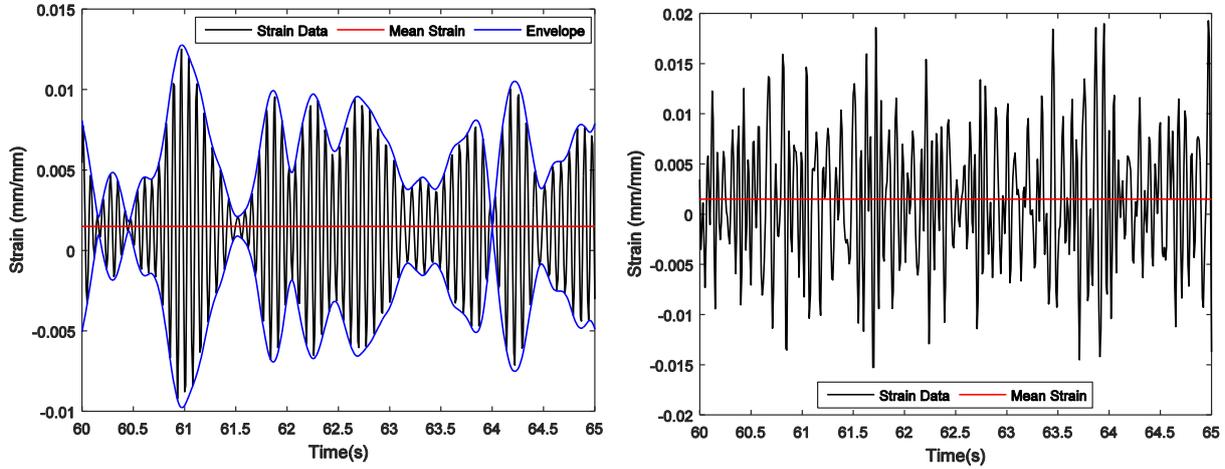
Figure 2 – On the left, a section of the history with narrow-band frequency spectrum (with envelope). On the right, a section of the history with wide-band frequency spectrum.

The PSD calculation, obtained by using Pwelch Function, is showed in Fig 3. (MathWorks Signal Processing Toolbox, 2018).
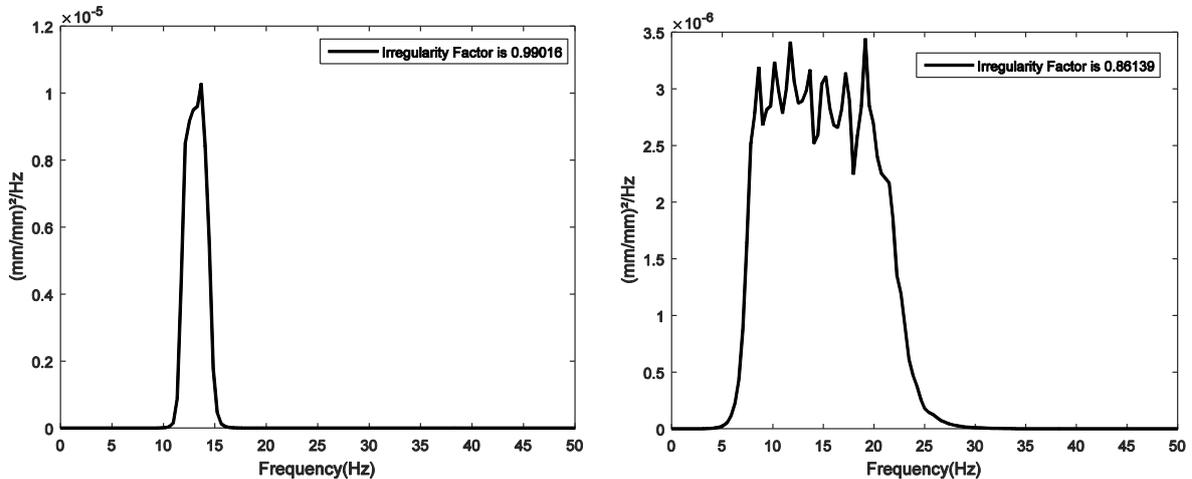

Figure 3 – On the left, the narrow-band PSD. On the right, the wide-band PSD.

As we can see in Fig. 3, the PSD for narrow-band signal has irregularity factor of 0.9916. For wide-band signal, the parameter moves away from 1.0 with value of 0.86139.

The PDF of strain amplitudes is shown in Fig. 4. The PDF of strain amplitude shows the probability of occurrence of a certain level of strain existing on the signal. The spectral methods results have a good agreement against Rainflow histogram prediction. Note that the x-axis is normalized by the maximum strain amplitude value.
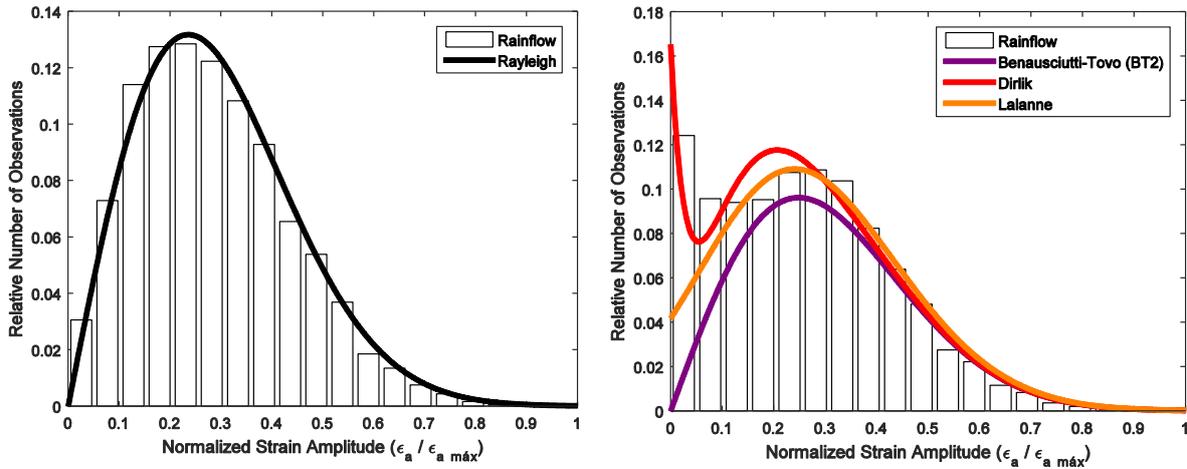

Figure 4 – On the left, the PDF for narrow-band signal. On the right, PDF for wide-band signal.

The strain distributions of cycles for narrow-band and wide-band spectrums are shown in Fig. 5. As we can see for the PDF of strain amplitude, the frequency domain methods count the strain cycles in a reliable way compared to Rainflow for both type of spectrums. The best description for wide-band signal is obtained with Dirlik method.

The blue highlighted region in Fig 5 shows the number of cycles with dominant plasticity. As we can see, the signal presents an elasto-plastic behavior with elastic dominance in the distribution of cycles for both narrow-band and wide-band spectrums.
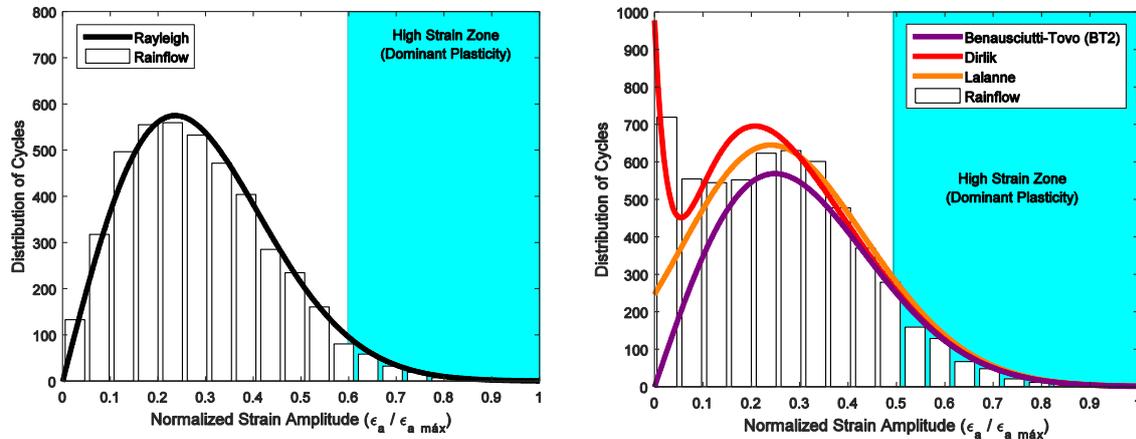

Figure 5 – On the left, the distribution of cycles for narrow-band signal. On the right, the distribution of cycles for wide-band signal.

The results for fatigue intensity damage for both types of spectrum is shown in Tab. 2 and Tab. 3. The frequency domain with mean stress correction approaches have good results compared with traditional time domain method. The spectral methods are an alternative against Rainflow. As we can see in Tab. 2 and Tab. 3, SWT is more conservative than Morrow predictions. Moreover, the results show that not considering the mean stress effect brings an expressive error in fatigue design.

Table 2 – Fatigue damage intensity (per second) for narrow-band strain signal

Rainflow			Rayleigh		
Zero-Mean	Morrow	SWT	Zero-Mean	Morrow	SWT
0.0170	0.0242	0.0282	0.0184	0.0259	0.0302

Table 3 – Fatigue damage intensity (per second) for wide-band strain signal

Rainflow			Benausciutti-Tovo (BT2)			Dirlik			Lalanne		
Zero Mean	Morrow	SWT	Zero Mean	Morrow	SWT	Zero Mean	Morrow	SWT	Zero Mean	Morrow	SWT
0.0358	0.0482	0.0543	0.0373	0.0488	0.0550	0.0391	0.0515	0.0582	0.0419	0.0548	0.0618

As we can see above, the spectral methods results are more conservative than Rainflow both for narrow and wide-band spectrums. Nevertheless, the spectral methods have good estimates and small errors against Rainflow, as we can see in Tab. 4 and Tab. 5 for both types of spectrums.

Table 4 – Solution error against Rainflow for narrow-band signal

	Zero-Mean	Morrow	SWT
Rayleigh	8.10%	7.32%	7.16%

Table 5 – Solution error against Rainflow for wide-band signal

	Zero-Mean	Morrow	SWT
Benausciutti-Tovo (BT2)	4.27%	1.16%	1.29%
Dirlik	8.03%	6.79%	7.19%%
Lalanne	17.06%	13.65%	13.82%

The results of Tab. 4 show that Rayleigh method is sufficient for analyzing narrow-band signals, with a maximum error of 7.32% for cases with corrected mean. According to Tab. 5, Benausciutti-Tovo (BT2) method is the best method for fatigue intensity damage of wide-band signal. We can notice that Lalanne approach is the worst for wide-band characteristics.

The results of fatigue damage intensity as a function of the strain amplitude bins for wide-band signal are presented in Fig. 6 and Fig. 7. If we sum the contribution of all the strain bins, we have the total damage intensity value for the signal. The zero mean and non-zero mean stress cases are shown below, presenting Dirlik, Benausciutti-Tovo (BT2) and Lalanne spectral methods against traditional Rainflow.
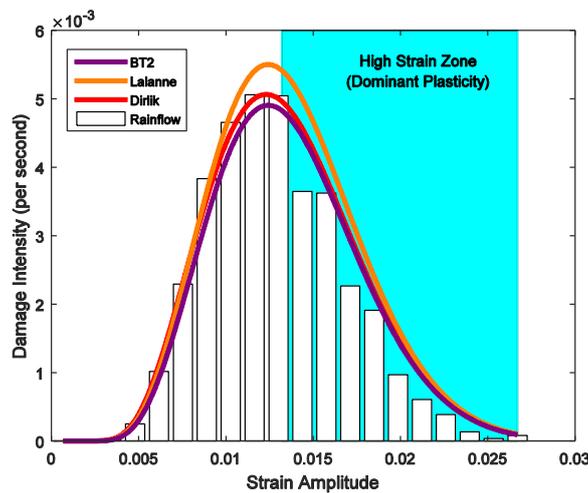

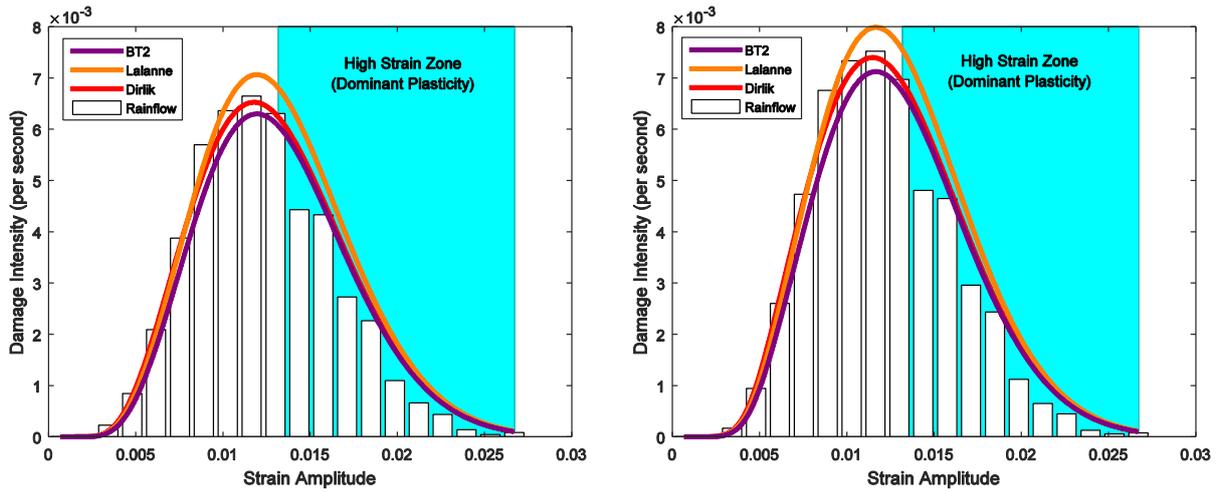**Figure 6 – Wide-band damage intensity for zero mean case**

Figure 7 – On the left, the wide-band damage intensity with Morrow mean stress correction. On the right, the wide-band damage intensity with SWT mean stress correction.

As we can see in figures above, Dirlik and BT2 approaches have a good agreement against Rainflow for all range of strain bins. We can verify that Lalanne method is very conservative compared to Rainflow. All the spectral results are more conservative for the blue highlighted region that displays the zone of high strain bins of the signal, where the plastic component is dominant. Nevertheless, all frequency domain methods have reasonable results against time domain one and can be an alternative to evaluate fatigue damage.

As the literature shows for time domain, the mean stress effect is also crucial for frequency domain since the results are non-conservative by neglecting the effect of the mean load, as shown by the comparison between Fig. 6 and Fig. 7.

As shown for wide-band signal, the results of fatigue damage intensity as a function of the strain amplitude bins are presented in Fig. 8 and Fig. 9 for narrow-band signal. Rayleigh method is the only one used for this case. Zero Mean Rayleigh method underestimated the damage by not considering the Morrow and SWT corrections.
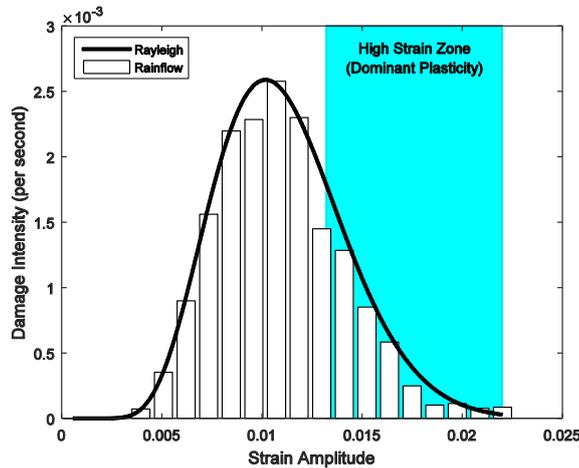

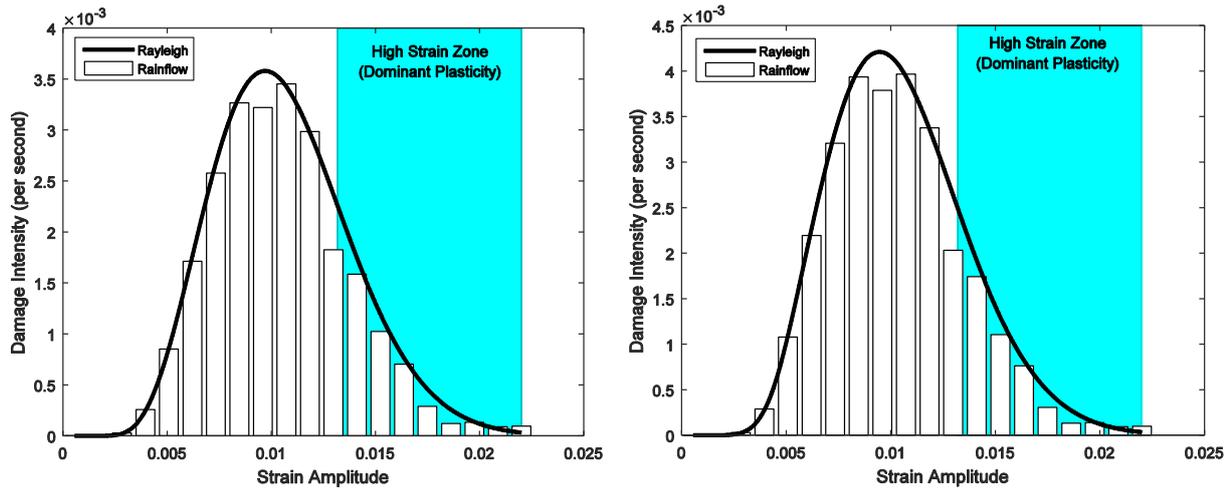**Figure 8 – Narrow-band damage intensity for zero mean case**

Figure 9 – On the left, the narrow-band damage intensity with Morrow mean stress correction. On the right, the narrow-band damage intensity with SWT mean stress correction.

Figure 10 presents the results for damage intensity for each strain amplitude bin. For each bin of Rainflow results, the top of each color bar (Zero-Mean, Morrow and SWT) presents its damage intensity value. The white bars represent the increase in damage intensity by correcting the mean load by Morrow approach. The magenta and green bars show the correcting increment by SWT method. As we can check clearly in Fig. 10, the mean stress correction is crucial in strain-life spectral method since despising its effect is non-conservative and underestimates the fatigue intensity damage.
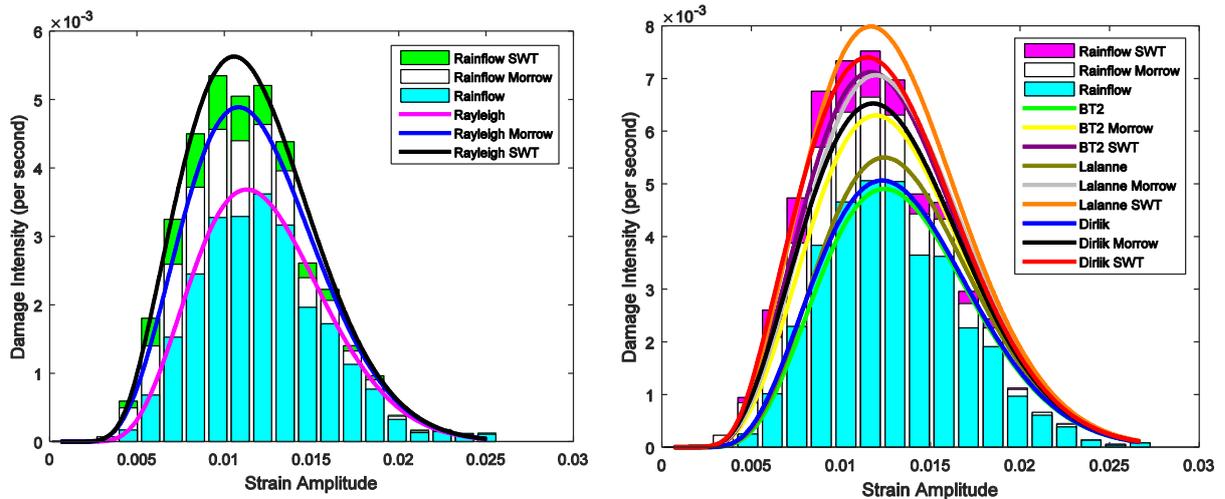

Figure 10 – On the left, the narrow-band damage intensity for each strain amplitude bin. On the right, the wide-band damage intensity for each strain amplitude bin.

CONCLUSIONS

Based on the literature research, there are no papers about spectral strain-based methodology that take in account the mean stress effect in fatigue damage. The proposed algorithm has shown that is very important to correct the mean stress effect in frequency domain, as shown in previous studies for time domain. Based on discussion of results, we can conclude that:

- The algorithm of fatigue analysis in frequency domain by strain-life approach with mean stress correction has been shown as an alternative to Traditional Rainflow time domain method,
- The spectral mean stress correction was validated for narrow-band and wide-band spectrums,
- Manson-Coffin and Ramberg-Osgood have presented acceptable results for frequency domain strain-life approach,
- The application of the mean stress correction algorithm for frequency domain method has brought good results against time domain approach, both for Elastic Morrow and SWT approaches,
- The spectral method results for nonzero mean stress were more conservative than Rainflow. Nevertheless, the frequency domain approach with mean load correction have brought acceptable results,
- The algorithm can be used for various spectral methods. In this paper, Rayleigh, Dirlik, Benausciutti-Tovo (BT2) and Lalanne methods have been applied to the proposed method,
- As showed for time domain, the zero mean spectral method underestimate the fatigue intensity damage,
- Benausciutti-Tovo (BT2) method have presented the best results for fatigue damage in wide-band characteristics. On the other hand, Lalanne method is the worst estimate for the same spectrum.

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