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Spectral fatigue analysis for Al 2024-T3 with nonzero mean stress

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Abstract: This paper consists of an overview of fatigue studies carried out on an Aluminum Alloy 2024-T3 in both time domain and frequency domain. A random signal of strain and stress are analyzed in time domain using usual Rainflow mothod and the damage is accumulated with the Palmgren-Miner rule, according to mean stress equations. The spectral domain analysis does not consider the prejudicial effect of the mean stress in metal life under fatigue, so it is used the correction factors for mean stresses developed by Goodman, Morrow, and Walker, changing the power spectral density and, thus, the damage calculated by the probability density functions postulated by Dirlik and Tovo and Benasciutti.

Keywords: fatigue damage, Rainflow, frequency domain, mean stress, power spectral density

INTRODUCTION

The effect of variable loading on materials causing mechanical fatigue is one of the main reasons of material failure. The study in the frequency domain, often called Spectral Fatigue, is a procedure to calculate fatigue damage and life for random vibrations, which are complicated to be analyzed within the time domain, through usual Rainflow.

The Rainflow cycle counting can be dispendious for long time signals because the whole set of data, during the whole test, must be used to compute the damage generated in the material due to the loading. The Spectral analysis is carried out in the frequency domain. For this reason, it's possible to calculate the damage through statistic properties of the Power Spectral Distribution (PSD) of the time signal. These properties are equal for the whole signal as well for smaller samples, due to the ergodic feature of the signal. (Newland, Newland, and Newland 1993)

The Fatigue damage is calculated by the Palmgren-Miner accumulation rule where the number of occurrences is taken from the Power Distribution Function (PDF) of Dirlik. However, this method does not include the effects of the mean stress, because the PSD is computed from the Fast Fourier Transform (FFT) of the strain signal in time. Thus, only the levels of amplitude are considered. The mean values are ignored, and they are not taken into account when calculating the damage.

It is well discussed the negative effect in fatigue for metals when loaded with a tractive mean stress. Ignoring the values of mean stress can underestimate the level of damage accumulated and overestimate the life time under fatigue. So, it's necessary to correct the amplitude values using existent correction factors. (Niesłony and Böhm 2015)

In this paper, the correction factors proposed by Goodman, Morrow and Walker are used to increase the amplitude signal. The Walker correction factor is used to represent the Smith-Watson-Topper factor. The calculated damage for the time-based analysis is compared to the frequency-based studies, in the next sections.

Time Domain Analysis

The time domain analysis is carried out using the Rainflow method. The Stress time signal was generated on MATLAB using the normal distributed random numbers and filtered by a Chebyshev filter type I to generate narrow band and broadband signals. The Rainflow cycle counting is performed by the Rainflow function on MATLAB. The result of the function is a Matrix with the numbers of occurrences for each cycle and its values of range of stress and mean stresses. The damage is calculated by the Palmgren-Miner where the life is calculated using the S-N curve. The properties of the aluminum alloy are given in Tab. 1. (Dowling 2013)

Table 1 – Material properties of Aluminum alloy 2024 T3.

Ultimate stress σ_u	Fatigue Strength σ_f'	True Facture Strength $\sigma_{\tilde{fb}}$	Fatigue exponent b
758 (MPa)	782 (MPa)	558 (MPa)	-0.118

The damage accumulation can be determined by Eq. 1.

$$D = \sum \frac{n_i}{N_{f_i}} \tag{1}$$

Where N_{f_i} is the fatigue life calculated by the S-N curve with the parameters from Tab. 1 using Eq. 2.

$$N_{f_i} = \frac{1}{2}\left(\frac{\sigma_f'}{\sigma_{ar_i}}\right)^{1/b} \tag{2}$$

Being σ_{ar_i} the alternating stress for each cycle counting. As the generated signal presents a nonzero mean stress, the alternating stress becomes the all reversed alternating stress, calculated by a mean stress correction. In this paper, three factors are discussed: Goodman, Morrow and Walker, this last being in the form of the Smith-Watson-Topper factor. Each factor is introduced as a constant that multiplies the σ_{a_i} and they are shown by Eq. 3, Eq. 4 and Eq. 5. (Niesłony and Böhm 2012)

$$K_{Morrow} = \frac{1}{1-\frac{\sigma_{m_i}}{\tilde{\sigma_{fb}}}} \tag{3}$$

$$K_{Goodman} = \frac{1}{1-\frac{\sigma_{m_i}}{\sigma_u}} \tag{4}$$

$$K_{walker} = \left(\frac{2}{1-\frac{\sigma_{min}}{\sigma_{max}}}\right)^{1-\gamma} \tag{5}$$

Equation 5 becomes the SWT correction factor using the value of $\gamma = 0.5$

Frequency Domain Analysis

The spectral analysis is carried out on a different domain, the frequency domain. For this reason, the time of the total signal is no longer important. This method is based on statistic properties extracted from the Power Spectral Density (PSD) of the original signal, without the mean stress, obtained by eliminating the global mean stress from the whole signal. The PSD in the paper is obtained using the Welch's method of modified periodograms. It's a method for the application of the fast Fourier transform algorithm to estimate power spectra.

The Welch's method involves sectioning the record, taking modified periodograms of these sections, and averaging these modified periodograms. In most of the cases, this method involves fewer computations than other methods. In addition, it involves the transformation of sequences which are shorter than the whole record. (Welch 1967)

From the PSD, the spectral analysis obtains the probabilistic properties in the form of its spectral moments. The general form for the i-th spectral moment m_i is given by: (Mršnik, Slavič, and Boltežar 2013)

$$m_i = \int_0^\infty f^i G(f) df \tag{6}$$

Where i represents the order of the moment, f is the frequency vector generated and $G(f)$ is the Welch's PSD. Still, Welch's method uses the Finite Fourier Transform of the zero-mean signal, which only takes into account the values of amplitude. As this procedure excludes the mean stress, the final damage calculated would not include the effect caused by these stresses, resulting in an underestimated fatigue damage.

To work around this situation, (Niesłony and Böhm, 2015) presented a simple yet reliable way to consider the mean stress effect, multiplying the PSD by the square of the mean stress correction factor, Eq. 7. This procedure changes the PSD function values and its statistical properties, altering the value of the damage.

$$G(\sigma(t) - \widehat{\sigma_m})_{corrected} = G(\sigma(t) - \widehat{\sigma_m}) * K^2 \tag{7}$$

Where G is the PSD function and the value of $\widehat{\sigma_m}$ is the global mean stress. The damage is calculated through the Probability Density Functions (PDF) of Dirlik and Tovo-Benasciutti. These Methods have been considered to be the best spectral methods.

$$D = E[p]T \int_0^\infty \frac{p(\sigma_a)}{N_f(\sigma_a)} d\sigma_a \tag{8}$$

The $E[p]$ is the total amount of peaks in the signal, that can be obtained as a function of the PSD's spectral moments from Eq. 6, T is the total time of the signal, N_f is the fatigue life calculated by the Eq. 2 and $p(\sigma_a)$ is the probability function. Dirlik and Tovo-Benasciutti's formulations can be seen in (Mršnik, Slavič, and Boltežar 2013)

NUMERICAL RESULTS

The numerical signals generated for the results presented here are created using random numbers in MATLAB that represents a stress signal in time. After, the random numbers were filtered on a Chebyshev filter type 1 to generate the broad band curve with peaks between $20Hz$ and $80Hz$, and a narrow band curve with peaks between $40Hz$ and $45Hz$. The wave generated has a global mean stress of $120MPa$.

The Rainflow method, performed by the algorithm in MATLAB, generates the histogram of counts, around means and ranges of stress. Both a sample from the time signal and the Rainflow histogram are presented in Fig. 1 and Fig. 2, respectively.

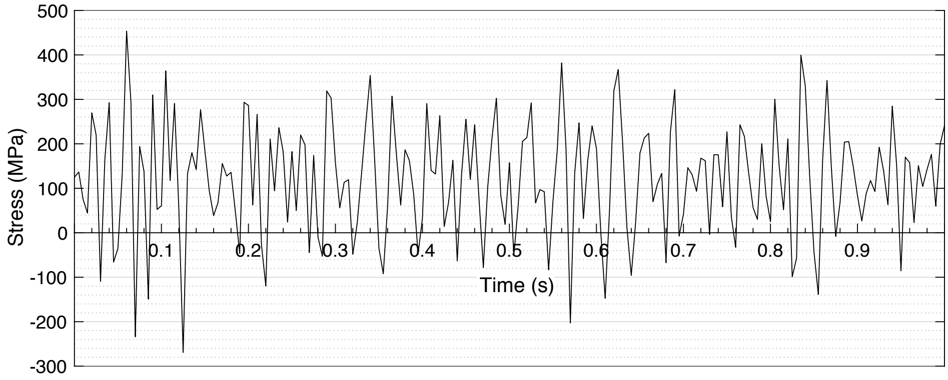


Figure 1 – Sample of the broad band stress-time signal.

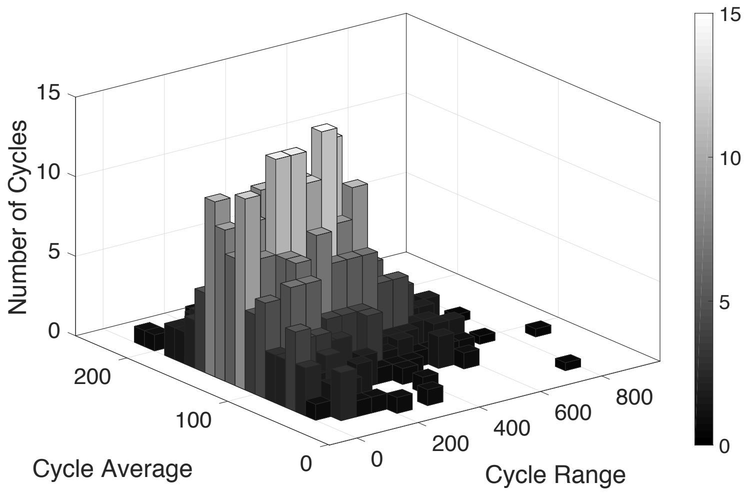


Figure 2 – Histogram of Rainflow cycle count.

With the cycles count, the calculation of the damage follows through Eq. 1, considering the material properties from Tab. 1 and the mean stress correction factors from Eq. 3, Eq. 4 and Eq. 5. The damages are presented ahead, in Tab. 2.

The first step within the spectral analysis is to obtain the PSD of the signal without mean stresses. For that reason, a global mean stress is obtained, and, after that, this value is subtracted from the signal. After, the PSD is calculated by means of fast Fourier transformation, FFT and the flat top windowing. To correct the PSD and to take into account the effect of mean stress, the PSD is multiplied by the correction factor, as shown in Eq. 7. The value of σ_m in Eq. 3 and Eq. 4 are replaced by $\widehat{\sigma_m}$ and the σ_{max} e σ_{min} in the Eq. 5 are taken as mean values from peaks and valleys of the original signal, respectively.

After, the Dirlik and Tovo-Benasciutti functions are calculated and the damage is accumulated by Eq. 8. The PDF functions are made for both PSD values, the one without the mean stress correction and the one corrected.

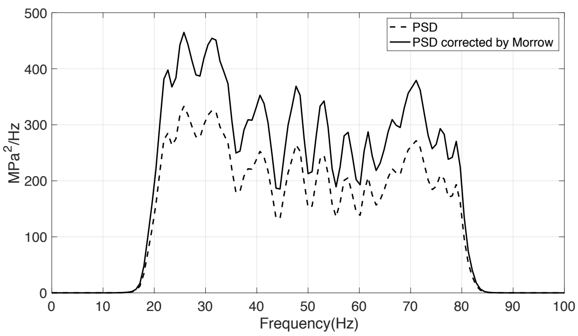


Figure 3 – The original Welch's PSD and the corrected PSD using Morrow mean stress relationship.

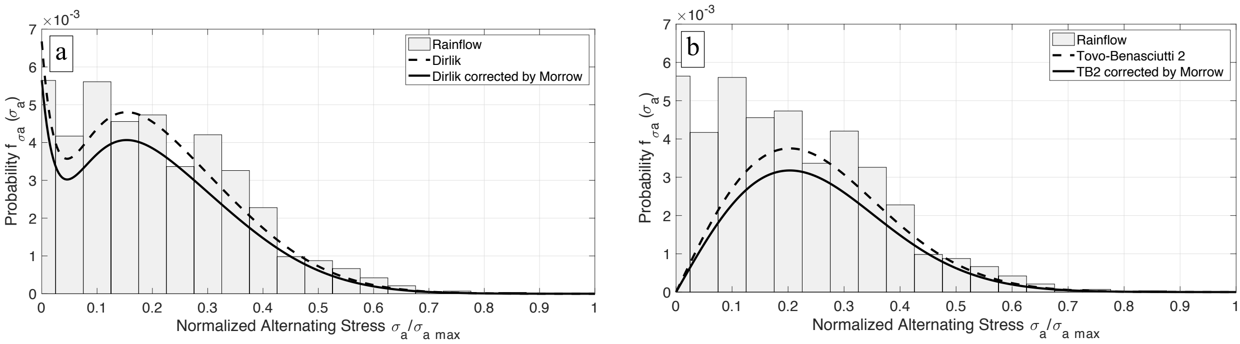


Figure 4a – Dirlik PDF for both PSDs and Rainflow Cycle probability density function. Figure 4b – Tovo-Benasciutti 2 PDF for both PSDs and Rainflow Cycle probability density function.

The PSD shows wide band characteristic, with the peaks between 20Hz and 80Hz, the same values of frequency set in the Chebyshev filter, with an irregularity factor of 0.82. The maximum value of the corrected PSD is $464{,}9 MPa^2/Hz$ at $25{,}78\ Hz$. The PDF shows a difference between the original PDF functions and the ones obtained from the statistics of the corrected PSD. The bars represent the probability of each specific bin of alternating stress. The alternating stress is normalized by the value of 5 times the standard deviation of the signal. The same figures can be created for the SWT and Goodman method, but they are not shown in this paper. The Tab. 2 presents the damage accumulated in Rainflow and the Spectral method for each correction factor, as well as the spectral value of the zero-mean signal, ignoring the mean stress.

Table 2 – Damage calculated for each method for a wide band signal.

Method	Goodman	Morrow	SWT
Rainflow (time domain)	0.2562	0.2461	0.1265
Dirlik (without correction)	0.0749	0.0749	0.0749
Dirlik (corrected)	0.3229	0.3075	1.5956
Tovo-Benasciutti 2 (without correction)	0.0629	0.0629	0.0629
Tovo-Benasciutti 2 (corrected)	0.2711	0.2582	1.3396

Table 2 shows that without the mean stress correction, the damage calculated on the spectral domain is smaller than the damage calculated on time domain's Rainflow cycle counting, which represents an underestimated level of fatigue damage, compared to this usual method. This result makes it clear that ignoring the mean stress from the damage calculus can be non-conservative and could result in an overestimated fatigue life, even more with a high value of mean stress. All the results show that the correction presented is conservative and the closest result between Rainflow and spectral method is obtained using Morrow factor with TB2 PDF equation. The difference is of 4,9%. On the other hand, for the SWT relationship, TB2 and Dirlik become extremely conservative.

Another signal was generated, a narrow band signal. The same normal random set of numbers were used but filtered with the Chebyshev Filter between $20Hz$ and $25Hz$. With this bandwidth, the signal generated is characterized as narrow band by the symmetry around the horizontal axis, being able to envelope the wave using two symmetric curves, as represented in Fig. 5a.

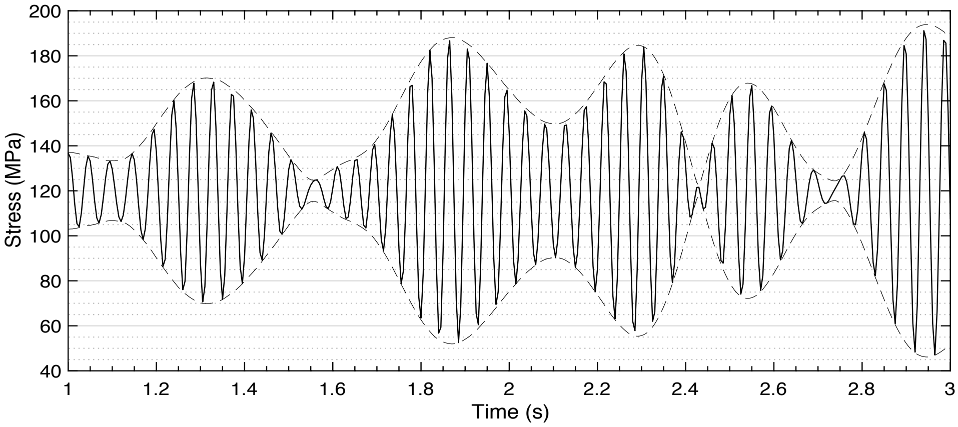


Figure 5 – Sample of the narrow band stress-time signal

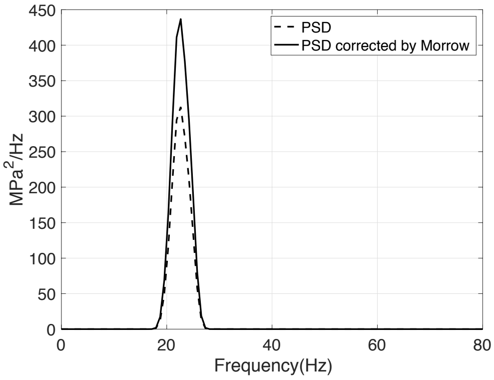
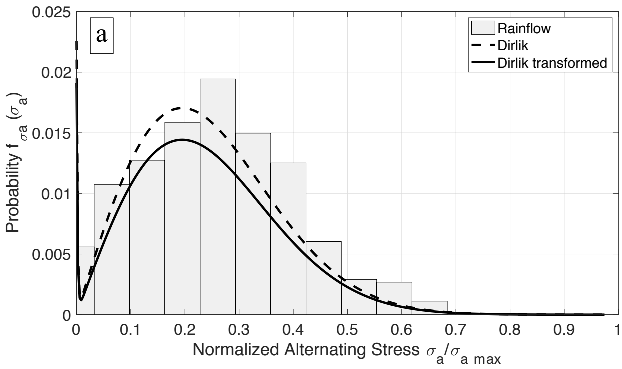


Figure 6 – Welch's PSD of the narrow band signal.



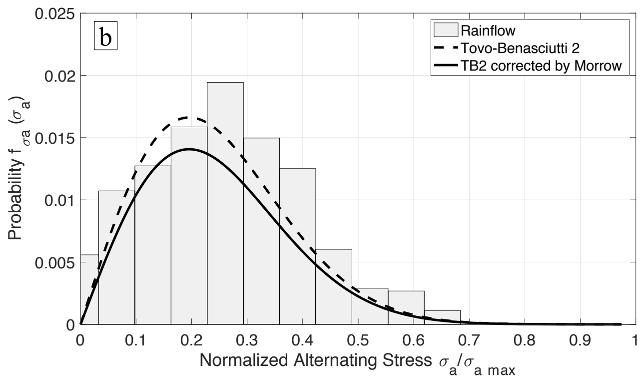


Figure 7a – Dirlik PDF for the narrow band signal. Figure 7b – Tovo-Benasciutti 2 PDF for the narrow band signal.

The maximum value of the PSD is $436.80\ MPA^2/Hz$ at $22.66Hz$, within the band width of $20Hz$ and $25Hz$ filtered. The damage accumulation data presented in Tab. 3 shows that for the same signal, a narrow band filtered history of load generates much less damage when it's compared to the wide brand results presented on Tab. 2.

Table 3 – Damage calculated for each method for a narrow band signal

Method	Goodman	Morrow	SWT
Rainflow (time domain)	2.525e-6	4.5615e-6	2.4253e-5
Dirlik (without correction)	1.2034e-6	1.2034e-6	1.2034e-6
Dirlik (corrected)	5.1872e-6	4.9401e-6	3.001e-4
Tovo-Benasciutti 2 (without correction)	1.1859e-6	1.1859e-6	1.1859e-6
Tovo-Benasciutti 2 (corrected)	5.1115e-6	4.8680e-6	2.9568e-4

From the results of damage obtained in this test, it is possible to conclude that the TB2 method is more accurate when compared to the time domain usual Rainflow method. The smallest difference is, again, using the Morrow relationship, 6.8%. And, again, both TB2 and Dirlik are extremely more conservative for SWT.

Considering the results for the SWT formulation, a different approach is followed. In spite of correcting the PSD with the factor K, the SWT formulation is applied direct in the damage accumulation in the PDF, Eq. 8. The $Nf(\sigma_a)$ becomes the $Nf(\sigma_{ar})$ as Eq. 9 shows.

$$D = E[p]T \int_0^{\infty} \frac{p(\sigma_a)}{N_f(\sigma_{ar})} d\sigma_a \qquad \text{Eq. 9}$$

$$\sigma_{ar} = \sqrt{\sigma_a^2 + \sigma_a \widehat{\sigma_m}} \qquad \text{Eq. 10}$$

This way, a global mean stress $\widehat{\sigma_m}$ is used to correct the level of stress in life estimation within the damage accumulation. The results of this method are shown in Tab. 4

Table 4 – Damage calculated for SWT using Eq. 9 and 10

Method	Broad Band Signal	Narrow Band Signal
Rainflow (time domain)	0.1265	2.4253e-5
Dirlik (corrected)	0.2623	3.3980e-5
Tovo-Benasciutti 2 (corrected)	0.5147	4.2043e-4

Figure 7 represents the relationship between the damage calculated by Rainflow and Dirlik, considering different levels of mean stresses. The graph shows that, even if the Dirlik method is more conservative for zero-mean stresses, the result is still conservative for low values of mean stresses. But for higher values, the Dirlik method doesn't change and it becomes non-conservative meanwhile the corrected Dirlik represents a more conservative result.

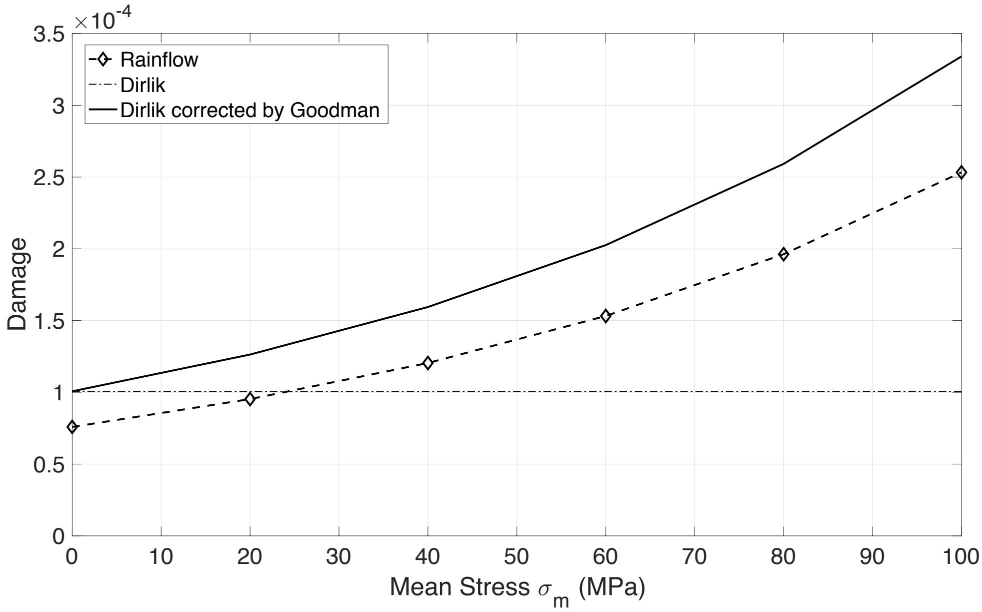


Figure 8 – Relationship between the damage by Rainflow in time domain and Dirlik in spectral domain.

CONCLUSIONS

The fatigue analysis within the time domain is the more traditional and common way to calculate the damage accumulation and estimate fatigue life. The traditional Rainflow cycle counting method is a reliable tool to count the cycles and evaluate the amount of damage accumulated in each cycle alongside the Palmgren-Miner rule.

The frequency domain comes as an option bringing a methodology that is faster to be implemented and calculated when compared to traditional Rainflow cycle count algorithms and yet reliable in results. But the limitation of not including the mean stress effects make it difficult to use in components where a high level of tractive mean stress is present.

The methodology proposed in this paper shows good results, specially considering Morrow equation and the PDF proposed by Tovo and Benasciutti, this last in agreement with (Mršnik, Slavič, and Boltežar 2013),. The differences between the damage calculated on the time domain and the spectral domain are 4.9% (Broad Band Signal) and 6.8% (Narrow Band). The Smith-Watson-Topper mean stress equation delivers a much more conservative result but when the correction is made directly in the damage accumulation, the results are closer to the ones found on Rainflow.

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