MECSOL 2019 - Proceedings of the 7th International Symposium on Solid Mechanics,
V. Tita, A. Ferreira, M.L. Ribeiro, A. Vieira and A.T. Beck (Editors), ABCM, Sao Carlos, SP, Brazil, April 15th to 17th, 2019

Influence of Failure on the Effective Properties of Laminate Composites via Finite Element Analyses

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Abstract: This work aims to investigate the influence of failure on the effective properties of composites by the use of Finite Element Analyses in a commercial package. A 0-90 periodically laminated composite, modeled as a representative volume element (RVE), is chosen for the analysis. The homogenization is performed by analyzing six load cases with prescribed loads, or displacements, and boundary conditions and by the use of a theorem of averages. Initially, three analyses are performed. The first one with an intact material and prescribed loads. The second one with fails and prescribed loads. The third one with a crack in the 90 layer and prescribed displacements. The results show a consistency for the first and third cases and a physical inconsistency for the second case. The results show that the method must be use with some caution. An alternative approach is proposed, the use of asymptotic homogenization within a commercial finite element package. This alternative shows very consistent results by comparison with the commonly homogenization approaches found into literature. And, there is a discussion about the size of the RVE as well as about the application of adequate boundary conditions. The main difficulties and advantages of all approaches are pointed all.

Keywords: homogenization, effective properties, laminate composites, finite element analyses

INTRODUCTION

Composite materials are a reality in a wide range of engineering applications, mainly in the aerospace industry. Despite its excellent specific properties, when compared with more traditional engineering materials, many challenges exist in composite analysis. In the past decades both academia and industry spent a lot of time and resources on trying to understand and improve its mechanical behavior. On the other hand, the inherent complexity associated to this kind of material renders it a topic of hard studies still.

One of the challenges faced by the researches is the correct prediction of the mechanical properties in composites and the research group in which the present work is inserted (Group of Aeronautical Structures) has been made a great effort in this issue. Several works have been made within this scope, such the works of Rodriguez-Ramos et al. (2013), Brito-Santana et al (2018) and Sartorato (2018) for the determination of the elastic properties of composites and the works of Medeiros (2012), Medeiros et al. (2015), Tita et al. (2016) and Brito-Santana et al. (2016) for the determination of the effective properties of piezoelectric composites. Despite all the effort, only few works were made considering cracks in plies of the composites, its effect on the effective elastic properties and the difficulties found in the numerical analyses.

Thus, this work aims to investigate the the effective properties of laminate composites when cracks are present in the composite layers. The load cases are applied in two distinct ways: as load control and as displacement control. The composite is modelled as a representative volume element (RVE) and all simulations are performed via finite element package ABAQUS™ alongside Python scripts for the post processing of the data. An alternative approach, not so commonly used, is also proposed, the use of asymptotic homogenization via ABAQUS™. The method requires the use of an ABAQUS™ subroutine, written in Fortran, and the use of Python scripts for pre- and post-processing. There is a discussion concerning the mesh convergence and the size of the RVE, as well as a brief discussion concerning the difficulties found in the approaches.

THEORETICAL BACKGOUND

The homogenization procedure follows the work of Otero et al. (2015). It consists in the solution of six independent load cases, chosen as the simplest possible: three uniaxial pure tractions in each of the principal directions and three pure shear in each of the principal planes. Figure 1 shows the nomenclature used in the procedure and table 1 shows a summary of the loads and boundary conditions used in all of the six load cases. The stresses can be either applied as a distributed load or as a prescribed displacement.

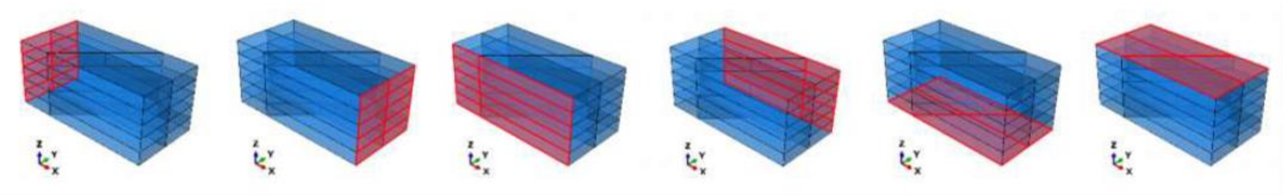

Figure 1 – Nomenclature of the faces. -x, +x, -y, +y, -z, +z, respectively.

Table 1 – Load cases and boundary conditions for the homogenization procedure.

Load Case	Face x+	Face x-	Face y+	Face y-	Face z+	Face z-
1	$u_x = 0$	$+\sigma_x$	-	-	-	-
2	-	-	$u_y = 0$	$+\sigma_y$	-	-
3	-	-	-	-	$u_z = 0$	$+\sigma_z$
4	$u_x, u_y, u_z = 0$	$+\tau_{xy}$	$+\tau_{yx}$	$-\tau_{yx}$	-	-
5	$u_x, u_y, u_z = 0$	$+\tau_{xz}$	Material Continuity		-	-
6	Material Continuity		$u_x, u_y, u_z = 0$	$+\tau_{yz}$	-	-

The continuity relation, also known as parallelism relation, is given by

$$u_i^{+j} - u_i^{-j} = c_i^j \quad (i, j = x, y, z), \tag{1}$$

where c_i^j is a constant and u_i^{+j} and u_i^{-j} are displacements at opposite faces of the RVE. This condition ensures the parallelism between opposite faces of the RVE (Jim et al. (2009)).

For each load case, the volumetric average of the stress field,

$$\bar{\sigma}_{ij} = \sum_{p=1}^{n} \frac{1}{V^p} \frac{\sum_{q=1}^{ng} \sigma_{ij}^{pq}}{ng} \tag{2}$$

and the average of the strain field

$$\bar{\varepsilon}_{ij} = \sum_{p=1}^{n} \frac{1}{V^p} \frac{\sum_{q=1}^{ng} \varepsilon_{ij}^{pq}}{ng}, \tag{3}$$

where n is the number of elements in the finite element mesh, ng is the number of Gauss Points used in the numerical integration and V is the volume of the RVE are obtained. The homogenized fourth order elasticity tensor is related to the average stresses and strains by

$$\begin{Bmatrix} \bar{\sigma}_{11} \\ \bar{\sigma}_{22} \\ \bar{\sigma}_{33} \\ \bar{\sigma}_{23} \\ \bar{\sigma}_{13} \\ \bar{\sigma}_{12} \end{Bmatrix} = \begin{pmatrix} C^H_{1111} & C^H_{1122} & C^H_{1133} & C^H_{1123} & C^H_{1113} & C^H_{1112} \\ & C^H_{2222} & C^H_{2233} & C^H_{2223} & C^H_{2213} & C^H_{2212} \\ & & C^H_{3333} & C^H_{3323} & C^H_{3313} & C^H_{3312} \\ & & & C^H_{2323} & C^H_{2313} & C^H_{2312} \\ & & & & C^H_{1313} & C^H_{1312} \\ sym. & & & & & C^H_{1212} \end{pmatrix} \begin{Bmatrix} \bar{\varepsilon}_{11} \\ \bar{\varepsilon}_{22} \\ \bar{\varepsilon}_{33} \\ \bar{\varepsilon}_{23} \\ \bar{\varepsilon}_{13} \\ \bar{\varepsilon}_{12} \end{Bmatrix}. \quad (4)$$

By analyzing the six load cases, equation (4) provides 36 linear equations. The coefficients of the homogenized tensor are obtained by a least square method.

The commercial finite element package ABAQUS™ is used to solve the equilibrium problem. Python scripts are written in order to both automate the procedure and perform the post processing, that is, the calculation of the effective elastic coefficients.

PRELIMINARY RESULTS AND DISCUSSION

A 0-90 periodically laminated composite and three cases are chosen for the preliminary analyses. The first one is an intact composite and in the second and third ones, a crack in the 90 ply is considered. For the intact model and for first damaged model the shear load cases are performed by using applied forces on the faces of the RVE, while in the second damaged model, displacement fields are applied. The representative volume elements (RVEs) of the intact and damaged models are shown in figure 1. Table 2 shows the mechanical properties used in the simulations.

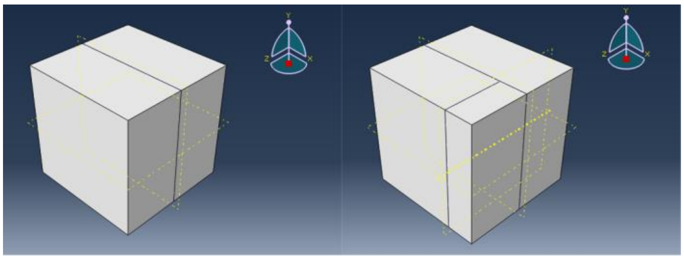

Figure 2 – RVE models representing a 0-90 periodically laminated composite. Intact (left) and damaged (right).

Table 2 – Mechanical properties

E_1 [GPa]	E_2 [GPa]	ν_{12}	ν_{23}	G_{12} [GPa]	G_{23} [GPa]
127.0	10.3	0.34	0.306	5.4	3.05

Figure 3 shows the displacement field of the RVE (qualitative) for load case 5 for, respectively, the intact composite (case 1), the damaged composite with load control (case 2) and the damaged composite with displacement control (case 3). Table 3 shows the numerical results for the normal and shear components of the homogenized fourth order elasticity tensor.

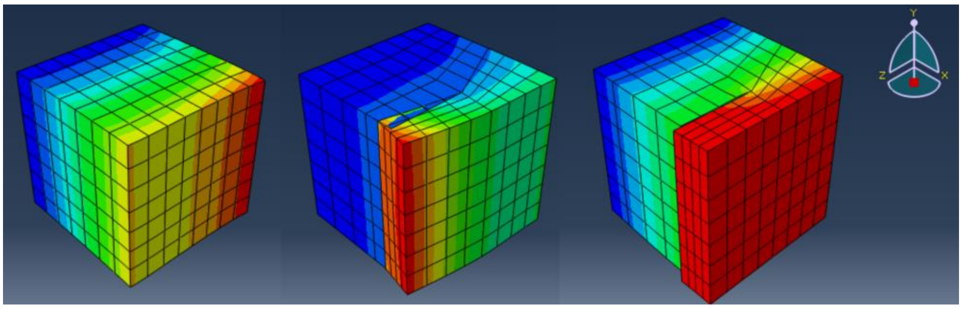

Figure 3 – Displacement field for the load case 5. Intact material (case 1), damaged material with load control (case 2) and damaged material with displacement control (case 3), respectively.

Table 3 – Numerical results. All results in Pa.

	C^H_{1111}	C^H_{2222}	C^H_{3333}	C^H_{2323}	C^H_{1313}	C^H_{1212}
Case 1	7.347×10^{10}	7.401×10^{10}	1.566×10^{10}	1.566×10^{10}	4.689×10^{09}	5.421×10^{09}
Case 2	1.868×10^{10}	7.114×10^{10}	1.338×10^{10}	4.864×10^{09}	3.930×10^{09}	5.438×10^{09}
Case 3	1.873×10^{10}	7.169×10^{10}	1.350×10^{10}	4.230×10^{09}	3.691×10^{09}	5.667×10^{09}

The analysis of the intact model was made in order to verify the consistency of the procedure. There were no issues found during this analysis. It is important to stress that, at a first glimpse, the numerical results are not the main concern of this work.

By analyzing the damaged models, one can see that there is a big difference between the displacement fields of cases 2 and 3. Yet, for the damaged case with load control, one can see a physical inconsistency in the model, once there is a superposition in the elements of the finite element mesh. Therefore, this approach only can be used with a strict caution concerning the adopted models. The superposition of elements has occurred also in other load cases. On the other hand, for the case in which a displacement control was applied, there was no superposition of elements in the RVE. Thus, for this particular case, the approach with displacement control is more consistent. However, it is unlikely that an ill-behavior does not occur in any shape of crack. Thus, this issue must be further investigated.

In the numerical results, one can see that the normal elastic coefficients for both damaged models are similar. On the other hand, in the shear components, the difference between the damaged models are significant. Although the similarity of the numerical results between cases 2 and 3, the homogenization procedure must be carried out with caution. The similarity on the normal components is due to the ease of applying both load and displacement conditions for the homogenization procedure, even when the crack is present in the model. Conversely, the application of the shear conditions is not that easy.

ALTERNATIVE APPROACH

As an alternative approach, the two scale asymptotic homogenization, implemented in a commercial finite element package, is proposed. Despite being widely used in well established in literature, its use within commercial software is not widespread. Although the full implementation of an in-house Asymptotic homogenization software is quite straightforward, the use of commercial finite packages bring some advantages, such as pre- and post-processing facilities. The principal disadvantage in using commercial software codes is the difficulty in access data stored within the software.

The asymptotic homogenization method is well established in literature and, as shown by Sanchez-Palencia (1986), Hassani and Hinton (1998a), Guedes and Kikuchi (1990) and Muñoz-Rojas et al (2011), states that the homogenized fourth order elasticity tensor of a three dimensional media can be found by

$$C^H_{ijkl} = \frac{1}{|Y|}\int_Y \left(\mathrm{C}_{ijkl} - C_{ijpq}\frac{\partial \chi^{kl}_{ij}}{\partial y_q} \right) \mathrm{dY} \tag{5}$$

where Y is the dimension vector of the RVE, $\mathbf{y}$ is the coordinate vector and $\boldsymbol{\chi}^{kl}$ is the periodic solution of the equilibrium problem given by

$$\int_Y C_{ijpq}\frac{\partial \chi^{kl}_p}{\partial y_q}\frac{\partial v_i}{\partial y_j} dY = \int_Y C_{ijkl}\frac{\partial v_i}{\partial y_j} dY \tag{6}$$

where $\mathbf{v}$ is a vector of virtual displacements. In order to obtain all of 21 independent constants of the homogenized tensor, the equilibrium problem must be solved for six load cases kl. Now, in a finite element method context, Fang et al (2004) and Hassani and Hinton (1998b) show that equations (5) and (6) can be written, respectively, by

$$C^H_{ijkl} = \frac{1}{|Y|}\int_Y \left(C_{ijkl} - \mathbf{c}^T_{ij}\mathbf{B}\boldsymbol{\chi}^{kl} \right) dY \tag{7}$$

and

$$\int_Y \mathbf{B}^T\mathbf{C}\mathbf{B}dY\boldsymbol{\chi}^{kl} = \int_Y \mathbf{B}^T\mathbf{c}_{kl}dY \tag{8}$$

where $\mathbf{B}$ is the strain-displacement matrix and $\boldsymbol{\chi}^{kl}$ is the nodal solution of the equilibrium problem. Equation (8) is very similar to a standard finite element stiffness equation, and can be written as

$$\mathbf{K}\boldsymbol{\chi}^{kl} = \mathbf{f}_{kl} \tag{9}$$

where $\mathbf{K}$ is the global stiffness matrix and $\mathbf{f}_{kl}$ represents a load vector. It is interesting to notice that, as shown in Christoff (2016), the homogenized load vectors induce a unitary strain in the kl case analyzed. This result is used in the implementation of the method within a commercial finite element package.

The numerical implementation of this kind of problem in commercial finite element packages is not a straightforward task. With a closer look in equation (7) and in equation (8), one can see that the access to the strain-displacement matrix is crucial in the procedure in order to assemble the homogenized load vector and, subsequently, to obtain the homogenized fourth order elasticity tensor. However, ABAQUS™ does not allow the user to access this information, thus, the homogenization theory alongside other ABAQUS™ facilities must be investigated.

In ABAQUS™, it is allowed to implement user subroutines (Simulia, 2011) to perform several different tasks and, as shown by Yang and Fish (2008), the subroutine UEXPAN can be used in order to enforce unitary strains to the RVE. As aforementioned, the homogenized load vectors enforce unitary strains in the load case analyzed, thus overcoming the problem in accessing the strain-displacement matrices.

The UEXPAN subroutine can be used to define incremental thermal strains as functions of temperature, predefined field variables and state variables (Simulia, 2013). For the present case, for each load case needed to obtain the effective constitutive tensor, a pre-defined field is used to enforce a unitary strain in this particular direction. For instance, for the load case $kl = 11$, a unitary strain is enforced in the direction 11, while in all other directions the strains remain constant and equal to zero. Likewise for the load cases $kl = 22$, $kl = 33$, $kl = 23$, $kl = 13$ and $kl = 12$.

Another point is the periodicity consideration. In opposite faces of RVE, the displacement fields must be the same in order to enforce the periodicity constraints, as shown by Hollister and Kikuchi (1994). Nevertheless, ABAQUS™ does not have any pre-implemented facility that allows this periodicity constraint to be enforced. Thus, a Python code is written in order to enforce these constraints. It is used the Multiple Point Constraint (MPC) (Simulia, 2011) in order to enforce opposite nodes of the mesh to have the same displacement field thus enforcing the periodicity constraint.

The post-processing, ie, the evaluation of the fourth order elasticity tensor, equation (7), is also made by Python scripting. ABAQUS™ provides the solution of the six linear systems, and the displacement and stress data are used in order to evaluate the tensor.

COMPARISSONS AND DISCUSSION

Figures 4 to 9 show the influence of all stress components on the RVE on the asymptotic homogenization. In these figures one can see how each one of the load cases affect the effective properties of the media. In figure 4, it is seen that the load case $kl = 11$ highly influences the 11 component of the effective tensor. This load case also affects the 22, and 33, components of the homogenized tensor, but in a smaller order of magnitude. Also, it can be seen that the load case $kl = 11$ does not affect any of the shear components. This is expected since the material is orthotropic and, in this kind of material, there is no coupling between normal and shear components. The same analysis can be made for the load cases $kl = 22$ and $kl = 33$. Here it is interesting to notice that the load cases $kl = 11$ and $kl = 22$ induces symmetric stress fields. Since the plies are reinforced in directions 1 and 2, it is expected that the homogenized components in these directions to be the same. Also, this symmetry of the stress fields shows the consistency of the load cases applied.

In figures 7 to 9, it can be seen that the load cases $kl = 23$, $kl = 13$ and $kl = 12$ are affected only by the respective stresses, ie, the load case $kl = 12$ only induces stresses in the 12 direction and so on. Here, it is also can be notice the consistency of the load cases since, for this kind of material, one shear component cannot affect the other two.

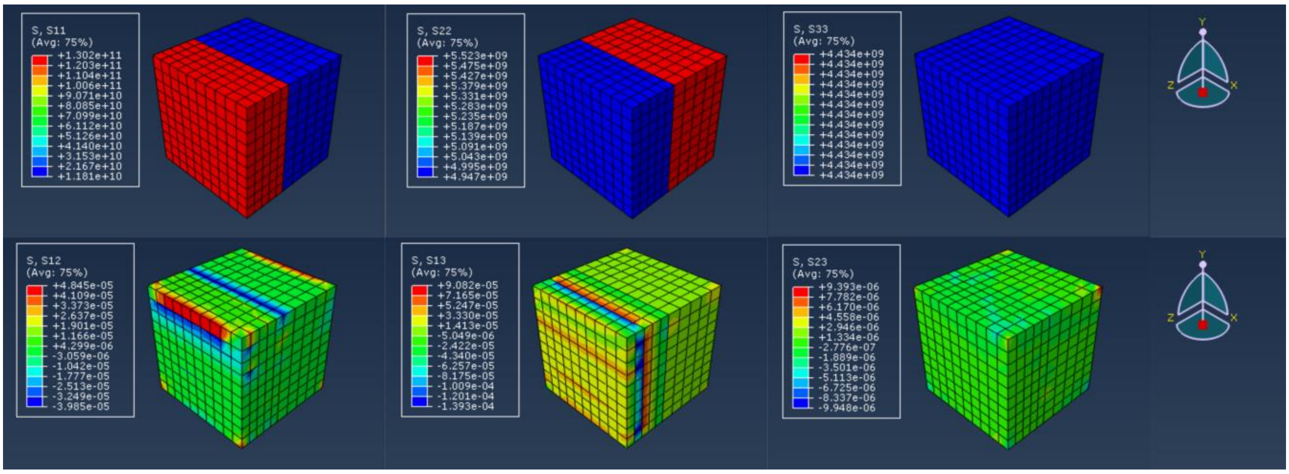

Figure 4 – Stress influence for the load case kl=11. Components 11, 22, 33, 12, 13 and 23, respectively.

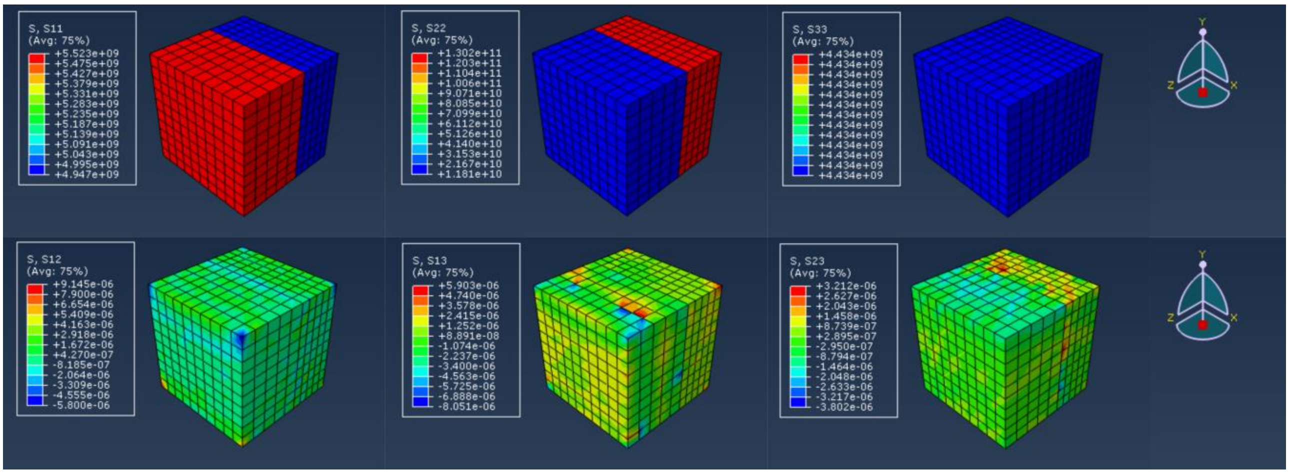

Figure 5 – Stress influence for the load case kl=22. Components 11, 22, 33, 12, 13 and 23, respectively.

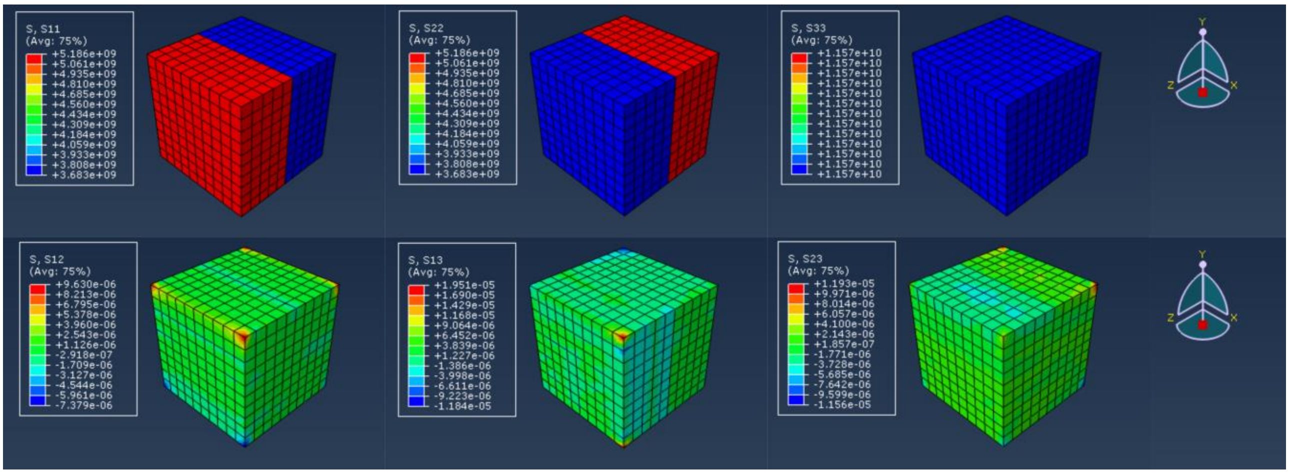

Figure 6 – Stress influence for the load case kl=33. Components 11, 22, 33, 12, 13 and 23, respectively.

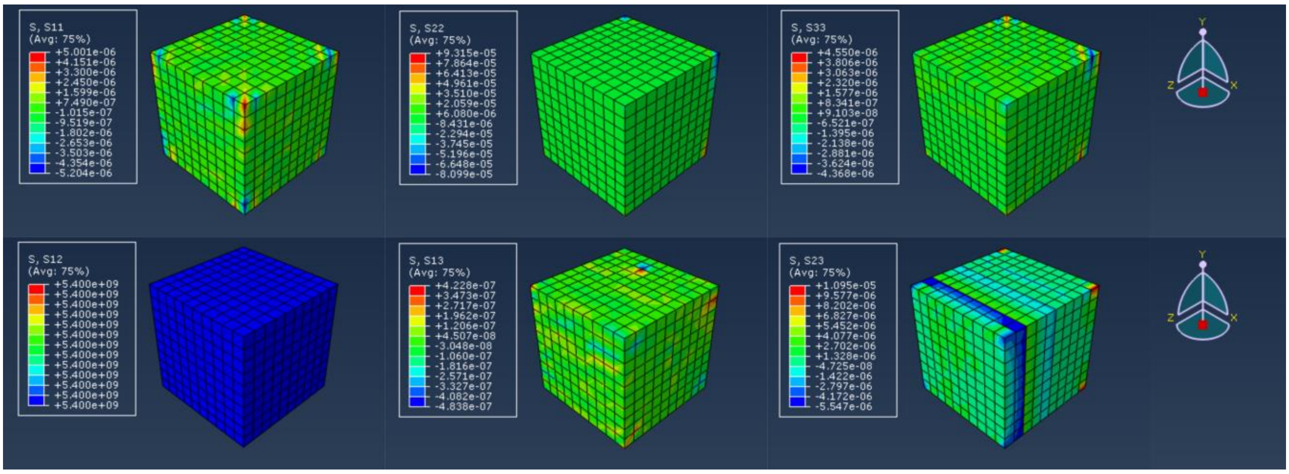

Figure 7 – Stress influence for the load case kl=12. Components 11, 22, 33, 12, 13 and 23, respectively.

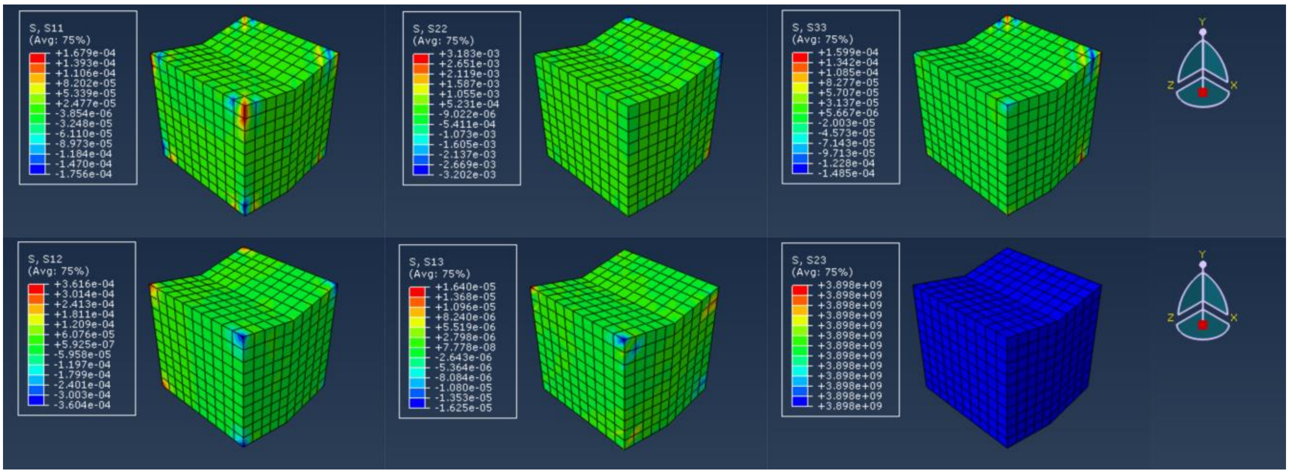

Figure 8 – Stress influence for the load case kl=23. Components 11, 22, 33, 12, 13 and 23, respectively.

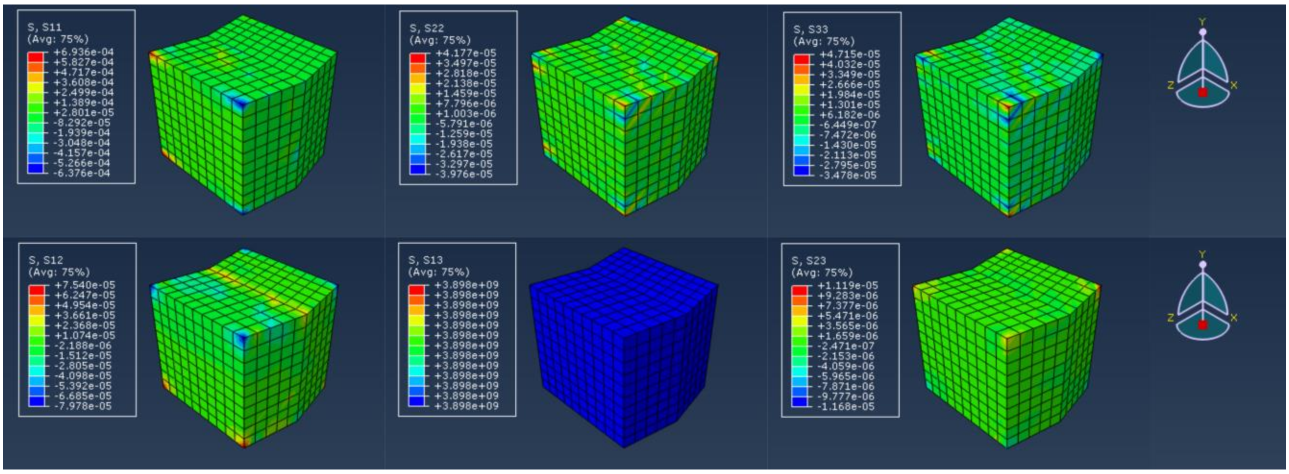

Figure 9 – Stress influence for the load case kl=13. Components 11, 22, 33, 12, 13 and 23, respectively.

In table 4 one can see the mesh convergence analysis for the asymptotic homogenization. As it can be seen, there is no influence of the mesh for this particular case. The type of element chosen for the discretization, the linear 8-node brick with incompatible nodes (C3D8I), is sufficient to solve the equations exactly. Table 5 shows the convergence analysis for the by the displacement control method. The element chosen to solve the equilibrium problems is the quadratic 20-node brick (C3D20), and, as it can be seen, even with a quadratic interpolation, there is influence of the finite element mesh. For both cases a unitary RVE is used. Another point is the symmetry of the tensor. Due to the geometry of the problem, it is expected the components C^H_{1111} and C^H_{2222} and the components C^H_{2323} and C^H_{1313} to be the same numerical value. This was found to be true for the asymptotic homogenization procedure, but not for the homogenization via displacement control, in which a small deviation was found in these components.

Table 4 – Convergence Analysis. Asymptotic expansion.

Mesh Density	C^H_{1111}	C^H_{2222}	C^H_{3333}	C^H_{2323}	C^H_{1313}	C^H_{1212}
$2\times2\times2$	7.099×10^{10}	7.099×10^{10}	1.157×10^{10}	3.898×10^{09}	3.898×10^{09}	5.400×10^{09}
$4\times4\times4$	7.099×10^{10}	7.099×10^{10}	1.157×10^{10}	3.898×10^{09}	3.898×10^{09}	5.400×10^{09}
$10\times10\times10$	7.099×10^{10}	7.099×10^{10}	1.157×10^{10}	3.898×10^{09}	3.898×10^{09}	5.400×10^{09}

Table 5 – Convergence Analysis. Displacement control.

Mesh Density	C^H_{1111}	C^H_{2222}	C^H_{3333}	C^H_{2323}	C^H_{1313}	C^H_{1212}
$2\times2\times2$	7.392×10^{10}	7.402×10^{10}	1.569×10^{10}	4.413×10^{09}	4.318×10^{09}	5.426×10^{09}
$4\times4\times4$	7.325×10^{10}	7.373×10^{10}	1.535×10^{10}	4.256×10^{09}	4.309×10^{09}	5.402×10^{09}
$10\times10\times10$	7.311×10^{10}	7.339×10^{10}	1.483×10^{10}	4.204×10^{09}	4.295×10^{09}	5.40×10^{09}

Another comparison is the RVE size. Table 6 and table 7 show the results obtained for an RVE with dimensions $1\times1\times1$ and $2\times2\times2$ for the two scale homogenization and the method of displacement control, respectively. For both cases a mesh comprised by $10\times10\times10$ elements is used. As it can be seen, for both cases, there is no influence of the RVE size, which is expected, since the linear regime is adopted.

Table 6 – Influence of RVE size. AHM. Mesh of 10x10x10 elements.

RVE size	C^H_{1111}	C^H_{2222}	C^H_{3333}	C^H_{2323}	C^H_{1313}	C^H_{1212}
$1\times1\times1$	7.099×10^{10}	7.099×10^{10}	1.157×10^{10}	3.898×10^{09}	3.898×10^{09}	5.400×10^{09}
$2\times2\times2$	7.099×10^{10}	7.099×10^{10}	1.157×10^{10}	3.898×10^{09}	3.898×10^{09}	5.400×10^{09}

Table 7 – Influence of RVE size. Displacement control. Mesh of 10x10x10 elements.

RVE size	C^H_{1111}	C^H_{2222}	C^H_{3333}	C^H_{2323}	C^H_{1313}	C^H_{1212}
$1\times1\times1$	7.311×10^{10}	7.339×10^{10}	1.483×10^{10}	4.204×10^{09}	4.295×10^{09}	5.40×10^{09}
$2\times2\times2$	7.311×10^{10}	7.339×10^{10}	1.483×10^{10}	4.204×10^{09}	4.295×10^{09}	5.40×10^{09}

Although the numerical results are very similar between the asymptotic homogenization and the displacement control, one can argue that this difference results from the application of true homogeneous fields in the RVE domain. In figure 10 a comparison between the component 22 of the stress, obtained for the load case $kl = 11$. As it can be seen, the stress field for the asymptotic homogenization is completely homogeneous. The same cannot be told about the displacement control, in which one can find small variations in the stress field, mainly between the two plies of the RVE. This finding shows that the displacement control is not sufficient to guarantee a homogeneous stress field through the RVE. These variations certainly affect the final result when the effective properties are calculated. Also, it is worth mentioning that is even harder to apply homogeneous fields for the shear load cases. This figure shows, again, the consistency in the fields applied by the use of asymptotic homogenization.

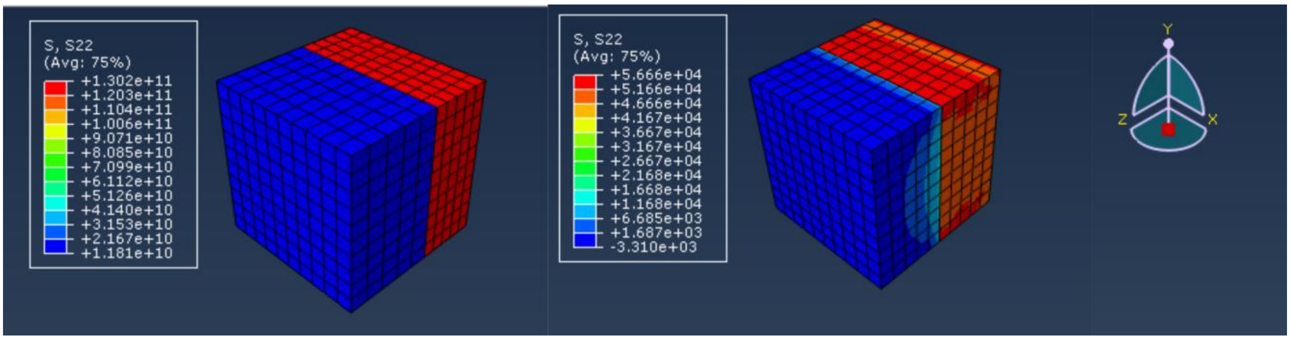

Figure 10 – Comparison between the stress field obtained by asymptotic expansion and by using displacement control. Load case kl=11, stress component 22.

CONCLUSIONS AND FUTURE WORKS

Homogenization procedures are well established into the numerical mechanics context. However, some precautions must be taken in order to assess complex problems, such as cracks, or failures, in layers of composites. This work aimed the investigation of the effects cracks on a periodically laminated composite on the homogenization analysis procedure carried out by a commercial finite element package. The main objective was to point out the possible struggles in the well-known homogenization procedures. It was found that, for the homogenization conditions applied via load control, a superposition of elements of the mesh appear, showing a physical inconsistency on the adopted model. On the other hand, this issue was not noticed when the homogenization conditions were applied by the use of displacement control.

Despite the widespread use of homogenization procedures via application of pre-described boundary conditions and loads, it is very difficult to guarantee a homogeneous stress field in the RVE. This issue was overcome by the use of asymptotic homogenization, a well-established method found in literature, but not so commonly used alongside commercial finite element packages. The results showed a good consistency in the use of this particular kind of homogenization. It was possible to apply completely homogeneous stress fields in the RVE, as well as guarantee the symmetries in the components of the homogenized fourth order elasticity tensor. On the other hand, the implementation of asymptotic homogenization alongside ABAQUS™ is not straightforward, and this can be pointed out as a disadvantage of the method.

For the future works, it should be noted that a more careful investigation must be carried out in order to improve the comprehension of the effects of cracks in the models. Yet, as soon as the Homogenization by Asymptotic Expansion becomes well developed and fully consistent, one can use it to investigate much more complex problems, such as composites with complex stacking sequences and composites with cracks in specific plies. By using the most common forms of homogenization, it is very difficult to handle complex problems and the asymptotic homogenization procedure seems to be promising in overcoming this gap. The final goal is to obtain numerical results of damages composites and comparing them with experimental data.

ACKNOWLEDGMENTS

The authors are thankful to Coordination for the Improvement of the Higher Level Personnel (CAPES/PNPD) and National Council for Scientific and Technological Development (CNPq process number: 401170/2014-4 and 310094/2015-1).

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