

Uncertainty Quantification of McEvily Model using the Fast Crack Bounds Method

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Abstract: In this work the randomization of the McEvily Model is carried out through its definition parameters, in order to perform the quantification of uncertainty through the Fast Crack Bounds method, FCB. The FCB and Monte Carlo Simulation, MCS methods are used together to obtain the estimators. Proposed by Ávila et al. 2016, the FCB method consists in obtaining lower and upper bounds for the crack size function, which are obtained by appropriate enhancements from the Taylor series retaining the second order term with Lagrange remainder. The performance of the proposed methodology through the comparison of the estimators obtained through FCB-MCS and Runge Kutta-MCS was efficient. The bounds for the first and second statistical moments enveloped the numerical solution. The maximum relative deviation was 10.84% and the computational times of the methodology were up to 4450 times smaller than the numerical solution of RK4.

Keywords: Linear Elastic Fracture Mechanics, McEvily Model, Fast Crack Bounds Method, Runge-Kutta Method.

INTRODUCTION

According to Dieter (1981), fatigue failures can be conceptualized as being due to the conditions of dynamic loading in a repetitive or floating tension much lower than the tension to cause the fracture due to a static load. About 80% of machine failures occur due to fatigue failure (SMITH, 2006).

Linear Elastic Fracture Mechanics (LEFM) is a method to predict fatigue and is used to evaluate the propagation of a crack in a component where there is no significant plastic deformation during the fracture (ANDERSON, 2005).

In LEFM the methods can be divided into two categories: Constant Amplitude Loading (CAL) and Variable Amplitude Loading (VAL). For CAL there are several models, such as Paris-Erdogan (1963), Walker (1970), Forman (1972), Collipriest (1972), Priddle (1976), Wang (1994), among others. Usually these models were developed based on an Initial Value Problem (IVP), which details the evolution of the crack size during load cycles.

In order to prove the efficiency of the FCB method, it was used together with the Monte Carlo Simulation (MCS), to establish upper and lower bounds of the stochastic process statistical moment estimators. The methodology efficiency was evaluated from the combination of the MCS with the numerical response of the 4th order Runge Kutta method.

RESEARCH METHOD

First, the mathematical formulation was used to determine the upper and lower bounds for the crack evolution of the McEvily model through the FCB method. The uncertainty quantification of the McEvily propagation model was used together with the FCB and Monte Carlo Simulation methods, right after implementing an algorithm in MATLAB software for the example of the plate with finite width and central crack. The result of this iteration within the computational environment resulted in an algorithm for comparison between the numerical solutions acquired through the RK4 method and the bounds estimators.

CRACK PROPAGATION MODEL

The study of the evolution of a crack is made based on the crack growth rate as a function of the stress variation, which can be represented by the diagram $\log\left(\frac{da}{dN}\right) \times \log(\Delta K)$ where there are three regions corresponding to the life stages of the crack: region I (nucleation) where the crack growth is almost linear; region II (propagation), in this region the propagation of the crack is proportional to K, in logarithmic scale and region III (fatigue failure), accelerated crack growth leading to failure of the component Figure 1.

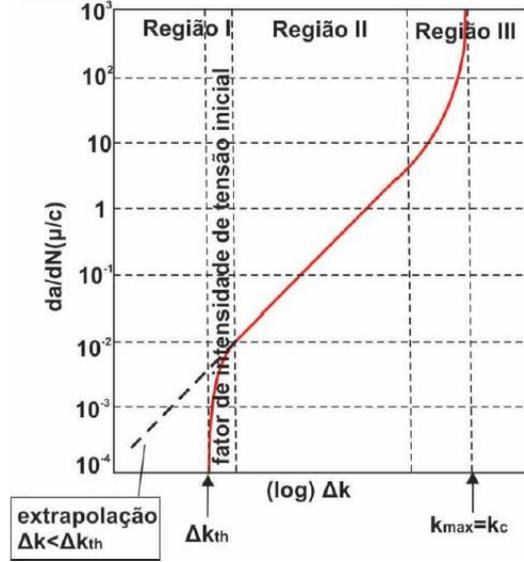


Figure 1 – The three regions of the crack growth rate as function of ΔK (Schijve, 2001)

McEvily Crack Propagation Model

McEvily and Groeger (1977) proposed a model to describe the regions from I to III, according to the equations:

$$\frac{da}{dN} = C_M (\Delta K - \Delta K_{th})^2 \left(1 + \frac{\Delta K}{K_c - K_{max}} \right). \quad (1)$$

Where C_M is the constant of McEvily, ΔK is the variation between the maximum and minimum values of the stress intensity factor where the crack in formation propagates and ΔK_{th} is the threshold stress intensity factor.

FAST CRACK BOUNDS METHOD

The Fast Crack Bounds method, FCB, consists in obtaining upper and lower bounds for the crack size function. These quotas envelop the solution, have a computational cost lower than necessary to obtain solutions through numerical integrations (Machado, 2015) and present results close to those obtained by numerical methods such as RK4. In general, the mathematical models for crack evolution are defined by an initial value problem, IVP:

$$\begin{cases} \text{Encontrar } a \in C^1([N_0, N_1]; \mathbb{R}^+), \text{ tal que:} \\ \frac{da}{dN}(N) = h(a, \Delta K), \forall N \in (N_0, N_1) \\ a(N_0) = a_0 \end{cases} \quad (2)$$

Being $\frac{da}{dN}$ the crack growth rate, N the number of cycles, h the evolution law.

The bounds of FCB method were obtained through Taylor series expansion, order through the Lagrange remainder:

$$a(N) = a_0(N_0) + \left(\frac{da}{dN}(N_0) \right) (N - N_0) + \frac{1}{2} \left(\frac{d^2a}{dN^2}(\eta) \right) (N - N_0)^2, \text{ com } \eta \in [N_0, N] \quad (3)$$

Being $\bar{a}(N)$ the upper bound function, $\underline{a}(N)$ the lower bound function, $a(N)$ crack size and N the number of cycles.

$$\underline{a}(N) \leq a(N) \leq \bar{a}(N), \forall N \in (N_0, N_1) \quad (4)$$

The mathematical equations of the dimensions for the crack propagation model will be elaborated from the following hypotheses described by Avila et al. 2016,

$$H1) \begin{cases} f \in C^1\mathbb{R} \\ 0 < f(a_0) \leq f(x) \leq f(y), x \leq y, A x, y \in [a_0, a_1]; \\ f'(a_0) \leq f'(x) \leq f'(y), x \leq y \in [a_0, a_1]. \end{cases} \quad (5)$$

$$H2) \{m \geq 1$$

Determination of the bounds for the McEvily model via FCB method

The functions that will define the upper ((16) and lower ((17) bounds for the McEvily model are determined from equation ((6 with the initial value problem (IVP):

The IVP for McEvily crack propagation model is:

$$\left\{ \begin{array}{l} \text{Determine } a \in C^1([N_0, N_1]; \mathbb{R}^+) \text{ such that,} \\ \left(\frac{da}{dN} \right) (N) = C_M (\Delta K - \Delta K_{th})^2 \left(1 + \frac{\Delta K}{K_c - K_{max}} \right), \forall N \in (N_0, N_1) \\ a(N_0) = a_0. \end{array} \right. \quad (6)$$

By the Taylor expansion of 2nd order, with Lagrange remainder:

$$a(N) = a_0(N_0) + \frac{da}{dN}(N_0)(N - N_0) + \frac{1}{2} \frac{d^2a}{dN^2}(\eta)(N - N_0)^2, \text{ com } \eta \in [N_0, N] \quad (7)$$

Hypotheses:

$$H1) \begin{cases} f \in C^1(\mathbb{R}) \\ 0 < f(a_0) \leq f(x) \leq f(y), x \leq y, \forall x, y \in [a_0, a_1]; \\ f'(a_0) \leq f'(x) \leq f'(y), x \leq y, \forall x, y \in [a_0, a_1]; \end{cases} \quad (8)$$

Through this hypothesis, the following inequalities can be written:

$$a(s) \leq a(t), s \leq t, \in [N_0, N] \quad (9)$$

$$(a(s))^m \leq (a(t))^m, s \leq t, \in [N_0, N] \quad (10)$$

thus,

$$(f(s))^m \leq (f(t))^m, s \leq t, \in [N_0, N] \quad (11)$$

$$(a^{1/2}f)^m(s) \leq (a^{1/2}f)^m(t), s \leq t, \in [N_0, N] \quad (12)$$

so,

$$(\Delta K)^m(a(s)) \leq (\Delta K)^m(a(t)) \quad (13)$$

being $C > 0$:

$$\frac{da}{dN}(s) \leq \frac{da}{dN}(t), s \leq t, \in [N_0, N] \quad (14)$$

The second derivative of IVP described by McEvily:

$$\frac{d^2a}{dN^2} = C_M^2 \Delta K [\Delta K - \Delta K_{th}]^3 \cdot \left\{ \begin{array}{l} \left[2 \left(1 + \frac{\Delta K}{K_c - \frac{\Delta K}{(1-R)}} \right) + \frac{K_c (\Delta K - \Delta K_{th})}{\left(K_c - \frac{\Delta K}{(1-R)} \right)^2} \right] \\ \cdot \left(1 + \frac{\Delta K}{K_c - \frac{\Delta K}{(1-R)}} \right) \left[\frac{1}{2a} + \left(\frac{f'}{f} \right) (a) \right] \end{array} \right\} \quad (15)$$

Thus, replacing the first and second order derivatives of the model's IVP in Taylor's 2nd-order expansion with the Lagrange remainder, and making use of the hypotheses proposed by Ávila et al. (2016), you get the functions that will define the upper and lower dimensions for the McEvily model:

$$a(N) - a^* \leq C_M \left\{ \begin{array}{l} \left[\Delta K(a_0) - \Delta K_{th} \right]^2 \left[1 + \frac{\Delta K(a^*)}{K_c - \frac{\Delta K(a^*)}{(1-R)}} \right] + \frac{1}{2} C_M \Delta K(a^*) [\Delta K(a^*) - \Delta K_{th}]^3 \\ \left[2 \left(1 + \frac{\Delta K(a^*)}{K_c - \frac{\Delta K(a^*)}{(1-R)}} \right) + \frac{K_c (\Delta K(a^*) - \Delta K_{th})}{\left(K_c - \frac{\Delta K(a^*)}{(1-R)} \right)^2} \right] \\ \left[1 + \frac{\Delta K(a^*)}{K_c - \frac{\Delta K(a^*)}{(1-R)}} \right] \left[\frac{1}{2a^*} + \left(\frac{f'}{f} \right) (a^*) \right] (N - N_0) \end{array} \right\} (N - N_0) \quad (16)$$

$$a(N) - a_0 \geq C_M \left\{ \begin{array}{l} \left[\Delta K(a_0) - \Delta K_{th} \right]^2 \left[1 + \frac{\Delta K(a_0)}{K_c - \frac{\Delta K(a_0)}{(1-R)}} \right] + \frac{1}{2} C_M \Delta K(a_0) [\Delta K(a_0) - \Delta K_{th}]^3 \\ \left[2 \left(1 + \frac{\Delta K(a_0)}{K_c - \frac{\Delta K(a_0)}{(1-R)}} \right) + \frac{K_c (\Delta K(a_0) - \Delta K_{th})}{\left(K_c - \frac{\Delta K(a_0)}{(1-R)} \right)^2} \right] \\ \left[1 + \frac{\Delta K(a_0)}{K_c - \frac{\Delta K(a_0)}{(1-R)}} \right] \left[\frac{1}{2a_0} + \left(\frac{f'}{f} \right) (a_0) \right] (N - N_0) \end{array} \right\} (N - N_0) \quad (17)$$

$$\forall N \in [N_0, N] \forall N \in [N_0, N]$$

RUNGE-KUTTA METHOD

The Runge-Kutta method is easy to program. Generally, it produces precise approximations, with a number of iterations being reasonably small (Nagle, 2005), being easy computational implementation.

A single-step, 4th order precision numerical method for solving ODEs is the Runge-Kutta Explicit method, which encompasses the Euler and Improved Euler methods described in the equations below (BOYCE et al., 2006).

$$\left\{ \begin{array}{l} y_{n+1} = y_n + \left(\frac{1}{6}\right)(k_{n1} + 2k_{n2} + 2k_{n3} + k_{n4}); \\ k_{n1} = hf(x_n, y_n); \\ k_{n2} = hf\left(x_n + \frac{1}{2}hy_n + \frac{1}{2}hk_{n1}\right); \\ k_{n3} = hf\left(x_n + \frac{1}{2}hy_n + \frac{1}{2}hk_{n2}\right); \\ k_{n4} = hf(x_n + hy_n + hk_{n3}). \end{array} \right. \quad (18)$$

UNCERTAINTY QUANTIFICATION

The model parameters are random and continuous variables in $(\Omega, \mathfrak{F}, P)$, a probability space, where Ω is a sample space, \mathfrak{F} is a σ -algebra of events and P is a probability measure (JAMES, 1981). For the quantification of McEvily's model, the Paris-Erdogan law equation will be used, as described in the paper by Lopes and Ávila (2015), which is presented as follows:

$$\left\{ \begin{array}{l} \text{Encontrar } a: (N_0, N_1) \times (\Omega, \mathfrak{F}, P), \mathbb{R}^+, \text{ tal que:} \\ \frac{da}{dN} = h(\alpha, \Delta K), \forall (N, \omega) \in (N_0, N_1) \times (\Omega, \mathfrak{F}, P); \\ a_0(N_0) = a_0(\omega)(N_0) \forall \omega \in (\Omega, \mathfrak{F}, P). \end{array} \right. \quad (19)$$

For statistical moments, the upper and lower bounds of the crack size function are considered. The estimate of the sum of the product of each probability of output of the experiment by its respective value must satisfy the definition given below, in ((4, when this condition is reached the equation is rewritten:

$$\hat{\mu}_{\underline{a}}(N) \leq \hat{\mu}_a(N) \leq \hat{\mu}_{\bar{a}}(N), \forall (N_0, \omega) \in (N_0, N_1) \times (\Omega, \mathfrak{F}, P) \quad (20)$$

Where $\hat{\mu}_{\underline{a}}, \hat{\mu}_a, \hat{\mu}_{\bar{a}}$, are the values of the lower bound, crack size and upper bound, respectively.

MONTE CARLO SIMULATION

Monte Carlo Simulation (MCS) is a statistical simulation method that can be defined as a methodology that uses a sequence of random numbers to generate a simulation (SOUZA, 2014). The method is developed in four stages:

1. Generate samples from a probability law;
2. Get the realization of the problem for each sample;
3. Analyze result set statistics
4. To obtain histograms and estimators of the statistical moments of the stochastic process of crack size.

The MCS method is used to estimate the statistical moments of the 'crack size' stochastic process using the fourth-order Runge-Kutta method and the Fast Crack Bounds methodology. The samples are generated through a predefined algorithm in MATLAB.

In the work of Lopez and Ávila (2015), the numerical approximation of the stochastic process realizations describing the crack propagation is evaluated using an RK4 method. Thus, for the i -th accomplishment of the parameters $\{\alpha(\omega_i), a_0(\omega_i)\}$, the approximate numerical solution of the i -th realization of the stochastic process 'crack size' is given by the equation below.

$$\left\{ \begin{array}{l} \text{Determinar } a_{k+1}(\omega_i) \in i^+, \text{ para cada } \omega_i \in \Omega \\ a_{k+1} = a_n + \left(\frac{\Delta N}{6}\right) (k_1 + 2k_2 + 2k_3 + k_4)(\omega_i), \forall k \in \{0, 1, 2, \dots, n\}; \\ k_1(\omega_i) = h(a(\omega_i), \Delta k); \\ k_2 = a_k(\omega_i) + \left(\frac{\Delta N}{2}\right) k_1(\omega_i); \\ k_3 = a_k(\omega_i) + \left(\frac{\Delta N}{2}\right) k_2(\omega_i); \\ k_4 = a_k(\omega_i) + (\Delta N)k_3(\omega_i); \\ a_0(\omega_i) = a(N_0, \omega_i). \end{array} \right. \quad (21)$$

NUMERICAL RESULTS

The results were generated using MATLAB software, version R 2015. To evaluate the performance of the dimensions for the McEvily model, the data from the Kuman work for an aluminum alloy Al 2024 T351 were applied. In the analysis, 10,000 samples were generated and the values for the execution of the work are described in Table 1:

Table 1 – Data used on McEvily model.

Parameter	Numeric value	Unit
Expected value of the random variable 'C'	1.811e-10	m/cycle
Expected value of the random variable 'K _c '	37	MPa√m
Stress range (Δσ)	55	MPa
Ratio of minimum load to maximum load (R)	0.5	Dimensionless
Plate with (b)	0.1	M
Number of cycles (N)	900000	Cycles

The value of star crack size for the upper dimension (a*) was determined by inspection according to Santos (2015). For this work a* = 1.13 was used. It must ensure that the upper bound is not violated by the approximate numeric solution. If the upper bound is violated by the numerical solution of RK4, a new a* is determined.

To obtain the results, the correction function ((22) was used for the classical finite-plate and central-crack problem proposed by Bannantine (Figure 2):

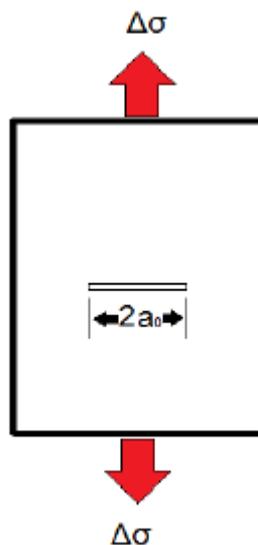


Figure 2- Plate with finite width and central crack (Bezerra, 2017).

$$f(a) = \sqrt{\sec\left(\frac{\pi a}{2b}\right)}, \text{ em } (N_0, N_1) \quad (22)$$

Figure 3 shows the graph for the first statistical moment of the crack size function of the random variable Kc:

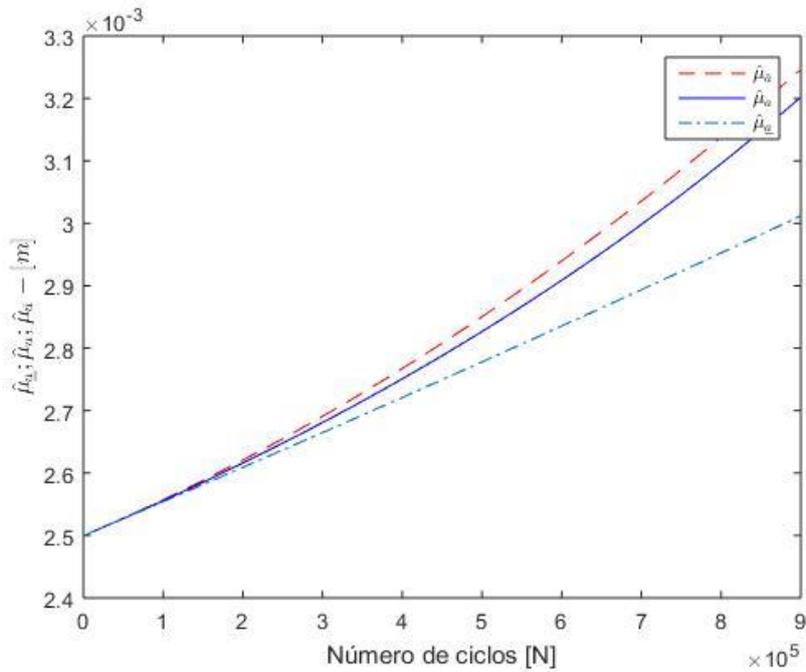


Figure 3 – First statistical moment for plate with finite width and central crack (the authors, 2018).

In Figure 4 it's possible to observe the second statistical moment for finite plate with central crack of the random variable Kc.

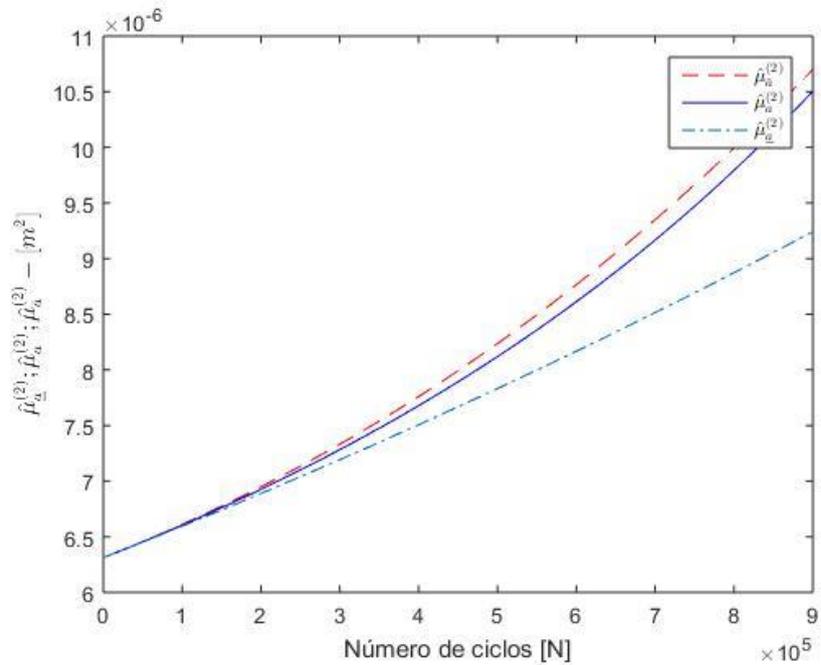


Figure 4 – Second statistical moment for plate with finite width and central crack (the authors, 2018)

Relative deviations

The estimate of the first statistical moment is the expected value and the estimate of the second statistical moment is the relation between variance and expected value.

To evaluate the methodology, relative deviation functions were defined for the first and second statistical moments, represented respectively by equations ((23 and ((24):

$$\begin{cases} \varepsilon_{\hat{\mu}_{\bar{a}}}(N) = 100 \left(\frac{\hat{\mu}_{\bar{a}} - \hat{\mu}_a}{\hat{\mu}_a} \right) (N) [\%], \forall N \in [N_0, N_1], \\ \varepsilon_{\hat{\mu}_{\underline{a}}}(N) = 100 \left(\frac{\hat{\mu}_{\underline{a}} - \hat{\mu}_a}{\hat{\mu}_a} \right) (N) [\%], \forall N \in [N_0, N_1]. \end{cases} \quad (23)$$

$$\begin{cases} \varepsilon_{\hat{\mu}_{\bar{a}}^{(2)}}(N) = 100 \left(\frac{\hat{\mu}_{\bar{a}}^{(2)} - \hat{\mu}_a^{(2)}}{\hat{\mu}_a^{(2)}} \right) (N) [\%], \forall N \in [N_0, N_1], \\ \varepsilon_{\hat{\mu}_{\underline{a}}^{(2)}}(N) = 100 \left(\frac{\hat{\mu}_{\underline{a}}^{(2)} - \hat{\mu}_a^{(2)}}{\hat{\mu}_a^{(2)}} \right) (N) [\%], \forall N \in [N_0, N_1]. \end{cases} \quad (24)$$

Table 2 shows the deviations related to the first and second statistical moments for the classic example ‘plate with finite width and central crack’.

Table 2- Relative deviation of the first and second statistical moment [%]

Parameter		First statistical moment			Second statistical moment		
		N [10 ⁵ cycles]					
		3	6	9	3	6	9
C _M	\bar{a} (N)	0.330	1.011	1.399	1.534	2.750	3.345
	\underline{a} (N)	-0.626	-2.527	-5.857	-0.378	-4.277	-10.842
K _c	\bar{a} (N)	0.448	1.459	2.260	1.776	3.686	5.187
	\underline{a} (N)	-0.562	-2.278	-5.405	-0.245	-3.764	-9.9145

Table 3 presents the computational times for the determination of the bounds and numerical solution, and the comparison between the runtimes is given by the following equation:

$$\rho = RK4/(FCB) * 100 \quad (25)$$

Table 3 - Runtime for 900,000 cycles for the McEvily model.

Parameter		Time [s]	ρ [%]
C _M	RK4	194.37	4162.10
	FCB	4.67	
K _c	RK4	207.33	4449.14
	FCB	4.66	

CONCLUSION

The uncertainty quantification of the crack propagation phenomenon of the McEvily method using the FCB method together with the MCS was applied in an aluminum alloy Al 2024 T351 in a plate with finite width and central crack, where the MCS and FCB methods were used together in order to estimate the statistical moments of the stochastic process ‘crack size’.

The bounds for the first and second statistical moments, according to the FCB method, enveloped the numerical solution. The maximum relative deviation was 10.84% and the methodology proved to be effective in presenting lower computational times than the RK4 numerical solution, being even 4450 times smaller.

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