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Rotor-blade vibration attenuation using a periodic $\mathscr{H}_\infty$ controller

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Abstract: The increasing demand for high speed and performance in rotor-blade systems have required the application of advanced control design techniques capable of considering the periodic nature of the system dynamics, since standard linear time invariant controllers usually cannot ensure closed-loop stability or acceptable performance. This paper proposes a periodic time varying full-state feedback $\mathscr{H}_\infty$ controller to attenuate the vibration of a rotor-blade system. Open-loop and closed-loop analyses are performed. Numerical results demonstrate the effectiveness of the proposed approach.

Keywords: Vibration control, rotor dynamics, periodic systems, $\mathscr{H}_\infty$ Controller

1 INTRODUCTION

Active vibration control of rotor-blade systems have been intensively used to improve system performance, suppress vibrations, and prolong machinery lifetime (Firoozian and Stanway, 1988; Szász and Flowers, 2000; Khulief, 2001; Christensen and Santos, 2005), since passive damping methods are not able to attain higher efficiency (Rao, 1983; Dimarogonas et al., 1983). However, most of the control design strategy found in the literature assume the system is linear and time invariant. However, even at constant speed, the system dynamics is periodic and the available results can not, in general, guarantee system performance and robustness (Sinha and Joseph, 1994; Jakobsen et al., 2013; Camino and Santos, 2018).

Some of the main results for periodic system are presented in Colaneri (2000a), where the notion of $\mathscr{H}_\infty$ norm is investigate. The $\mathscr{H}_\infty$ control design problem for linear time-varying systems (in continuous time) is formulated in Ravi et al. (1991), where it has been show that the solution of the dynamic output feedback controller exists if, and only if, two Riccati differential equations have a stabilizing positive definite solution. Synthesis conditions for the full-state feedback $\mathscr{H}_\infty$ controller for periodic system in continuous time can be found in Colaneri (2000b).

This paper presents the design of a periodic time varying full-state feedback $\mathscr{H}_\infty$ controller to attenuate the vibration of rotor-blade systems where the rotor lateral motion and the blade flexible motion are dynamically coupled. Numerical results shows the effectiveness of the proposed technique.

2 THE ROTOR-BLADE SYSTEM

Figure 1 shows a schematic representation of the rotor-blade system, borrowed from Christensen (2004) and Christensen and Santos (2005), in which the blades rotate in a flexible suspended rigid body, performing only lateral planar motion in the (x,y)-directions. Rotor angular displacement and gyroscopic effects are assumed to be negligible.

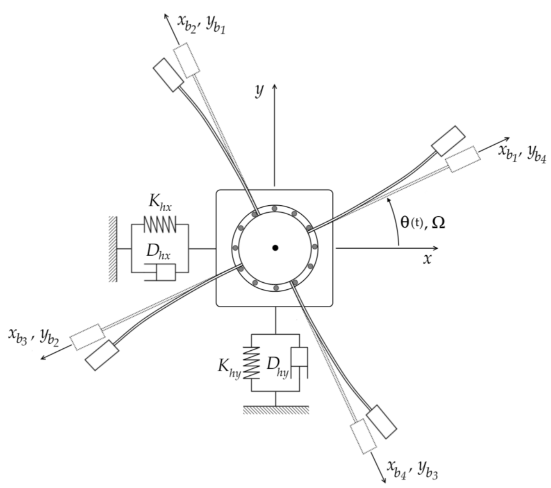

Figure 1 – Rotor-blade system.

The equation of motion for the coupled rotor-blade system, derived using Lagrange formulation, is a linear time-varying (LTV) differential equation, that depends on the rotor angular position $\theta(t)$ and on the angular velocity $\Omega(t)$, given by

$$M(\theta)\ddot{q}(t)+D(\theta,\Omega)\dot{q}(t)+S(\theta,\Omega)q(t)=Q\bar{u}(t)+p(t)+w(t) \tag{1}$$

with M the mass matrix, D the damping matrix, S the stiffness matrix, Q the input matrix, $q(t)$ the generalized coordinate, $\bar{u}(t)$ the control input, $p(t)$ the unbalanced, and $w(t)$ the exogenous disturbance. Considering the system rotates at a constant speed Ω, *i.e.*, $\theta(t)=\theta_0+\Omega t$, the equation of motion (1) becomes periodic:

$$M(t)\ddot{q}(t)+D(t)\dot{q}(t)+S(t)q(t)=Q(t)\bar{u}(t)+p(t)+w(t) \tag{2}$$

with the system matrices $M(t)=M(t+T)$, $D(t)=D(t+T)$, $S(t)=S(t+T)$, and $Q(t)=Q(t+T)$ periodic, with period $T=2\pi/\Omega$. Moreover, it is assumed that an appropriate feedforward control law $u_f(t)$, for the control input $\bar{u}(t)=u_f(t)+u(t)$, can be designed *a priori* to cancel out the internal conservative forces $p(t)$ due to unbalances (Jakobsen et al., 2013, and references therein). Thereby, the external force $p(t)$ is assumed to be zero.

By defining the state vector

$$x(t)=\begin{bmatrix}q^T(t) & \dot{q}^T(t)\end{bmatrix}^T$$

the equation of motion (2) can be written in the state-space form

$$\dot{x}(t)=A(t)x(t)+B_u(t)u(t)+B_w w(t)$$

where

$$A(t)=\begin{bmatrix}0 & I\\ -M^{-1}(t)S(t) & -M^{-1}(t)D(t)\end{bmatrix},\qquad B_u(t)=\begin{bmatrix}0\\ -M^{-1}(t)Q(t)\end{bmatrix},\qquad B_w(t)=\begin{bmatrix}0\\ M^{-1}(t)\end{bmatrix} \tag{3}$$

The displacement vector $q(t)$ has six generalized coordinates that account for the hub motion, in the (x,y)-directions, and the tip deflection of each one of the four blades. Thus, the system has 6 degrees of freedom and the state $x(t)$ has 12 entries. Moreover, the system has six actuators that can independently control all six degrees of freedom. Two actuators are located in the hub, providing forces in the (x,y)-directions, and the other four are located in each one of the blades.

3 PERIODIC FULL-STATE FEEDBACK $\mathscr{H}_\infty$ CONTROLLER

This section presents the periodic full-state feedback $\mathscr{H}_\infty$ controller. However, before presenting the control design, it is instructive to present the formulas for the upper bound γ on the $\mathscr{H}_\infty$ norm of a periodic linear time varying system. The characterization of the $\mathscr{H}_\infty$ norm using a Riccati differential equation, as show in Colaneri (2000a), is given in the next lemma.

Lemma 1 *Consider the periodic system H given by*

$$H=\begin{cases}\dot{x}=A(t)x+B(t)u\\ z=C(t)x+D(t)u\end{cases}$$

Let $\gamma>0$ be a given positive scalar. Then, the following statements are equivalent:

(i) $A(t)$ is stable and $\|H\|_\infty<\gamma$

(ii) $\gamma^2I-D'(t)D(t)>0$ and there exists a T-periodic stabilizing positive semidefinite solution to

$$-\dot{P}(t)=A'(t)P(t)+P(t)A(t)+C'(t)C(t)+(P(t)B(t)+C'(t)D(t))(\gamma^2I-D'(t)D(t))^{-1}(B'(t)P(t)+D'(t)C(t))$$

such that

$$A(t)+B(t)(\gamma^2I-D'(t)D(t))^{-1}(B'(t)P(t)+D'(t)C(t))$$

is asymptotically stable.

(iii) $\gamma^2I-D(t)D'(t)>0$ and there exists a T-periodic stabilizing positive semidefinite solution to

$$\dot{Q}(t)=A(t)Q(t)+Q(t)A'(t)+B(t)B'(t)+(Q(t)C'(t)+B(t)D'(t))(\gamma^2I-D(t)D'(t))^{-1}(C(t)Q(t)+D(t)B'(t))$$

such that

$$A(t)+(Q(t)C'(t)+B(t)D'(t))(\gamma^2I-D(t)D'(t))^{-1}C(t)$$

is asymptotically stable.

It is now possible to present the equations that provides a periodic full-state feedback controller $K(t)$ that guarantees an upper-bound γ on the $\mathscr{H}_\infty$ norm, as shown in Colaneri (2000b). The next Lemma 2 gives the Riccati differential equation that provides such controller. A bisection algorithm is used to approach the optimal $\mathscr{H}_\infty$ control design.

Lemma 2 *Consider the T-periodic system given by*

$$\dot{x}(t) = A(t)x(t) + B_u(t)u(t) + B_w(t)w(t)$$
$$z(t) = C_z(t)x(t) + D_{zu}(t)u(t)$$

where the pair $(A(t), B_u(t))$ is stabilizable, the pair $(A(t), C_z(t))$ is detectable, and the orthogonality conditions $C_z'(t)D_{zu}(t) = 0$ and $D_{zu}'(t)D_{zu}(t) = I$ are satisfied. Then, there exists a T-periodic controller $K(t)$ such that the closed-loop system, using the control law $u(t) = K(t)x(t)$, is stable and its $\mathscr{H}_\infty$ norm is bounded by γ if, and only if, there exists a T-periodic positive semidefinite solution X to the periodic Riccati differential equation

$$\dot{X}(t) + A'(t)X(t) + X(t)A(t) + X(t)\left(B_w(t)B_w'(t)\gamma^{-2} - B_u(t)B_u'(t)\right)X(t) + C_z'(t)C_z(t) = 0$$

such that

$$A(t) + (B_w(t)B_w'(t)\gamma^{-2} - B_u(t)B_u'(t))X(t)$$

is asymptotically stable. The set of all stabilizing full-state feedback controllers $K(t)$ is parameterized as follows:

$$K(t) = -B_u'(t)X(t) + \mathscr{Q}_{op}(\tau)\left(w(t) - \gamma^{-2}B_w'(t)X(t)\right)$$

where $\mathscr{Q}_{op}(\tau)$ is the input-output operator associated with a T-periodic system $\mathscr{Q}$ that satisfies $\|\mathscr{Q}\|_\infty < \gamma$ with $\mathscr{Q}$ stable. Setting $\mathscr{Q} = 0$, the full-state feedback gain is given by $K(t) = -B_u'(t)X(t)$.

4 NUMERICAL RESULTS

For the rotor-blade system (3), the angular velocity is set at $\Omega = 300$ [rpm]. The system matrices $A(t)$, $B_u(t)$, $B_w(t)$, are taken from Camino and Santos (2018), and the matrices $C_z(t)$ and $D_{zu}(t)$, for the performance channel, are given by

$$C_z(t) = 10\begin{bmatrix} N \\ 0_{6,12} \end{bmatrix}, \quad N = \text{blkdiag}(0_2, I_4, 0_2, I_4), \quad D_{zu}(t) = \begin{bmatrix} 0_{12,6} \\ I_6 \end{bmatrix}$$

Using a standard bisection algorithm, the periodic full-state feedback (optimal) $\mathscr{H}_\infty$ controller $K(t)$ is computed by solving the associated periodic Riccati differential equation given in Lemma 2. The control gain $K(t)$ is a time-varying matrix of size 6×12, which have up to 72 distinct (time-varying) elements, so it is impractical to plot all of them. Figure 2 shows some of the time-varying elements of $K(t)$: the entries $K_{1,j}$ for $j = 3, \ldots, 6$ (in solid line) and $K_{1,j} \times 10^2$ for $j = 9, \ldots, 12$ (in dashed line).

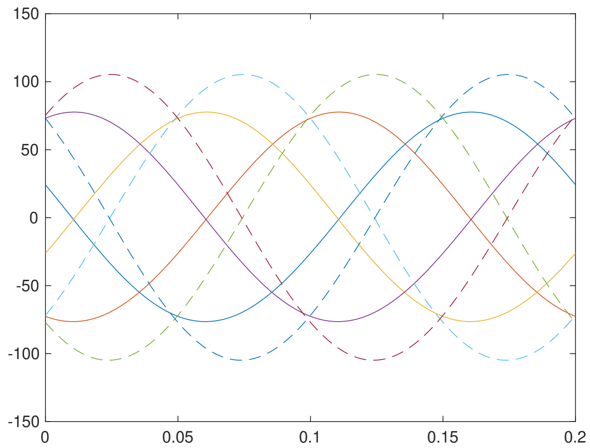

Figure 2 – Some of the time-varying elements of matrix $K(t)$: the entries $K_{1,j}$ for $j = 3, \ldots, 6$ (in solid line) and $K_{1,j} \times 10^2$ for $j = 9, \ldots, 12$ (in dashed line).

As shown in Camino and Santos (2018), the open-loop system is asymptotically stable, although lightly damped mainly due to the eigenvalues near the imaginary axis. Using Lemma 1, the $\mathscr{H}_\infty$ norm of the open-loop system is found to be 12.5055. The $\mathscr{H}_\infty$ norm of closed-loop system with the $\mathscr{H}_\infty$ controller $K(t)$ is 3.2307, providing a significant reduction.

A time domain simulation is performed using a Gaussian white noise input $w(t)$ with zero mean and covariance matrix equals to $100I$. The initial condition is $x(0) = 0$. Figure 3 and Figure 4 show, respectively, the hub position in the x-direction and the blade 1 tip deflection, for the open-loop (dashed black line) and the closed-loop (solid red line) systems. The vibration significantly decreased, as was expected. The other blades have similar behavior as blade 1.

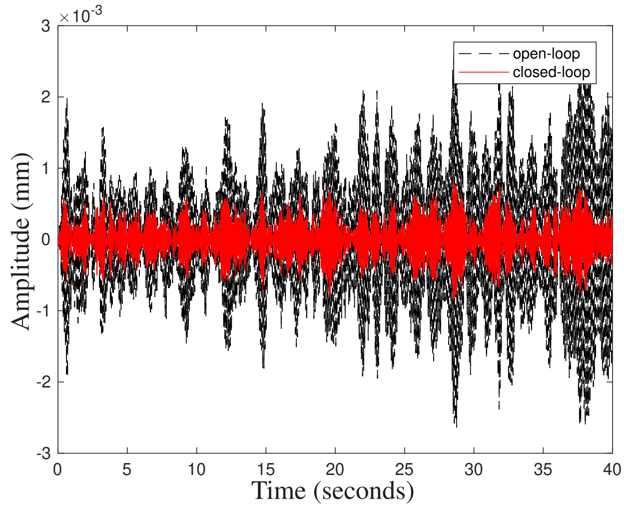

Figure 3 – Hub position in the x-direction.

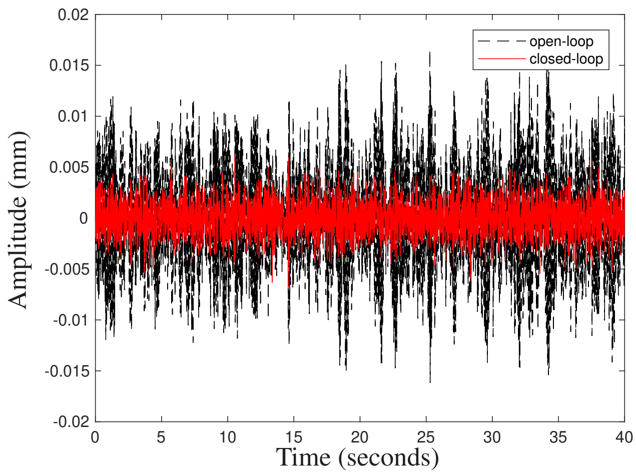

Figure 4 – Blade 1 tip deflection.

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