

Influence of pile-pile coupling conditions in the vibration of pile groups

A. C. A. Vasconcelos, J. Labaki

School of Mechanical Engineering, University of Campinas, SP, Brazil

This paper investigates the implications of two coupling conditions for embedded pile groups. The first is the condition in which the computation of the kinematic quantity in the direction of the loading disregards the influence of kinematic quantities in other directions. In this condition, for instance, the vertical motion of a pile in the group due to vertical loads does not take into account the influence of horizontal and rocking motion of the pile due to that vertical load. The second is the fully-coupled condition, which incorporates the effect of all degrees of freedom in each other's response. In this work, the soil is modeled as a viscoelastic homogeneous halfspace. The piles are described by finite beam elements. Time-harmonic external excitations are considered. Some representative cases are considered for different constitutive parameters and pile group distributions. The results show that the difference between the two coupling conditions is negligible for a wide variety of pile group systems. This enables analyses to consider the coupling condition with the lowest computational cost, without loss of physical consistency of the model.

Keywords: *Pile groups, coupling conditions, computational cost, soil-foundation interaction*

INTRODUCTION

Piles are structures widely used in geotechnical projects for their ability to stiffen the soil locally, imbuing it with additional static load bearing capacity. The dynamic interaction between pile and soil is a complex contact problem whose analyses often resort to numerical modeling. A number of soil models have been used in pile-soil interaction analyses. In the first, the soil is represented by distributed springs attached to the pile, named as dynamic Winkler model (Novak and Beredugo, 1972). The second type is the thin-layer or plain-strain model, which assumes that the soil is composed by infinitesimal thin elastic layers (Novak, 1974). The third are three-dimensional continuum models, which are more appropriate to study wave propagation in the soil (Rajapakse, Chen and Senjuntichai, 2005). The last type is the radially inhomogeneous model, in which the soil is a linear viscoelastic medium divided in two concentric regions, an inner annular of disturbed medium and an outer semi-infinite undisturbed one (Wang and Shang, 2006).

This work uses an implementation of Kaynia and Kausel's (1991) pile group model, in which the piles are modeled as one-dimensional beam elements, the soil is modeled through the superposition of Green's functions, and the pile-soil coupling is obtained through direct equilibrium and kinematic compatibility at the pile-soil interface. The contact between the pile and the soil can be described by two kinds of boundary conditions. In the relaxed bonding condition, kinematic compatibility and equilibrium at the pile-soil interface are enforced in the loading direction only. In the fully-bonded contact condition, kinematic compatibility and equilibrium are prescribed in all directions.

The fully-bonded contact condition demands a significantly larger amount of influence functions to be computed, which is expected to increase the computational cost of the solution considerably. Computing the influence functions is typically the most computationally expensive task in a boundary element model such as the present one (Labaki, Ferreira, and Mesquita, 2011). It has been shown that the difference between the two bonding conditions is negligible for the cases of surface plates (Labaki, Mesquita and Rajapakse, 2014; Labaki, Rajapakse and Mesquita, 2018) and single piles (Barros, 2006; Barros, Labaki and Mesquita, 2018). This work investigates how these bonding conditions affect the time-harmonic external vibration of groups of piles for different constitutive parameters and geometries. The goal is to verify whether or not the difference between the two conditions is negligible also for the case of pile groups, which would enable an analyst to choose that with the lowest computational cost.

FORMULATION

The present pile group model is based on a formulation proposed by Kaynia and Kausel (1991). The model consists of elastic piles embedded in a soil as shown in Fig. 1. The piles are discretized into N one-dimensional segments along the vertical direction. Each pile element has a corresponding soil layer of same length, with which it maintains bonded or relaxed contact. The pile has diameter d_p , length by l_p , Young's modulus E_p , and mass density ρ_p . The center-to-center distance between adjacent piles is denoted by s . The soil is a three-dimensional, layered, transversely isotropic, viscoelastic half-space. Its elasticity modulus, mass density, Poisson ratio, shear modulus and material damping

are denoted by E_s , ρ_s , ν_s , μ_s and β_s .

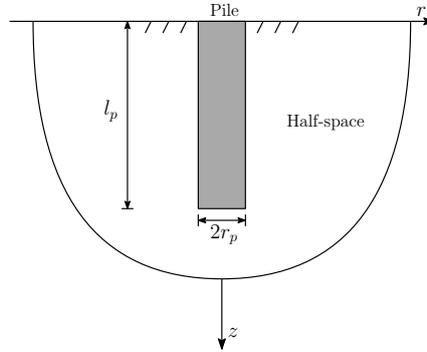


Figura 1 – Pile-soil interaction model.

The coupling of the pile-soil interface is obtained through direct equilibrium and kinematic compatibility between the N pile elements and the soil layers, and this coupling can be prescribed in all degrees of freedom or only those directly related to the loading direction. After extensive mathematical manipulation presented in detail by Kaynia and Kausel (1991), the discretized equation of motion for the pile-half-space coupled system results in the following algebraic system of equations:

$$\mathbf{P}_e = (\mathbf{K}_P + \Psi^T (\mathbf{F}_S + \mathbf{F}_P)^{-1} \Psi) \mathbf{U}_e = \mathbf{K}_e \mathbf{U}_e, \quad (1)$$

in which \mathbf{P}_e is the vector of external loads, \mathbf{K}_P is the dynamic stiffness matrix of the piles in the group, \mathbf{F}_P and \mathbf{F}_S are, respectively, the flexibility matrix of the pile group and the soil, \mathbf{K}_e is the dynamic stiffness for the ensemble of piles and \mathbf{U}_e is the vector of displacements of the pile heads. Matrix \mathbf{F}_S contains the displacements resulting from the interaction between the piles, which is the only term in Eq. 1 that is affected by different bonding conditions. For a full description of these quantities, please refer to Kaynia and Kausel (1991).

RESULTS

This section presents the dynamic response of pile groups under time-harmonic external excitations of circular frequency ω . These responses are computed considering both relaxed and fully-coupled contact conditions. The results are presented in terms of the normalized displacement $u_i^*(a_0) = u_i(a_0)/u_i(a_0 = 0)$, $i = x, z$ and normalized frequency $a_0^2 = \omega^2 \rho_s / \mu_s$. The soil medium is a viscoelastic, homogeneous, isotropic half-space. Throughout the study, the head of a pile i is subjected to vertical or horizontal loads p_z or p_x , and the effect of this load is measured at the head of pile j in terms of its vertical and horizontal displacements u_z and u_x . In all figures, continuous and dashed lines represent respectively the real and imaginary displacement for a coupled contact condition whereas discrete markers represent the same quantities for an uncoupled contact condition.

Pile group configurations

This section considers three different pile group configurations and the effect of coupling conditions in each case. The configurations are illustrated in Fig. 2. The first case considers a single pile, the second considers two piles along the x -axis, and the third considers four piles in a square grid centered at the origin of the coordinate system. In all cases, $l_p/d_p = 10$, $E_p/E_s = 100$, $\rho_p/\rho_s = 2$, and $s/d_p = 5$.

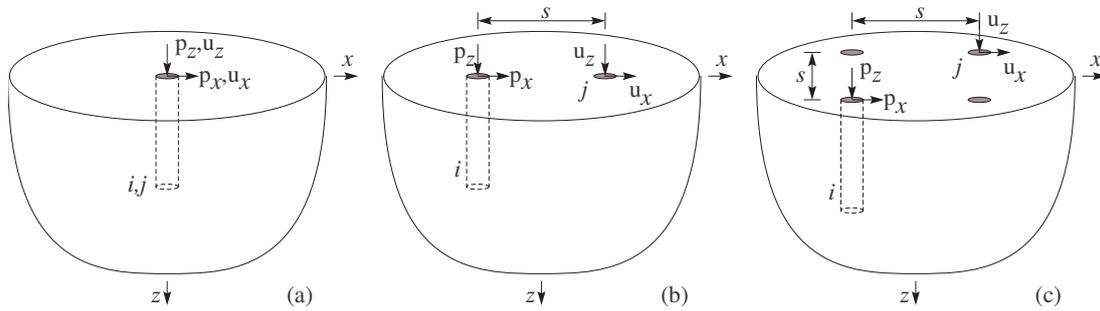


Figure 2 – Pile group configurations for (a) one, (b) two, and (c) four piles under vertical and horizontal loads

Figures 3 to 5 show the vertical and horizontal displacements of pile j due to vertical loads applied to pile i in each of the three configurations shown in Fig. 2. Note that the figures without markers indicate that the uncoupled contact condition is not applicable for the case. Moreover, due to the symmetry of the single pile case, no horizontal displacements result from the vertical loading case and vice-versa.

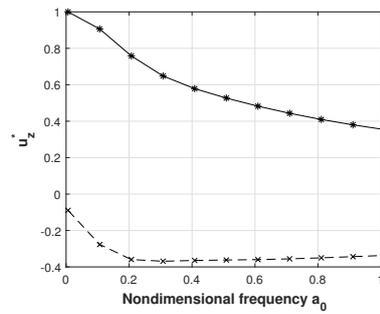


Figure 3 – Real and imaginary part of u_z^* for a single pile.

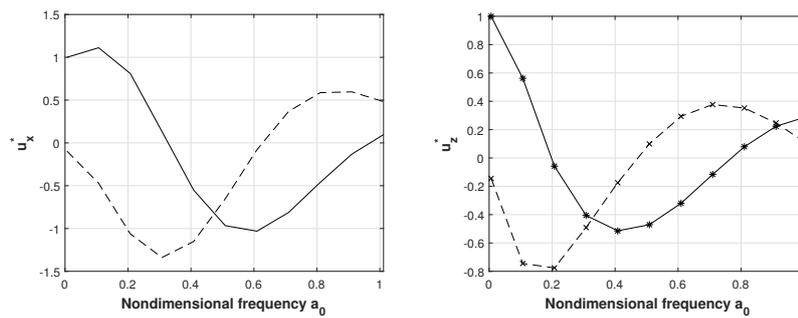


Figure 4 – Real and imaginary part of u_x^* and u_z^* for a group of two piles.

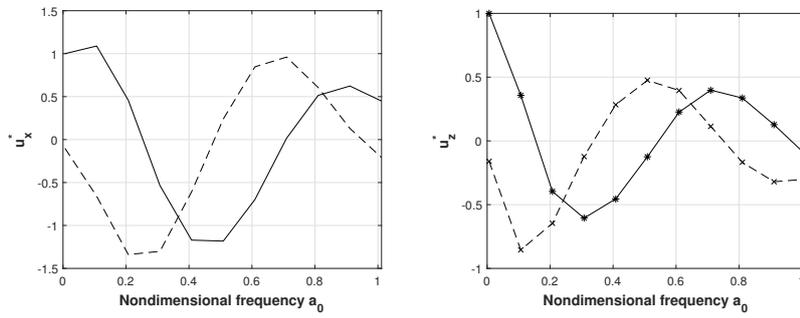


Figure 5 – Real and imaginary part of u_x^* and u_z^* for a group of four piles.

Analogously, Figs. 6 to 8 show the corresponding results for the case of horizontal loads applied to pile i in the three configurations.

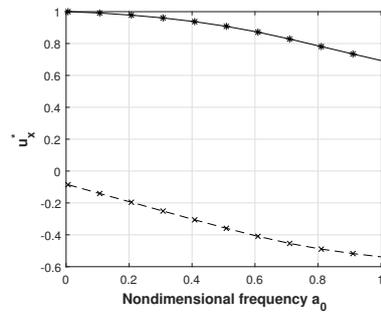


Figure 6 – Real and imaginary part of u_x^* for a single pile.

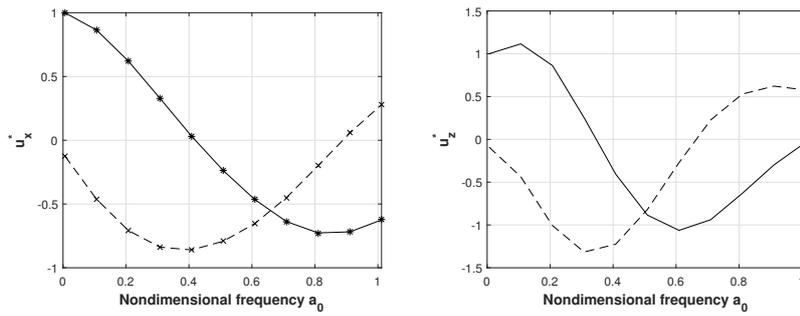


Figure 7 – Real and imaginary part of u_x^* and u_z^* for a group of two piles.

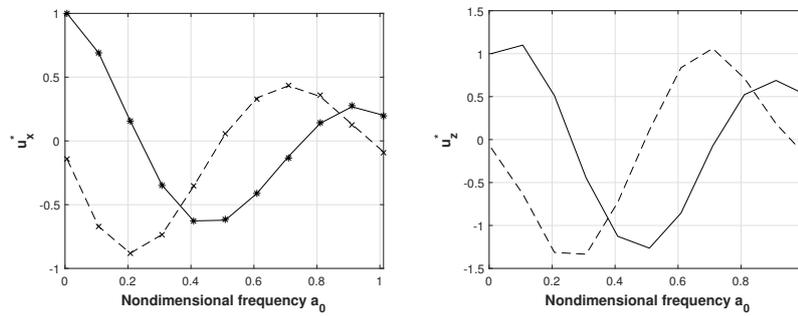


Figure 8 – Real and imaginary part of u_x^* and u_z^* for a group of four piles.

The results in this section indicate that there is no difference between the response of the piles in different group configurations when either relaxed or fully-bonded coupling conditions are used.

Pile parameters

This section presents a study on the influence of bonding conditions for different pile parameters. The parameters considered are the most relevant in pile group problems, such as pile length l_p , stiffness E_p , and mass density ρ_p , as well as distance s between piles. A system of two piles is considered in this section (Fig. 2b).

Figure 9 shows the effect of bonding conditions (continuous lines versus discrete markers) for three different pile stiffnesses. Square, star, and circle markers indicate $E_p/E_s = 1$, $E_p/E_s = 10$ and $E_p/E_s = 100$, respectively. In all figures, the diamond marker is used to identify the uncoupled solution. Frequencies $a_0 = 0.5$ and $a_0 = 1$ are considered. Normalized displacements $u_{kl}^* = u_{kl}(s)/u_{kl}(s = 2)$ indicate displacements at pile j in the $k = x, z$ direction due to loads at pile i in the $l = x, z$ direction (see Fig. 2). In these results, $l_p/d_p = 10$ and $\rho_p/\rho_s = 2$.

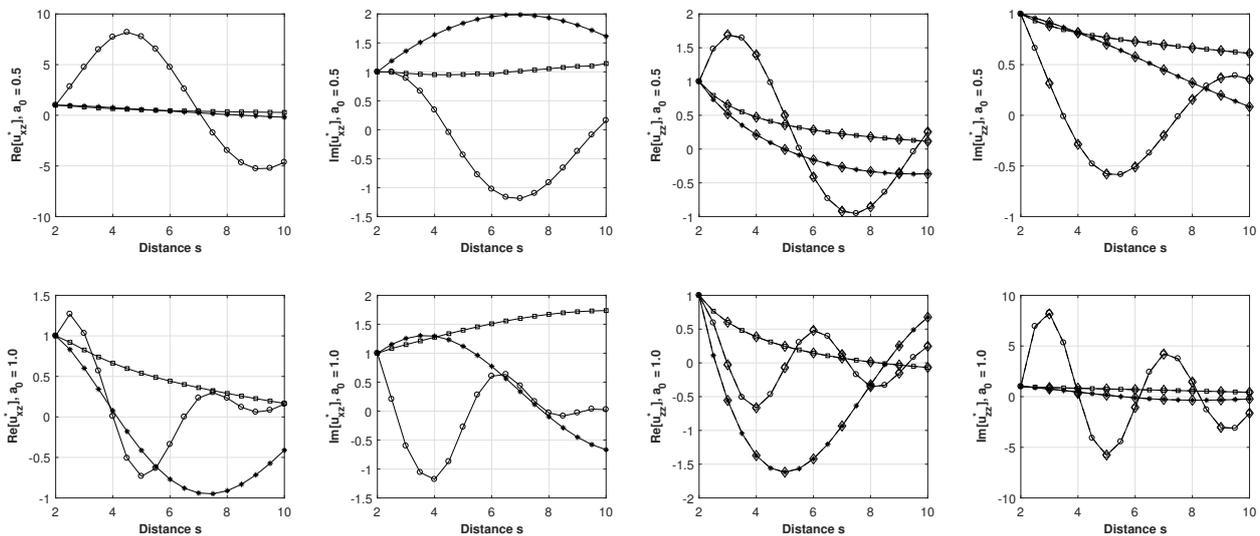


Figure 9 – Response of second pile for different elastic moduli.

Analogously, Figs. 10 considers three different pile mass densities. Square, star, and circle markers indicate respectively $\rho_p/\rho_s = 1$, $\rho_p/\rho_s = 10$, and $\rho_p/\rho_s = 100$. In these results, $l_p/d_p = 10$ and $E_p/E_s = 100$.

Influence of pile-pile coupling conditions in the vibration of pile groups

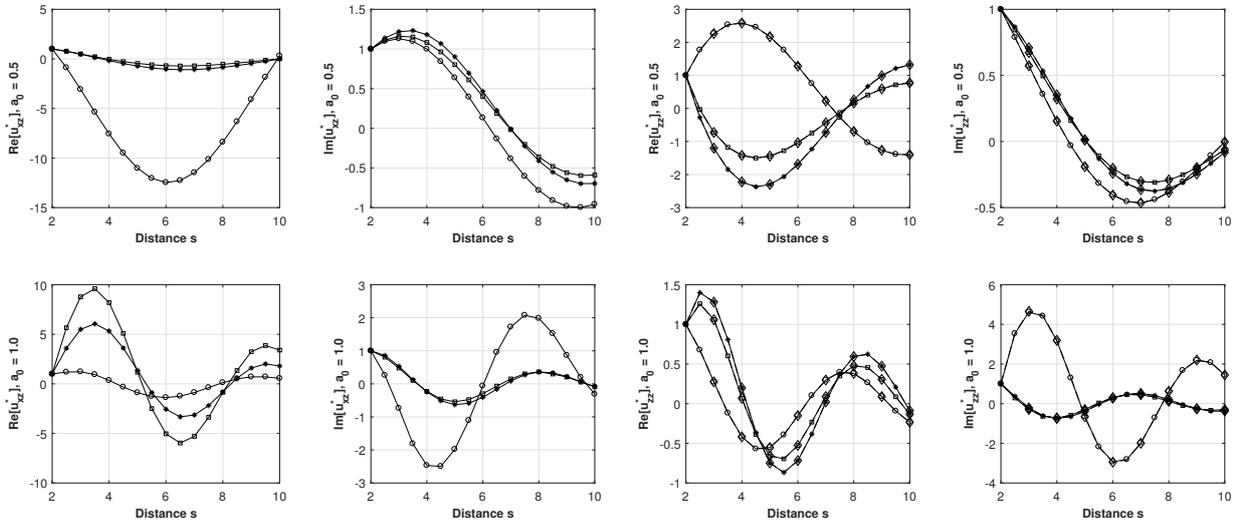


Figura 10 – Response from second pile for different mass densities.

Finally, Figs. 11 considers three different pile lengths. Square, star, and circle markers indicate respectively $l_p/d_p = 5$, $l_p/d_p = 10$, and $l_p/d_p = 20$. In these results, $\rho_p/\rho_s = 2$ and $E_p/E_s = 100$.

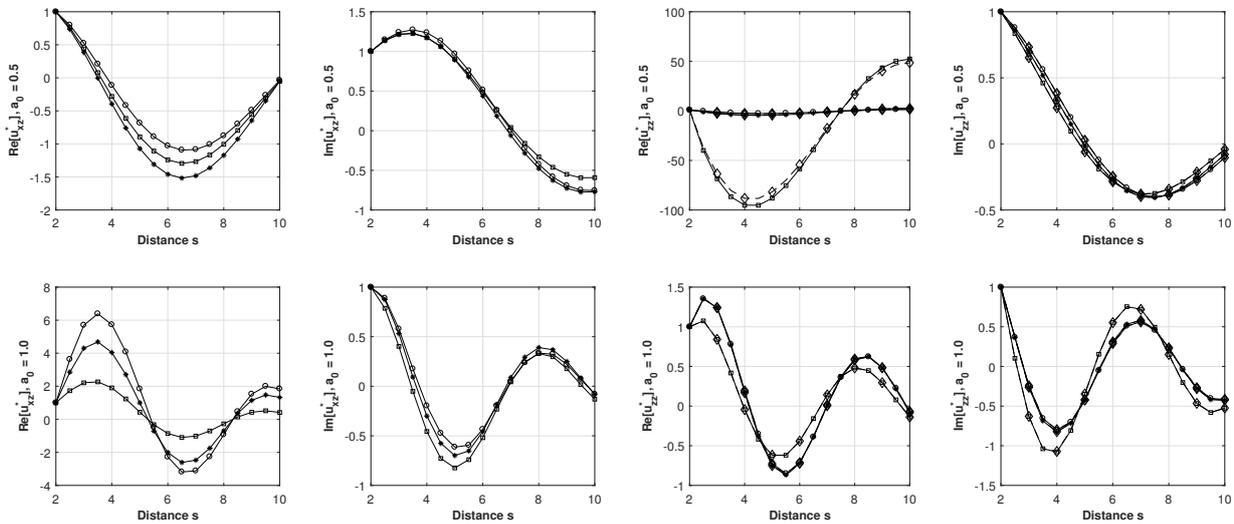


Figura 11 – Response from second pile for different lengths.

Once again, the results in this section indicate that the response of the pile group is the same for both coupling conditions. This indicates that one may choose to model a pile group with the coupling condition with the lowest computational cost, without loss of physical consistency of the model.

Computational cost

This section compares the computational cost of fully-coupled and relaxed bonding conditions for pile groups. For the analysis of computational cost, the three pile group configurations shown in Fig. 12 are considered. In all cases, all piles have length $l_p/d_p = 10$, $E_p/E_s = 100$, $\rho_p/\rho_s = 2$, and are discretized by $N = 20$ elements.

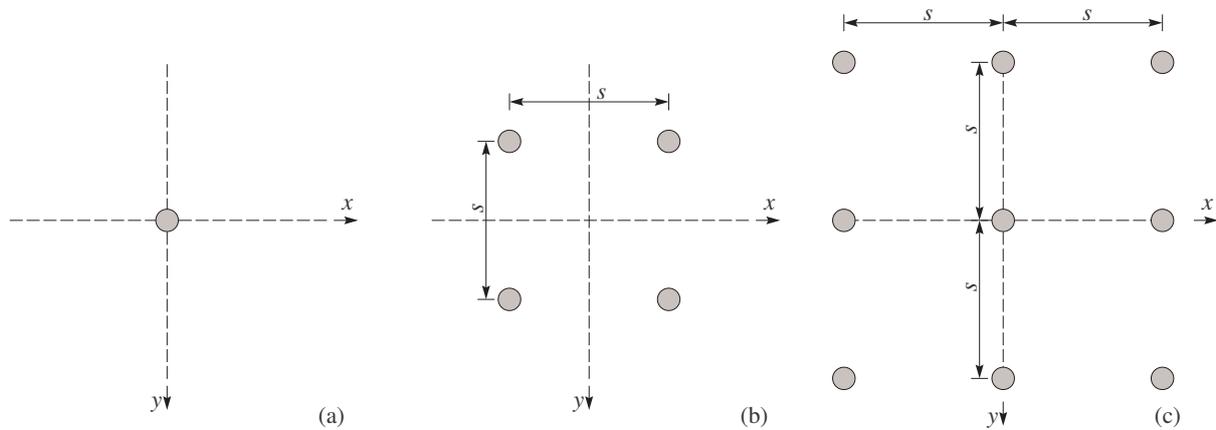


Figura 12 – Pile group configurations for (a) one, (b) four, and (c) nine piles

For the single pile problem, executed serially on a four-core i7-7700 3.6 GHz Intel processor, the solution was achieved in 28 s for the fully-coupled bonding condition model, and in 24 s for the relaxed bonding condition model. For the four-pile grid problem, these costs were 101 s and 87 s. For the nine-pile grid problem, 252 s and 212 s. In summary, for the one-, four-, and nine-pile grid problems, the fully-bonded condition has shown to be from 16 to 18 per cent more expensive than its relaxed condition counterpart.

CONCLUSION

This paper presented an analysis of the influence of pile-soil bonding conditions for pile groups. The pile group-soil interaction problem was modeled according to Kaynia and Kausel's (1991) thin-layer model. The results showed that results considering relaxed or fully-bonded contact conditions are indistinguishable from each other. The computational cost of the model, however, is significantly larger for the fully-bonded contact condition. These findings suggest that large pile group analyses may consider the relaxed contact condition between the piles without loss of physical consistency of the results.

ACKNOWLEDGMENTS

The research leading to this article has been funded in part by the São Paulo Research Foundation - Fapesp, through grant 2017/01450-0. The support of Capes, CNPq, and Faepex-Unicamp is also gratefully acknowledged.

REFERENCES

- Barros, P. L. A., 2006, Impedances of rigid cylindrical foundations embedded in transversely isotropic soils. *International Journal for Numerical and Analytical Methods in Geomechanics*, 30(7):683–702.
- Barros, P. L. A., Labaki, J. and Mesquita, E., 2018, IBEM-FEM Model of a Piled Plate Within a Transversely Isotropic Half-Space, *Engineering Analysis with Boundary Elements (in press)*.
- Kaynia, A. M. and Kausel, E., 1991, Dynamics of Pile and Pile Groups in Layered Soil Media, *Soil Dynamics and Earthquake Engineering*, Vol.10, pp. 386-401.
- Labaki, J., Ferreira, L. O. S. and Mesquita, E., 2011, Constant Boundary Elements on Graphics Hardware: a GPU-CPU Complementary Implementation. *Journal of the Brazilian Society of Mechanical Sciences and Engineering*, v. XXXIII, p. 475-482.
- Labaki, J., Mesquita, E. and Rajapakse, R.K.N.D., 2014. Vertical Vibrations of an Elastic Foundation with Arbitrary Embedment within a Transversely Isotropic, Layered Soil. *Computer Modeling in Engineering & Sciences*, 103(5), pp.281-313.
- Labaki, J. Rajapakse, R. K. N. D. and Mesquita, E., 2018. Dynamic Response of a Rigid Circular Foundation in Layered Anisotropic and Gibson's Soils, *Géotechnique (submitted)*.
- Novak, M., 1974, Dynamic Stiffness and Damping of Piles, *Canadian Geotechnical Journal*, Vol. 11, pp. 574-598.

Novak, M. and Beredugo, Y. O., 1972, Vertical Vibration of Embedded Footings, Journal of the Soil Mechanics and Foundations Division, Vol.98, pp. 1291-1310.

Rajapakse, R. K. N. D., Chen Y. and Senjuntichai, T., 2005, Electroelastic Field of a Piezoelectric Annular Finite Cylinder, International Journal of Solids and Structures, Vol. 42, pp. 3487-3508.

Wang, H. D. and Shang, S. P., 2006, Research on Vertical Dynamic Response of Single-Pile in Radially Inhomogeneous Soil During the Passage of Rayleigh Waves, Journal of Vibration Engineering, Vol. 19, pp. 258-264.

RESPONSIBILITY NOTICE

The authors are the only responsible for the printed material included in this paper.