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Influence of the support condition perturbations in the process of parameter identification using the Boundary Element Method

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Abstract: In the process of model parameter identification, the structural boundary conditions play a major role. Depending on the assumed support conditions, the searched parameter can change drastically. This fact can become problematic when particular experiments are designed for validation purposes. The present study proposes a numerical formulation for better understanding the effects that slight support perturbations can cause. For this end, a model updating scheme is proposed as an optimization problem based on the Constitutive Relation Error theory. The forward mechanical problem relies on the Boundary Element Method. Then, a series of controlled perturbations is utilized to verify the detrimental effects on the cost function that guides the identification. The preliminary results show two kind of perturbations with distinct impacts. One of them does not affect the identification process. The other one can degenerate completely the convergence process.

Keywords: Boundary Element Updating Model, Constitutive Relation Error, Boundary Conditions, Optimization

INTRODUCTION

The engineering community has long been concerned with quality of model predictions. The idea that has been receiving attention is improving the numerical predictions by computational model updating based on experimental measures. In this case, the model has finite dimension and it is assumed to be well known. The governing parameters are assumed to be doubtful and susceptible to be identifiable according to some criterion. The Model Updating has been developed since the middle of World War II in the airplane industry (Natke, 1988). The developments are mainly driven by domain methods such as Finite Element Method (FEM) (Azzouna, Feissel and Villon, 2015).

The first appearance of the direct Boundary Element Method (BEM) used for Model Updating is related to beam vibration using measured Frequency Response Function (FFR), instead of modal input data (Santos, Campos and Neto, 2000). The benchmark utilized is the Friswell and Mottershead's beam where the estimated parameters are the flexural rigidity, the translational stiffness and rotational stiffness of a flexible joint (Friswell and Mottershead, 1995). It is interesting to remark that the dynamical behavior of frame structures can be accurately modeled by BEM (Neto, Barretto and Pavanello, 2000). No more progress in this specific field has been communicated since then.

Concerning identification methods, the Constitutive Relation Error (CRE) approach is an important numerical tool. At its birth, it was proposed for quantifying the divergencies between numerical approximations and the exact solutions (Ladeveze and Leguillon, 1983). Posteriorly, it evolved for quantification of the stiffness and the mass matrices of linear dynamical structural systems basing on modal tests (Ladeveze, Nedjar and Reynier, 1994). The CRE problems are generally formulated as inverse forms, which also needs to include all available information. In this sense the augmented version (Modified Constitutive Relation Error - MCRE), an additional term acts as regularization parameter. It broadens the formulation in order to include the evidence measures, instead of making additional assumptions (Guchhait and Banerjee, 2016). The CRE technique has permitted the proposition of improved predictive algorithms for pressure levels decreasing the needs for prototyping which represent expressive saving costs (Decouvreur et al, 2004).

The present study proposes a numerical procedure for investigating the effect of variability at semi-rigid support conditions on the interface parameter identification. The numerical strategy is built from the coupling of the BEM and CRE theory.

PROBLEM FORMULATION

The present study focuses on two dimensional bodies occupying a domain Ω with boundary denoted by $\partial\Omega$. The strain tensor is locally defined as the symmetric part of the gradient displacement to assure only small perturbations. The scenario is presented in Figure 1. In any instant of time, t, the set of external actions can be outlined in the synthetic way:

A surface displacement field $\underline{u_i}$ on a portion Γ_u, a surface force density field $\underline{t_i}$ applied on Γ_t, a volume force density $\underline{b_i}$ on the domain Ω. Let Γ_j represent the interior elastic interface.

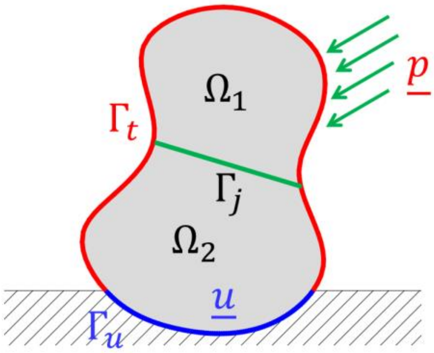

Figure 1 – Solid configuration

The problem is posed using the CRE theory. In this case, the boundary value problem is defined according to two categories. The *reliable fields* are the equations that must strictly be respected. The *unreliable fields* encompass all equations that are source of doubt and in general depend on the parameters to be identified. These categories are chosen by the designer. In the present case, the following distinction is made:

Reliable fields

Let U^{ad} be the displacement functional space preserving the necessary conditions of regularity such that:

$$U^{ad} = \left\{\forall y \in \partial\Omega, \forall t \in [0,T]: u_i|_{\Gamma_u} = \underline{u_i}, \dot{u}_i|_{t_0} = \dot{u}_{i_0}\right\} \quad (1)$$

Let S^{ad} be the stress functional space preserving the quasi-static admissible conditions:

$$S^{ad} = \left\{ \begin{array}{c} \forall t \in [0,T], \forall y \in \partial\Omega : \\ v_i(y) = \int_{\Gamma} \left[u_{ij}^*(x,y)s_j(x) - p_{ij}^*(x,y)v_j(x)\right] d\Gamma + \int_{\Omega} u_{ij}^*(x,y)b_j(x)d\Omega, \\ \forall y \in \partial\Omega : \sigma_{ik}n_k|_{\Gamma} = \underline{t_i} \end{array} \right\} \quad (2)$$

In this integral form, u_{ij}^* and p_{ij}^* are fundamental Kelvin solutions. Dotted quantities mean time derivative.

Unreliable fields

The force among the structural faces Γ_a and Γ_b (each side of the interface Γ_j) is calculated by the following expression:

$$p_i|_{\Gamma_a} = -p_j|_{\Gamma_b} = R_{ij}(u_j|_{\Gamma_a} - u_j|_{\Gamma_b}) \quad (3)$$

In expression 3, R_{ij} stands for the Hookean operator and it is dependent on a set of independent parameters $\{p\}$. The interior constitutive relation is considered to be reliable and it is already included in the admissible space S^{ad}. In principle, the kinematical and statically admissible fields are completely independent of each other. For this reason, it is necessary to have a meaningful quantity able to measure the quality of a given vector pair $(u,\sigma) \in U^{ad} \times S^{ad}$. This measure is the CRE norm having the following properties:

$$\begin{gathered} J: U^{ad} \times S^{ad} \longmapsto \mathbb{R} \\ J \geq 0 \\ J(u,\sigma) = 0 \Leftrightarrow (u,\sigma) \textit{ is the exact solution of the problem} \end{gathered} \quad (4)$$

The BEM problems are defined in terms of boundary fields. In this case the CRE norm will be defined such as the interface behaves independently from the remaining solid. For practical purposes, one can specify other material models than Hookean without changing the core formulation. Let $\breve{u}_k$ be the experimental measure vector, r is a real parameter between 0 and 1, and N is some real value to keep the same scale order compared to the first term. Let the functional norm be defined as follows;

$$J^*(u,t) = \frac{1}{2}\int_{\Gamma_j} \left(t_i - R_{ik}(u_k|_{\Gamma_a} - u_k|_{\Gamma_b})\right) \cdot [R_{ik}]^{-1}\left(t_i - R_{ik}(u_k|_{\Gamma_a} - u_k|_{\Gamma_b})\right) d\Gamma + \frac{r}{1-r}\frac{1}{N} \|u_k - \breve{u}_k\| \quad (5)$$

The problem of identifying the parameters can be written as follows:

Problem 1. *Find the set of parameters* $\{p\}$ *such as:*

$$\{p\} = \underset{p \in P}{\operatorname{argmin}} \left[\min_{(u_i|_\Gamma, t_i|_\Gamma) \in U^{ad} \times S^{ad}} J^*(u,t) \right] \tag{6}$$

APPLICATION: TWO-BAR STRUCTURE

Consider the structure composed by two independent solids showed in Figure 2. The material parameters are $E_1 = 100\ N/mm^2$, $E_2 = 100\ N/mm^2$, $v_1 = 0$, $v_2 = 0$. The geometric characteristics are $L = 100\ mm$, $H = 50\ mm$. The idea is identifying the joint parameter represented by a spring along the axis direction considering two distinct support conditions. The first case consists in maintaining the left side completely rigid (constrained). In the second case, a variable-rigidity is assumed at left support (semi-constrained). The surface load is constant $\bar{P} = 10\ N/mm^2$.

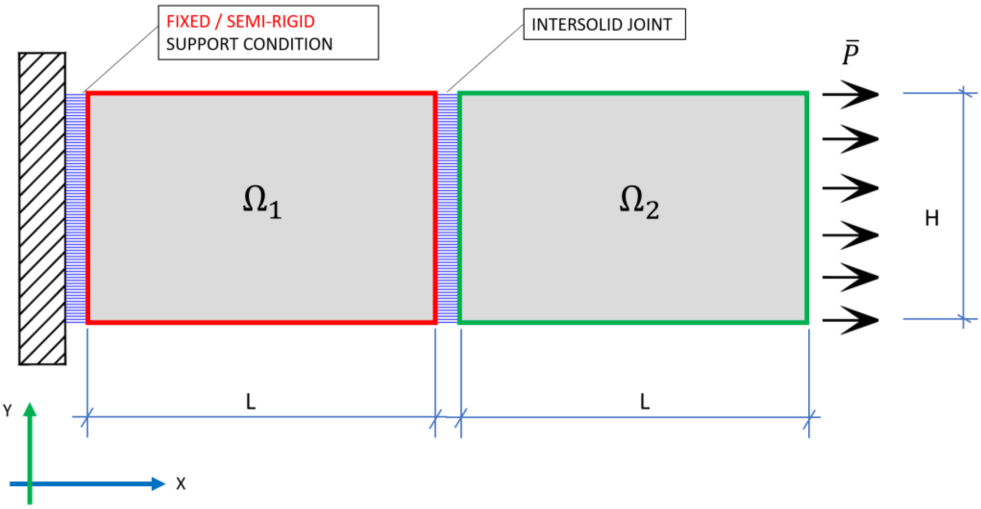

Figure 2 – Two-solid plane structure

Constrained case

The displacement field can be written analytically when the left support is fixed. The reference joint parameter is $k = 2N/mm$. The analytical solution is given by equation (7) which is utilized for synthetic generation of the experimental data.

$$u(x) = \begin{cases} \dfrac{\bar{P}x}{E_1} & ; 0 < x < L \\ \dfrac{\bar{P}(L+x)}{E_1} + \dfrac{\bar{P}}{k} & ; L < x < 2L \end{cases} \tag{7}$$

The solution of *Problem 1* is illustrated in Figure 3. This curve shows the evolution of the MCRE norm (Equation 5) as function of the stiffness parameter between the two bars. Each point of this curve is solution of a quadratic minimization problem. The local minimum corresponds to the wanted parameter value. Note how the MCRE varies with the design parameter (only one in this case). The formulation manifests quadratic-like behavior, which is very interesting from the minimization algorithmic point of view. No special treatment is necessary in the assembling of the classic H and G. No instabilities are observed in the solution of the BEM linear system of equations.

From Figure 3, it is possible to note that the reference parameter is correctly identified when the left side is perfectly rigid. Is that still valid when some flexibility is present at this support ending? To answer this question the left edge will be now transformed into semi-rigid.

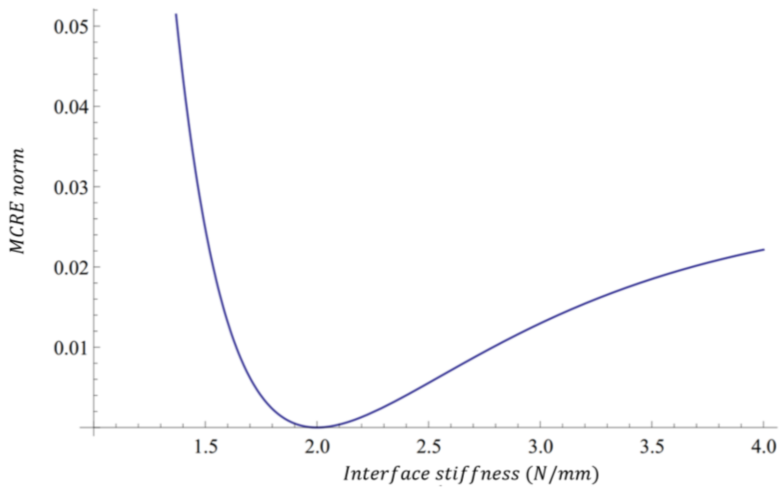

Figure 3 – Interface stiffness identification along x direction considering rigid support at the left side of domain Ω_1

Semi-constrained case

The left ending edge is now assumed flexible. Two variables are used for defining the support stiffness. The first one is the $BCXn$, which stands for Boundary Condition flexibility at X direction of value n. The second variable is the $BCYn$ meaning Boundary Condition flexibility at Y direction of value n. The idea is producing a series of controlled perturbation on the support stiffness and check if the reference parameter is still correctly identified. The following steps are followed:

Influence along the horizontal direction

1- Maintain $BCYn = 2N/mm$.

2- Make $BCXn$ vary through the set {*2N/mm*, *10N/mm*, *50N/mm*, *100N/mm*}.

3- For each pair (*BCY2*, *BCXn*) solve *Problem 1*.

Influence along the vertical direction

4- Maintain $BCXn = 2N/mm$.

5- Make $BCYn$ vary through the set {*2N/mm*, *10N/mm*, *50N/mm*, *100N/mm*}.

6- For each pair (*BCYn*, *BCX2*) solve *Problem 1*.

It is possible to remark the influence of the semi-rigid left support condition on the process of identification. The key information is obtained from the different aspects of the convergence history as function of the effective semi-rigid value adopted at the left support condition. The particular numerical investigation suggests the existence of two types of perturbation. The first type of perturbation is named LD (Figure 5) and acts on the direction of stiffness identification. In this case, the variation of the flexibility at the support conditions change drastically the results predicted by the algorithm. On the other hand, the second type of perturbation is named LI (Figure 6) and it is acting perpendicular to the direction of identification. The fluctuation of the parameter values along this direction does not impact the interface parameter identification.

Note that the formulation still maintains its quadratic-like behavior even in the presence of stiffness parameters with different orders of magnitude. This is an important aspect because it does not imply on loss of stability by the BEM matrices resulting from *Problem 1*. The discretization adopted in present example is composed of discontinuous second order elements and it is presented in Figure 4.

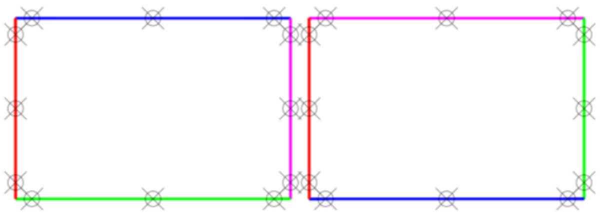

Figure 4 – Adopted Boundary Element mesh

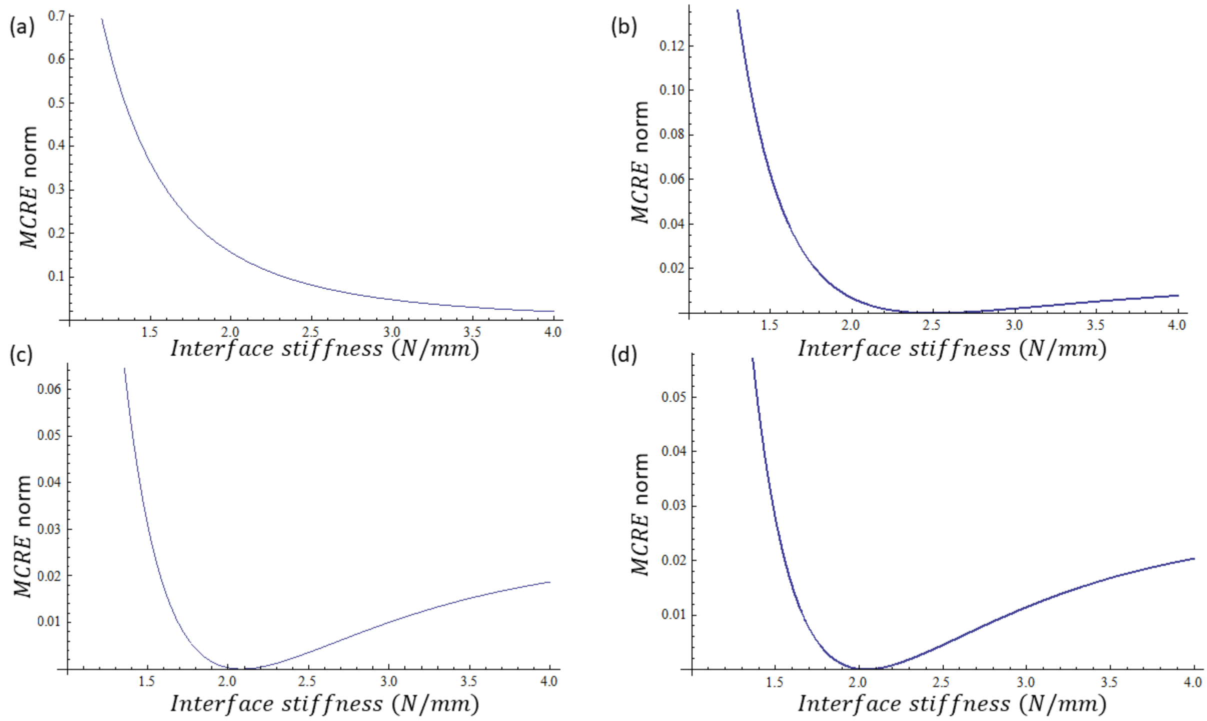

Figure 5 – Interface stiffness identification along x direction. (a) BCX2 (b) BCX10 (c) BCX50 (d) BCX100

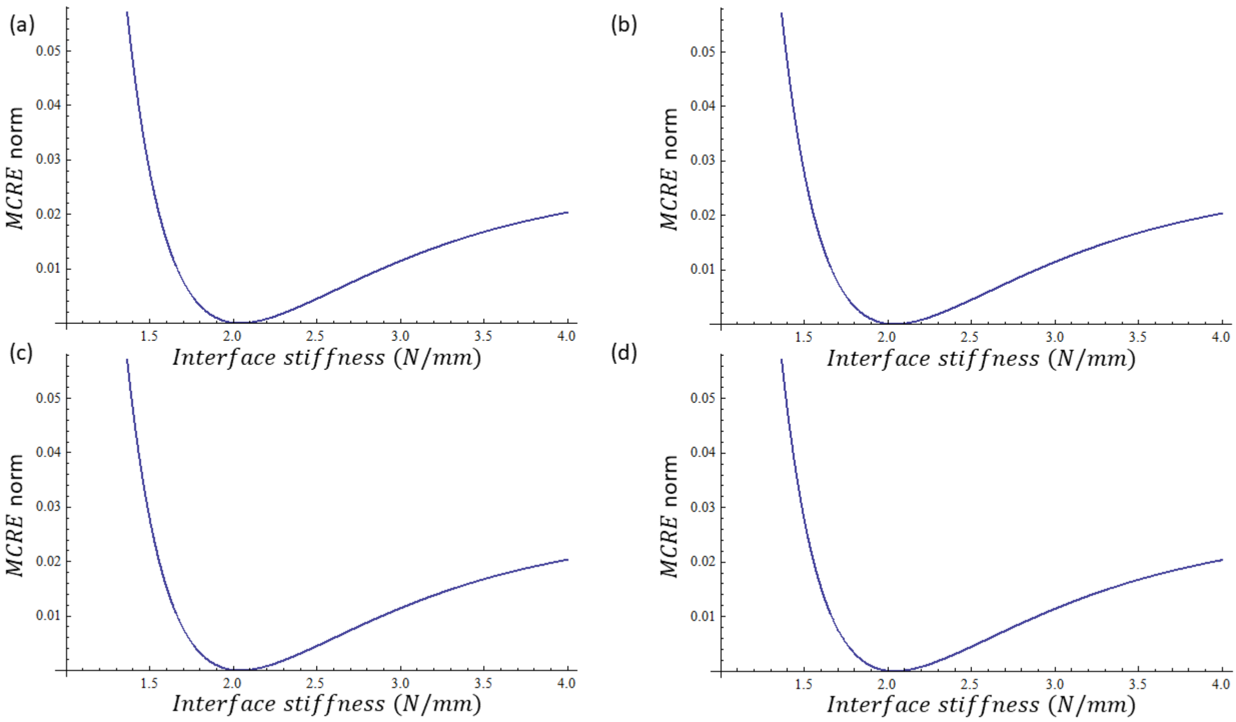

Figure 6 – Interface stiffness identification along x direction. (a) BCY2 (b) BCY10 (c) BCY50 (d) BCY100

An interesting information is obtained from the observation of how the degrees of freedom are connected. The present formulation provides an isolation of the interconnection behavior from the remaining deformable solids. This fact is illustrated using the matrix plot showed in Figure 7. Each pair of square block matrix corresponds to a sub-domain of the structure. Note that they are positioned after the degree of freedom 108. The square matrix situated between the position (0,0) to (108,108) corresponds to the interconnection behavior and it is shown in Figure 8. This feature is useful for implementing different types of interconnection behavior such as, sliding, plasticity or viscosity.

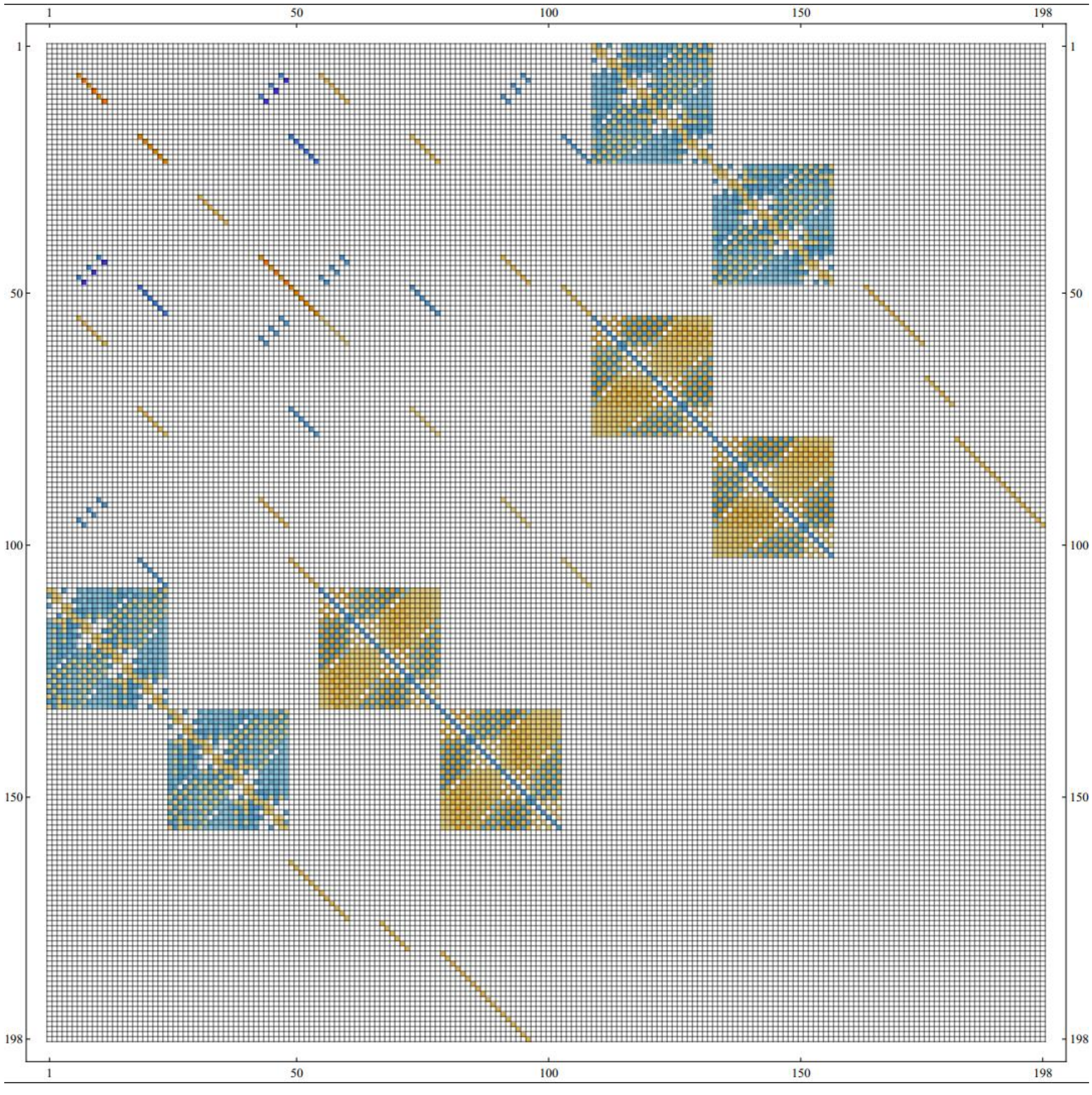

Figure 7 – Matrix plot of the global assembled system

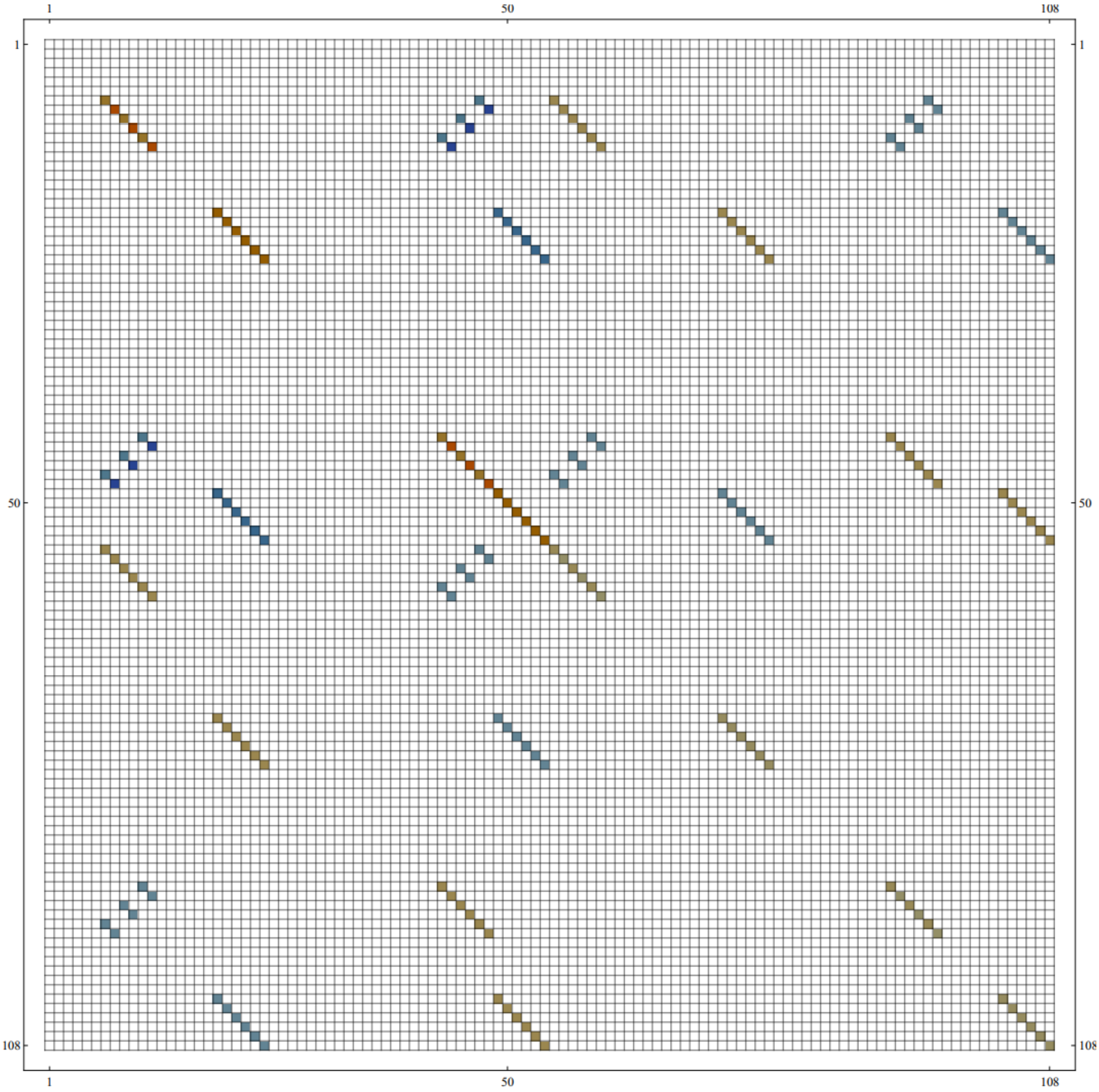

Figure 8 – Matrix plot of the interconnection assembled system

CONCLUDING REMARKS

Some interesting insights can be drawn from the particular numerical experiment presented in this study. The first of them is showing the existence of two types of boundary condition perturbations. One of them does not affect the identification of the interconnection parameters named LI. The second type, named LD, affects drastically the convergence process. This information is important for predicting possible sources of errors while doing experiments in real-world structures for parameter identification. It may be used for designing numerical procedures less sensitive to this type of perturbation because in real applications the clear separation between LI and LD is still not possible. This information is also useful for building numerical filters to improve numerical convergence.

The beneficial use of the BEM is highlighted by the clear distinction between interconnection behavior and solid behavior showed through isolated block matrix data structures. It permits to concentrate effort on the investigation of interface phenomena implementing more complex material behavior without changing the classical way of building and solving the BEM equations.

The perspectives of the present study cover the inclusion of quasi-static behavior and inertial effects. These sophistications may ameliorate the predictability of the present methodology to be tested in real-life structures.

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