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STATIONARY DYNAMIC RESPONSE OF A CIRCULAR RIGID FOUNDATION PARTIALLY SUPPORTED BY A FLEXIBLE PILE AND INTERACTING WITH A HALF-SPACE SUBJECTED TO A VERTICAL INCIDENT WAVE FIELD

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Abstract: This paper investigates the vertical response of a rigid circular foundation resting on the surface of soil supported by a flexible pile due to incident vertical waves. The soil is modeled as a three-dimensional, transversely isotropic homogeneous half-space. The surface foundation is a rigid, circular plate, presenting inertia properties. The pile is an elastic circular bar, in bonded contact with the soil throughout its length. The soil and the pile responses were obtained through the synthesis of a series of Green's function. The foundation is supported by the pile and by the soil-foundation interface. The introduction of a weighting parameter in the presented modelling allows the foundation support to vary from being totally supported by the pile to totally supported by the soil-foundation interface or any arbitrary combination of both supporting mechanisms. The numerical results show the influence of the foundation-pile-soil inertia ratios, the relative stiffness of the soil-pile system, on the dynamic response of the foundation due to an incident vertical wave field.

Keywords: Dynamic Soil-Structure Interaction, Foundations Dynamics, Pile Dynamics, Green's Functions.

INTRODUCTION

The dynamics of soil-structure interaction is an important consideration in the design of structures subject to loads such as earthquakes, winds and machine vibrations. According to Di Laora (2009), the response of the structure and of the soil in a soil-structure problem are coupled, and the process by which structure vibration influences soil response and vice versa is called dynamic soil-structure interaction (DSSI).

In the last years, dynamic response of circular plates has been studied using different types of numerical methods. One of the reason for the increase in these studies is because they have important practical applications, for instance, in vibration control of nano-facilities and nuclear plants. The case of a plate embedded resting on the surface of a transversely isotropic soil is of special interest to the analysis of vibration-sensitive such as synchrotron light sources (Labaki, Mesquita and Rajapakse, 2014).

The literature shows that there are two main categories of models of DSSI. The first is based on finite element discretization approach that provides sophisticated representation of the foundation and pile, but lacks accuracy in modelling the soil component. Another broad category of soil-foundation models is based on boundary element discretization (Labaki, Mesquita and Rajapakse, 2013 and 2014).

The soil is an unbounded domain presenting outgoing and non-reflected waves that withdraw energy from the excitation source. This effect is known as Sommerfeld radiation condition (Sommerfeld, 1949) and is also called radiation damping. The modeling of unbounded domains presenting radiation damping requires special techniques that incorporate this damping effect, such as the Boundary Element Method (BEM) or a semi-analytical method based on a Green's function approach (GF). The stationary dynamic behavior of soil have been successfully solve by the BEM and GF strategies (Carrion, Sousa and Mesquita, 2007; Labaki, Mesquita and Rajapakse, 2014).

In some DSSI problems, a surface foundation is supported partially by the soil and partially by a pile or group of piles. The function of the piles is to increase the support capacity of the soil by increasing its overall stiffness. An accurate analysis of a pile requires a sophisticated modeling that represents the energy it dissipates from external loads (coming from foundation) or takes in energy from internal loads (coming from the soil, such as earthquake), as well as the length, mass and stiffness characteristics of the pile (Rajapakse and Shah, 1989).

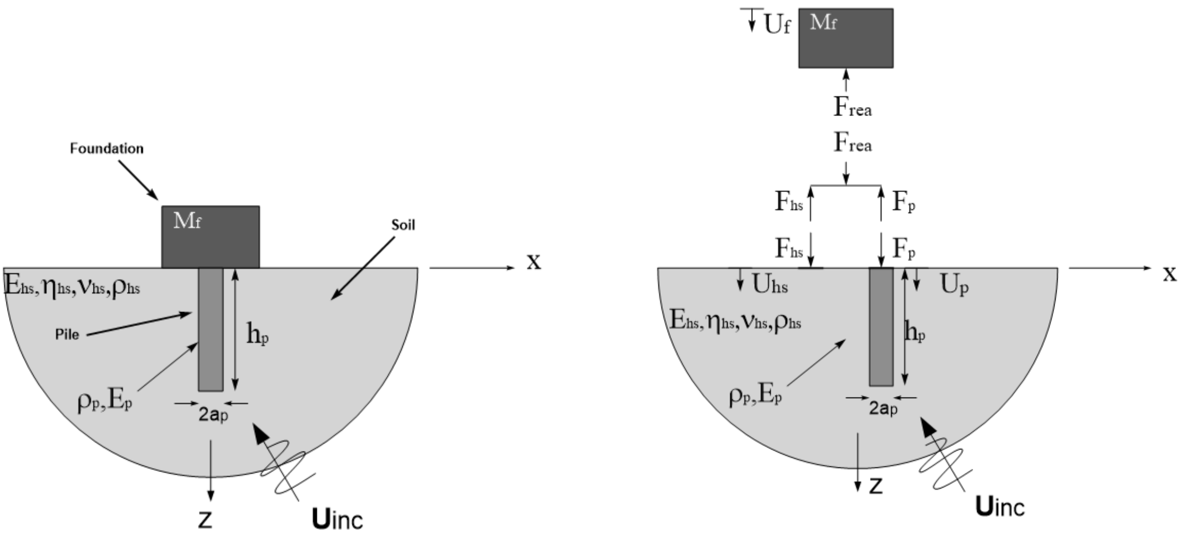
The present article considers the dynamic vertical response of a rigid foundation supported by its underlying soil with presence of the embedded pile under an incident vertical wave. Lima, Labaki and Mesquita (2016) studied this same problem with a foundation subject to external excitation. That study showed that the response of the coupled system depends significantly on the amount of load that is transferred from the foundation to the pile and the system and the introduction of a pile results in a decrease in the vibration of the coupled system in study. In the present work, the authors approach this complementary problem to study the dynamic behavior of the foundation when different values of incident vertical wave is transferred to the foundation by the soil and by the pile.

The method of coupling requires the determination of displacements and forces acting at the interface between the foundation and the pile due to an incident vertical wave. The displacement and force responses of the foundation and pile are determineded and must satisfy the criteria of kinematic compatibility and equilibrium of forces at the interface.

STATEMENT OF THE PROBLEM

Consider the problem of dynamic soil-pile-foundation interaction under incident vertical wave U_{inc}, show in Fig.1. The index ‘hs’ is related to the soil, modeled here as a 3D homogeneous, transversely isotropic half-space, with shear’s modulus G_{hs}, Poisson’s ratio ν_{hs}, mass density ρ_{hs} and material damping coefficient η_{hs}. The index ‘p’ is related to the pile, with Young’s modulus E_p, mass density ρ_p, radius a_p and length h_p. The index ‘f’ is related to the foundation with mass m_f.

The coordinate system is placed so that the x-y plane is aligned with the surface of the half-space, the pile is aligned along the z-axis and the center of the foundation coincides with the origin of the coordinate system (Fig.1a). The analysis is considered stationary, in the other words, in the frequency domain.



(a) (b)

Figure 1. (a) Foundation interacting with soil and pile and (b) equilibrium of forces

Although the soil is modelled as a 3D domain, only studied the vertical displacement of the foundation studies. The pile vertical response is obtained through a variational approach. For modelling purposes, the foundation mass is considered to be concentrated at the center of the soil-pile-foundation interface.

Three cases are studied is this work. In the first case, the foundation is considered to be perfectly bonded to the soil surface without the presence of the pile. In the second case, the foundation is only supported by the pile and all the load transfer from the incident vertical wave to the foundation occurs throught the pile. The third case is the intermediate case of the two extreme cases mentioned above, where the foundation is partially supported by the soil and partially supported by the pile. A vertical wave field impinges on the soil free surface and also on the pile head. The foundation reaction forces due to this incident wave field are divided between the soil-foundation interface F_{hs} and the pile-foundation interface F_p in a continously varying manner due to a parameter (α).

Figure 1b shows the forces acting on the structure. The reaction force (F_{rea}) is caused by the pile resistance and by the forces at the foundation-soil interface, as shown in Eq. (1):

$$F_{rea} = F_{hs} + F_p \tag{1}$$

The reaction force is a setting that changes continuously the pile force F_p to soil force F_{hs}.

How the soil displacement U_{hs} and the pile displacement U_p must to be the same, it is possible to change the soil force F_{hs} and the pile force F_p.

The soil-pile reaction force acting on the foundation F_{rea}, is given by Eq. (2):

$$F_{rea} = (1-\alpha)F_{hs} + \alpha F_p \quad (2)$$

in which case

$$0 \leq \alpha \leq 1 \quad (3)$$

When α=1, then force acting on the foundation is solely due to the soil reaction, F_{hs}. This corresponds to the first case in which the foundation is completely supported by the soil. When α=0, the incident vertical wave is balanced solely by the pile reaction and corresponds to the second case in which the foundation is completely supported by the pile. Any other value of α corresponds to the third case.

FORMULATION

Model of Pile and Soil

The model of pile used in this article was proposed by Rajapakse and Shah (1989) and is based on a variational formulation established from an Ansatz function for the vertical displacement of the body of the pile. An example of Ansatz function based on exponential functions is

$$w(z,t) = \sum_{n=1}^{N} \alpha_n(t) e^{-(n-1)z/h_p} \quad (4)$$

in which z is the vertical coordinate according to the Cartesian system shown in Fig. 1, h_p is the length of the pile, and α_n (n=1,N) are arbitrary constants to be determined according to the boundary conditions of a certain problem. For the case of the embedded pile in a homogeneous half-space, these constants arise from the solution of an algebraic system of equations, the terms of which involve the flexibility matrices of both the pile and the soil. A detailed description of this system is beyond the scope of this article and can be found at Rajapakse and Shah (1989). The solution of that system yields α_n, which can be substituted into Eq. (4) to obtain the displacement profile of the embedded pile. The soil model used in this article is a homogeneous, transversely isotropic, three-dimensional half-space, the Green's functions of which were derived by Rajapakse and Wang (1993).

First case: Soil-Foundation System

In the first case considered in this article, a rigid foundation is in perfectly bonded contact with its underlying soil (Fig. 2a). The plate-soil system is under the excitation of a stationary incident vertical wave U_{inc}. Figure 2b shows the forces at the foundation-soil interface. The equilibrium equation of the foundation in this system is

$$F_{hs} = -\omega^2 M_f U_f \quad (5)$$

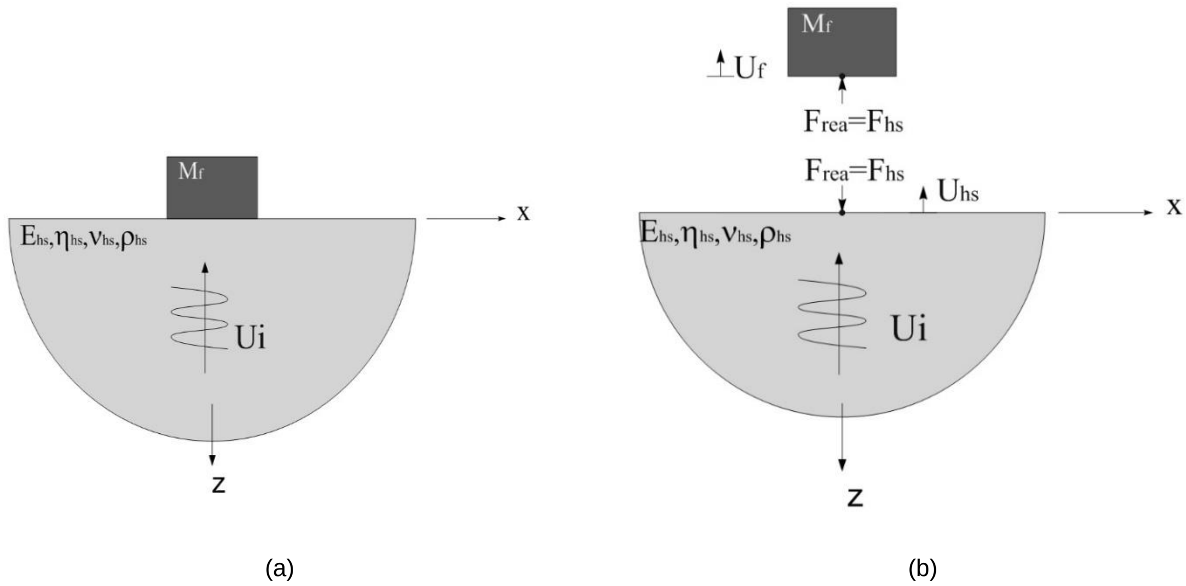


Figure 2. (a) Foundation interacting only with soil and (b) equilibrium of forces

The total displacement field of the surface of the soil (r=0, z=0) is

$$U_t = U_{inc} + U_\zeta \tag{6}$$

in which U_ζ is the scattered wave field, and the corresponding forces at the free surface due to U_{inc} are:

$$F_t = F_i + F_\zeta \tag{7}$$

in which the force at the free surface F_i due to U_{inc}, is zero. Hence, the force F_s at the foundation-soil interface equals the scatter force F_ζ,

$$F_{hs} = F_t = F_\zeta \tag{8}$$

The scattered wave field U_ζ and the scatter force F_ζ are related through the soil flexibility S_{hs},

$$U_\zeta = S_{hs} F_\zeta \tag{9}$$

The soil dynamic flexibility S_{hs} is obtained as described in Section “Model of pile and soil”.

In view of Eqs. (6) to (9), the dynamic equilibrium of the foundation (Eq. 4), can be written as:

$$U_t = U_i - \omega^2 S_{hs} M_f U_f \tag{10}$$

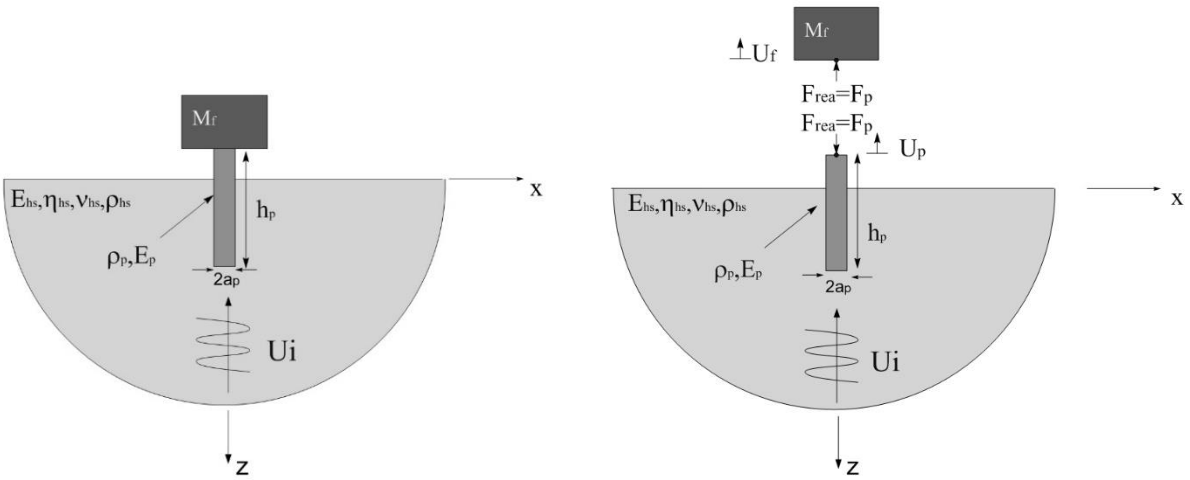
Finaly, in order to satisfy the criteria of kinematic compatibility at the foundation-soil interface, the total displacement of the system, U_t, must be equal to that of the foundation, U_f. This yield, from Eq. (10),

$$U_f = \left(1 + \omega^2 S_{hs} M_f\right)^{-1} U_{inc} \tag{11}$$

Equation (11) gives the displacement of a rigid foundation in bonded contact with the surface of the soil, in response to an incident vertical wave field with amplitude U_{inc}.

Second case: Pile-Foundation System

In second case, the surface foundation is fully supported by the pile, which in turn is in bonded contact with the soil throughout its length. There is no contact between the foundation and the soil (Fig. 3a). Figure 3b shows the forces acting at the foundation–pile interface.



(a) (b)

Figure 3. (a) Foundation interacting only with pile and (b) equilibrium of forces

The same reasoning leading to Eq. (4) from (9) applies in second case, except that the flexibility of the pile, S_p, is used instead of S_{hs}, and that the force applied on the foundation in Eq. (5) is F_p, instead of F_{hs}. The pile flexibility S_p is obtained as described in Section "Model of Pile and Soil". Hence, an analogous equation relates the displacement of the foundation, U_f, with the magnitude of the incident vertical wave field, U_{inc}:

$$U_f = \left(1+\omega^2 S_p M_f\right)^{-1} U_{inc} \tag{12}$$

Equation (12) gives the displacement of a rigid foundation that is fully supported by an embedded pile, in response to an incident vertical wave U_{inc}.

Third case: Soil-Pile-Foundation System

In this section, the soil-pile-foundation system is investigated. In this case, the foundation interacts partially with the soil and partially with the pile, as shown in Fig. 1.

Observing Fig.1b, it can be seen that the only force acting at the soil-pile-foundation interface is the reaction force (F_{rea}). However, that Eq. (2) is also the total force acting on the soil surface, then:

$$F_{rea} = F_t = F_\zeta \tag{13}$$

The scattered wave field U_ζ and the scatter force F_{rea} are related through the soil flexibility S_{hs} and pile flexibility S_p,

$$U_\zeta = S_{hs} F_{hs} + S_p F_p \tag{14}$$

Applying the dimensionless variable α in Eq. (14) to study the continuous change from one mechanism support to another,

$$U_\zeta = (1-\alpha) S_{hs} F_{hs} + \alpha S_p F_p \tag{15}$$

Figure 1b shows the forces at the foundation-soil interface. The equilibrium equation of the foundation in this system is:

$$F_{rea} = \alpha F_p + (1-\alpha) F_{hs} = -\omega^2 M_f U_f \tag{16}$$

In view of Eqs. (13) to (16), the dynamic equilibrium of the foundation, can be written as:

$$U_f = \frac{U_{inc} - \left(\frac{U_{inc}(1-\alpha)}{S_p} + \frac{U_{inc}\alpha}{S_{hs}}\right)\left(\frac{1}{S_{hs}} + \frac{1}{S_p}\right)^{-1}}{1 + \left(\omega^2 M_f - \frac{(1-\alpha)}{S_p} - \frac{\alpha}{S_{hs}}\right)\left(\frac{1}{S_{hs}} + \frac{1}{S_p}\right)^{-1}} \tag{17}$$

Equation (17) gives the displacement of the foundation subject to an incident vertical wave that is partially supported by the pile and partially supported by the soil.

In the Eq. (17), if the alpha value is equal to zero, the Eq. (17) becomes equal to the Eq. (11), that is the foundation is only supported by the soil (first case studied in this work). If the alpha value is equals to one, the Eq. (17) becomes equal to the Eq. (12), that is the foundation is only supported by the pile (second case studied in this work).

NUMERICAL RESULTS

This section presents numerical results to investigate the displacement of the foundation shown in Fig. 1a and study the influence of dimensionless variable α (Eq.17). For the purpose of presentation of numerical results, the following normalizations are defined: ratio of modulus of elasticity $E'=E_p/E_{hs}$, ratio of density $\rho'= \rho_p/ \rho_{hs}$ and mass ratio $B=M_f/M_{hs}$, in which M_f is the mass of the foundation and M_{hs} is the mass of the soil. The soil mass, M_{hs}, is defined as the mass comprised by a volume formed by the area of the soil-foundation interface possessing a unit depth.

The soil-pile-foundation system is subjected to an incident vertical wave with unit amplitude $U_{inc}=1$. In this section, the following parameters are considered: for the half-space, $E_{hs}=2.5$Pa, $\eta_{hs}=0.01$, $\upsilon_{hs}=0.25$, $\rho_{hs}=1$kg/m³; for the foundation, $a_f=1$m, and for the pile: $h_p/a_p=35$, $\rho_p=1$kg/m³. The proposed formulation in which the foundation interacts simultaneously with the pile and the soil (Fig. 1a), have been compared with a solution by Ji and Pak (1994).

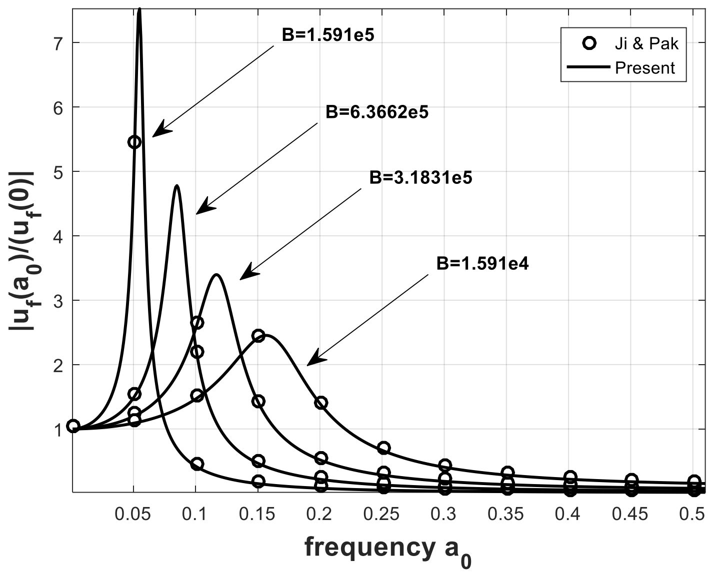
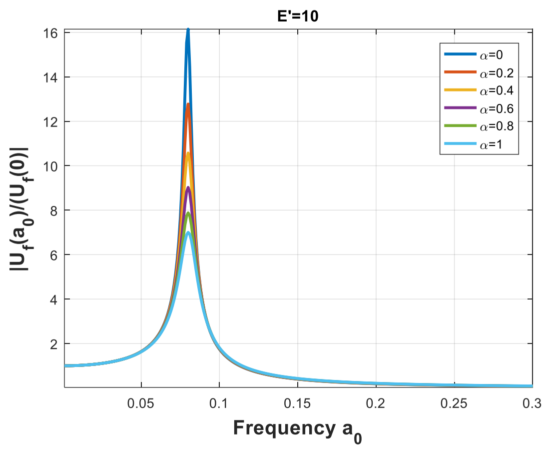


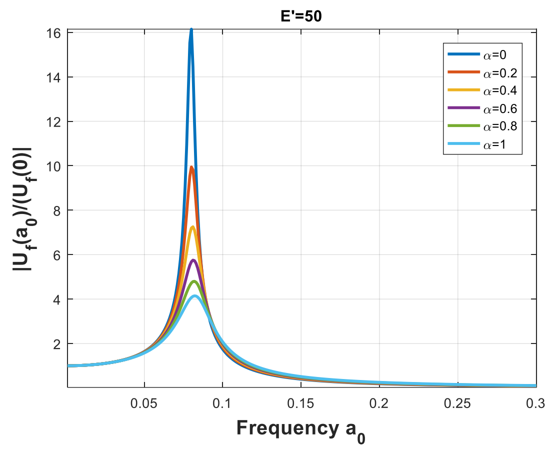
Figure 4. Validation of the pile-raft foundation with previous results

Figure 4 shows a good agreement of the present formulation with their alternative equivalents for different sets of data.

Figures 5a to 5d show the displacement and phase of the foundation for the case B=50. Different values of E' (E'=10 and E'=50) are considered. The effects of the parameter (α) are also shown.



(a)



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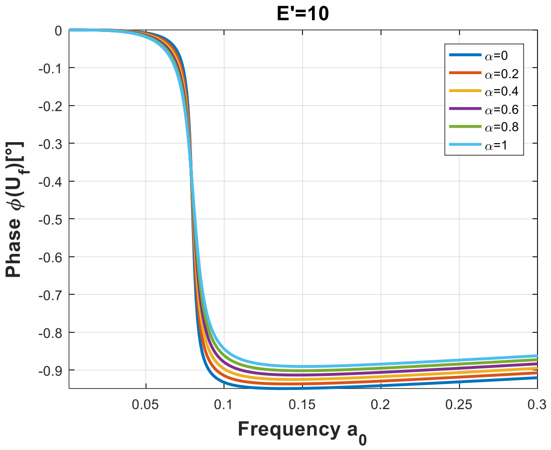
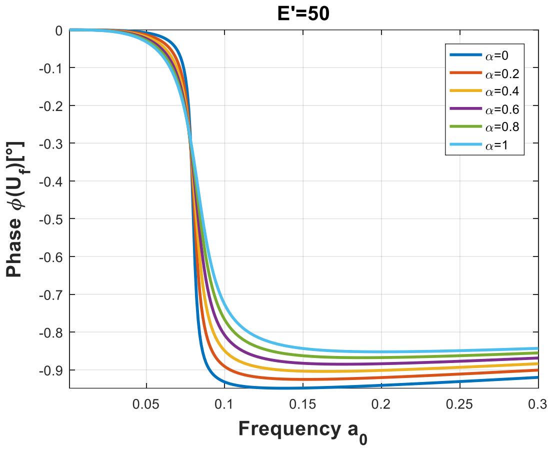


(c)



(d)

Figure 5. Absolute displacement and phase of the foundation with B=50 (E'=10 and 50)

Figure 6a to 6d shows the same analysis for the case of a foundation with B=50 and distinct values of E' (E'=100 and E'=150).

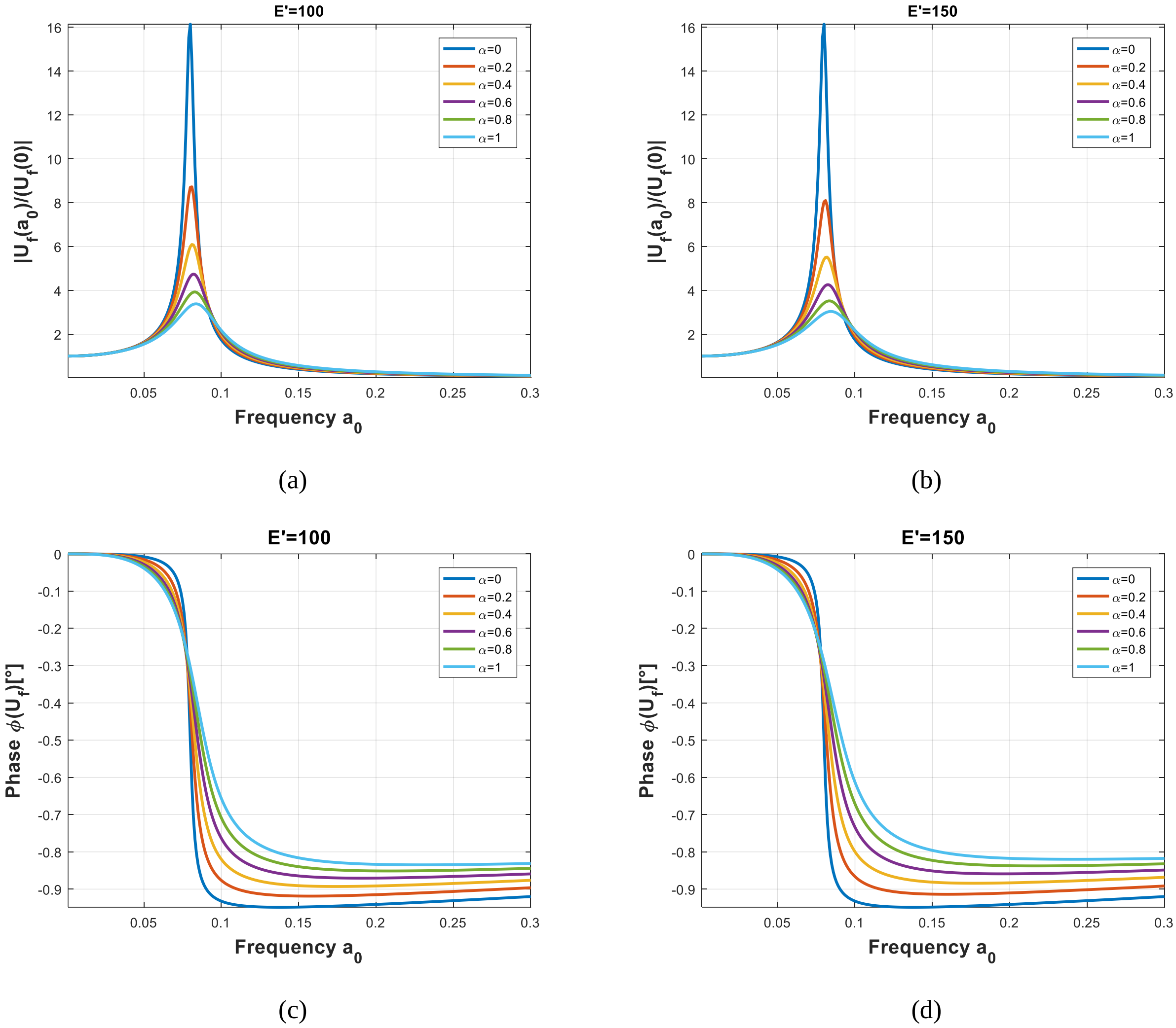


Figure 5. Absolute displacement and phase of the foundation with B=50 (E'=100 and 150)

An analysis of Fig.5 and Fig.6 show a well-defined resonance region for all cases considered. It also shows that when α=0, the displacements of the foundation is always the same independently of the value of E'.That is physically consistent because when α=0, the pile it not present in the coupled system (the foundation is fully supported by the soil). Besides that, it is observed that an increase in α corresponds to a decrease in the displacement of the foundation.

Figure 7 shows the influence of the relative density of the pile and soil in the displacement of the foundation in the foundation–pile–soil case with a mass ratio B=50. Two cases are shown. Upper figures shows the case with E' and ρ'=100 and the bottom figures shows the case with E'=10 and ρ'=100. The figures pictured at the right side are amplitude zooms of the left side figures.

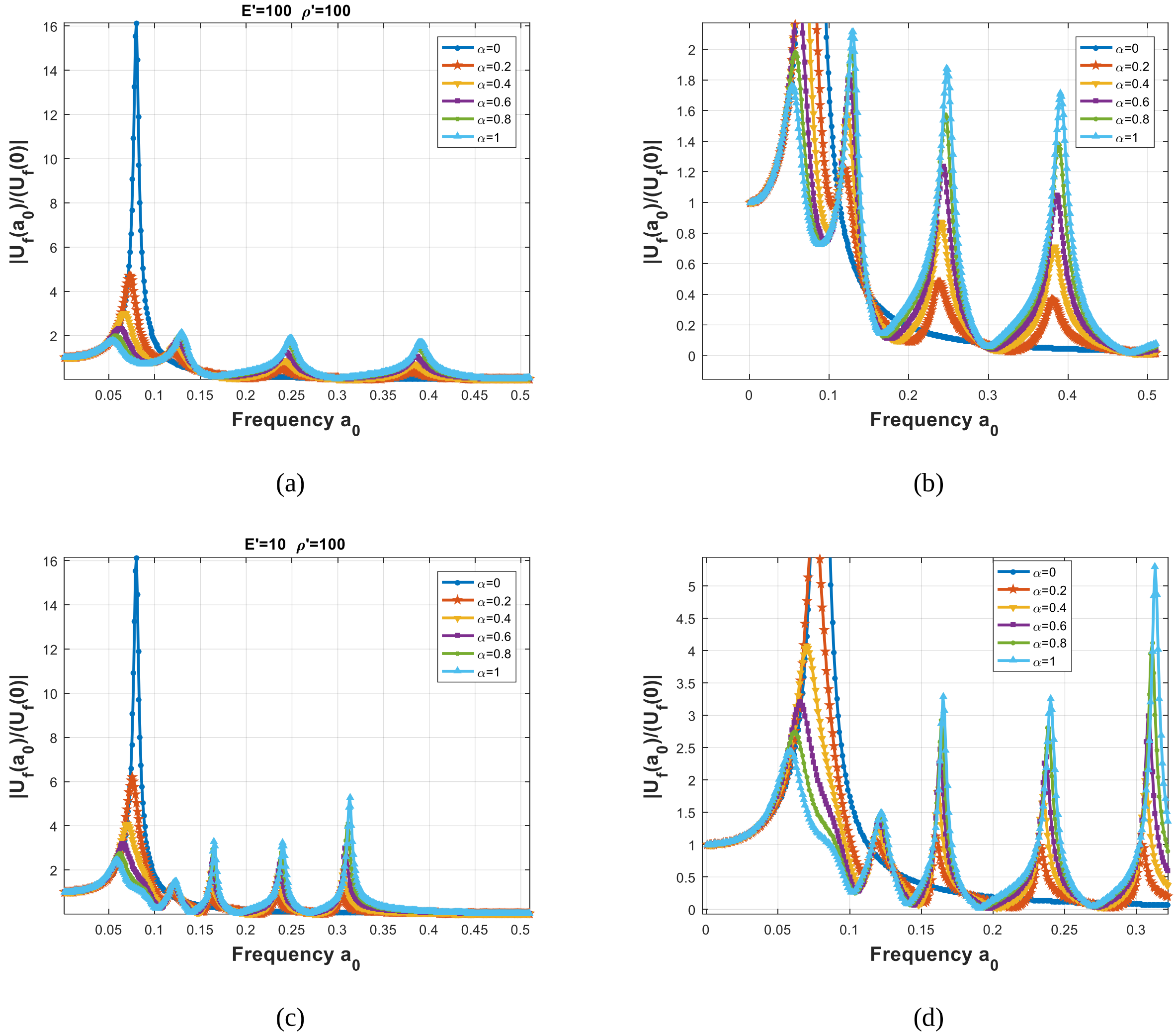


Figure 7. Vibration damping of surface foundation due to increase in pile mass.

Comparing Fig. 5, 6 and 7, it is noticed that as the density of the pile increases, besides the rigid mode resonance of the foundation, other resonances related to the flexible pile response are also observed. The only case that does not happen other resonances is when the alpha is zero, that is, when the foundation is fully supported by the soil. The ratio of the modulus of elasticity also influences resonances related to the flexible pile response.

Figure 8 shows a comparison of the displacement of the foundation when it is subjected to an external force or an incident wave field with B=50. The response of the foundation subjected to a unitary external force was obtained according to Lima, Labaki and Mesquita (2016).

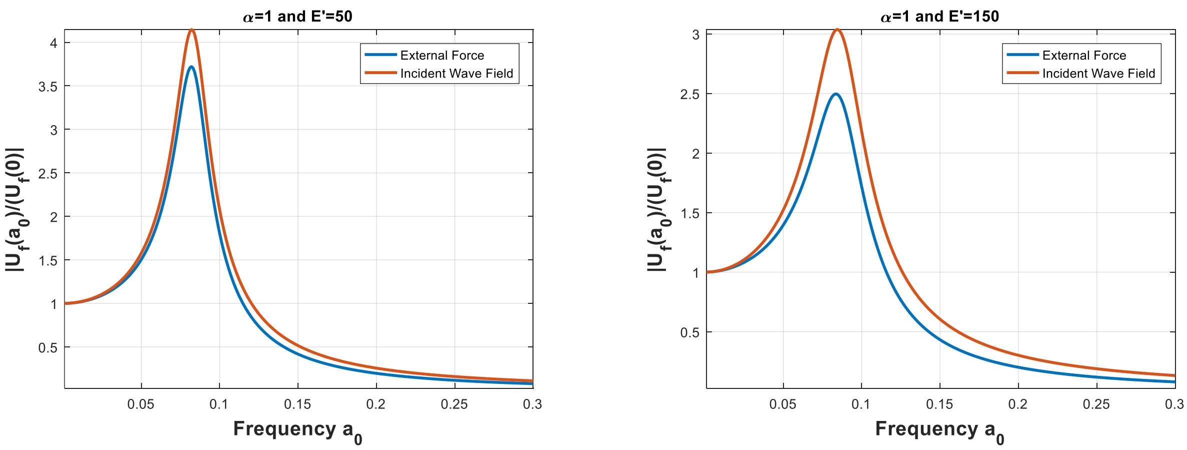


Figure 8. Comparison of the displacement of the foundation subjected to an external force or an incident wave field

As can be observed in Fig. 9, the behavior of the displacement of the foundation subjected to an external force or to an incident wave field is very similar. However, the normalized amplitude of the foundation displacement is slightly greater when subjected to an incident vertical wave field. But, as expected, the resonance frequency is the same for both exciting mechanisms, external force or incident wave field.

CONCLUSIONS

This paper presented a formulation for analyzing the vertical dynamic response of a circular rigid foundation interacting with an underlying pile and with a homogeneous 3D half-space subjected to a vertical incident wave field. A dimensionless parameter was introduced, which allows an investigation of coupling configuration between two extremes case: a) in which a rigid foundation is fully supported by a homogeneous half-space, and b) the case in which the foundation is fully supported by an underlying pile. The study has shown that the response of the coupled system depends significantly on the amount of load that is transferred from the foundation to the pile and the soil. For the representative cases considered in this model, the introduction of a pile results in an overall decrease in the vibration amplitude of the system. For all cases considered, an increase in the stiffness of the pile causes a slight increase in the resonance frequency of the system and a decrease in the displacement amplitude of the foundation. When relative stiffness and inertia parameters of the pile are increased the pile resonances are clearly depicted in the results. But in all studied cases, as the load is transferred from the soil-foundation interface to the pile-foundation interface (α: $0 \rightarrow 1$), the foundation response amplitude decreases.

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