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Dynamic iterative coupling scheme applied to the transient response of systems with soil structure interaction

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Abstract: This article describes an iterative coupling scheme to analyze the transient dynamic response of structures interacting with distinct soil profiles. Structures with lumped parameters and linear equations of motions are solved by the Newmark integration scheme. The soil, on the other hand, is an unbounded domain presenting outgoing and non-reflected waves that withdraw energy from the excitation source. This effect is known as Sommerfeld radiation condition and is also called radiation damping. The modeling of unbounded domains presenting radiation damping requires special techniques that incorporate this damping effect, such as the Boundary Element Method (BEM) or a semi-analytical method based on a Green's function approach (GF). The stationary dynamic behavior of soils has been successfully solved by the BEM and GF strategies. Nevertheless, the transient dynamic response of structures interacting with unbounded domains is still limited to a few cases. In this article the stationary dynamic response of the soil has been obtained by GF methods for very high frequencies, what, in conjunction with the FFT algorithm, allows to obtain soil impulse responses very accurately and for very small steps. The soil response to arbitrary time excitation is obtained by Duhamel's convolution integral. Special attention is given to higher order implementation of the Duhamel's integration scheme, what is a requirement to achieve accurate results for the soil response. In the proposed coupling scheme, the Newmark time integration algorithm is coupled with a discretized version of the Duhamel integral, used to obtain the transient soil response. A relaxation procedure is also applied to the iterative coupling scheme. The synthesized methodology is used to analyze the response of linear structures interacting with homogeneous and layered soil-profiles.

Keywords: Dynamic soil structure interaction, iterative coupling procedure, convolution integral.

INTRODUCTION

This article describes an iterative coupling scheme to analyze the transient dynamic response of structures interacting with distinct soil profiles. Structures with lumped parameters and linear equations of motions are considered. These time domain linear equations are solved by the Newmark integration Scheme. The soil, on the other hand, is an unbounded domain presenting outgoing and non-reflected waves that withdraw energy from the excitation source. This effect is known as Sommerfeld radiation condition and is also called radiation damping. The modeling of unbounded domains presenting radiation damping requires special techniques that incorporate this damping effect, such as the Boundary Element Method (BEM) (Carrion et al., 2007) or a semi-analytical method based on a Green's function approach (GF) (Labaki *et al.*, 2014). The stationary dynamic behavior of soils has been successfully solved by the BEM and GF strategies (Carrion et al., 2007; Labaki et al., 2014). The transient analysis of phenomena modelled by the BEM has experienced large progress as can be seen in the literature (Coda and Venturini, 1995; Gaul et al., 1992). Nevertheless, the transient dynamic response of structures interacting with unbounded domains is still limited to a few cases. This article aims to describe an iterative coupling strategy that allows to obtain the transient solution of linear structures interacting with unbounded soil profiles. The stationary dynamic response of the soil has been obtained for very high frequencies, what, in conjunction with the FFT algorithm, allows to obtain soil impulse responses very accurately and for very small steps (Mesquita et al., 2012). The soil response to arbitrary time excitation is obtained by a convolution integral, Duhamel's integral (Mesquita et al., 2012; Damasceno, 2013). Special attention is given to higher order implementation of the Duhamel's integration scheme, what is a requirement to achieve accurate results for the soil response. In the proposed coupling scheme, the Newmark time integration algorithm is coupled with a discretized version of the Duhamel integral, used to obtain the transient soil response.

STATEMENT OF PROBLEM

A very attractive scheme to determine the transient response of foundations and structures interacting with the soil is given by an iterative coupling of the subsystems. This is exemplified in Fig. 1. A foundation of mass, M_f, is interacting with an unbounded soil domain of Lamé constant G, mass density ρ, and material damping η. The system is excited by an external, transient force F_{ext}. Soil and foundation may be considered as two distinct systems and each separate system can be solved by the best available methodology. At the interface between the systems interface forces $F_{S1} = F_{S2}$ are observed. The displacement of the interface is U_F and U_S. The transient response of the foundation may be determined by a time integration algorithm, i.e., the Newmark integration scheme. The transient response of the soil may be obtained by performing a convolution between the current excitation F_{S2} and the response due to unit pulse excitation, also called transient Green's function. A typical example of a soil response to a unit impulse is shown in Fig. 2a. Figure 2a presents the impulse in frequency domain. This response can be obtained in time domain (Fig. 2b) through the application of the Inverse Fast Fourier Transformed (IFFT).

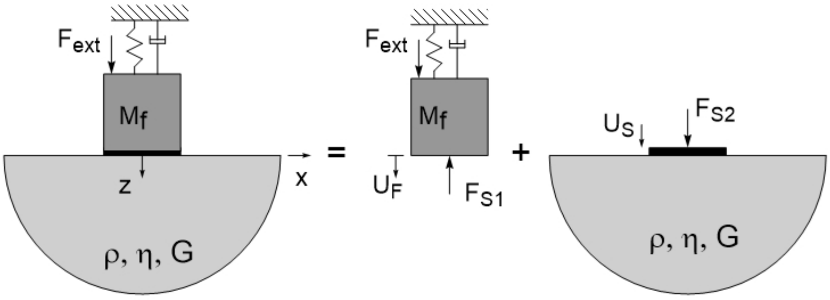
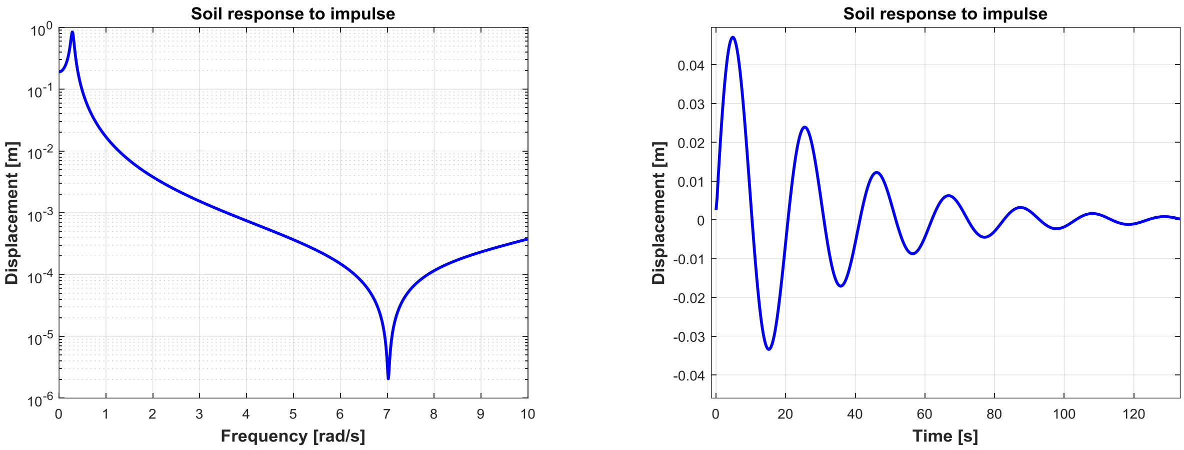


Figure 1 – Decomposition of soil-foundation into two subsystems.

The proposed strategy is to couple iteratively two subsystems, the response of the first, the structure, being determined by a classical time integration algorithm, and the soil response by a convolution integral. This scheme has been implemented by Damasceno (2013). One of the findings of that research is that the convergence of the iterative coupling procedure is highly dependent on the accuracy of the solution scheme used for each subsystem.



(a) **(b)**

Figure 2 – Soil response to impulse. (a) Frequency domain. (b) Time domain.

PROPOSED SOLUTION

The main idea is to solve the soil structure problem dividing the unique system into two subsystems. So, it´s possible to study each subsystem using the most appropriate method. As shown in Fig. 3, the structure, which has a bounded domain, can be solved through FEM (Finite Element Method) or Newmark algorithm, while the soil, which has an unbounded domain, can be solved through BEM (Boundary Element Method) or convolution integral. According to convolution theorem, if the impulse response is known for a linear system, it´s possible to determine the dynamic response of this system to an arbitrary external force (Cheng, 1972). For example, it is possible to apply the impulse response shown in Fig. 2b to determine the dynamic response for the soil through the convolution integral.

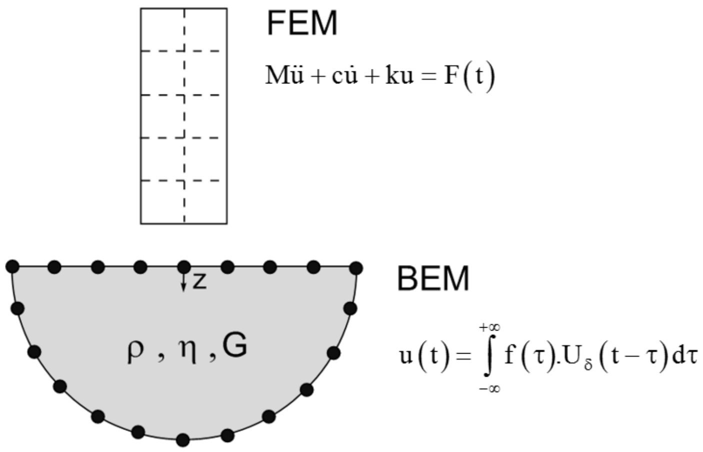


Figure 3 – Idealized soil-structure model solved by FEM x BEM (Tovo, 2018).

ITERATIVE COUPLING PROCEDURE

The problem with soil structure interaction can be studied through an iterative coupling procedure based on partitioned solution. Damasceno (2013) developed a methodology which consists in subdivide the unique system into two subsystems, and then, coupling them through an iterative process. In this way, it is possible to study each subsystem using the most appropriate method. In the case of soil structure problem, the structure, which has a bounded domain, can be treated with FEM (Finite Element Method) or Newmark Algorithm. On the other hand, the soil, which has an unbounded domain, can be treated with the numerical convolution integral. This methodology is shown in Fig. 4 below.

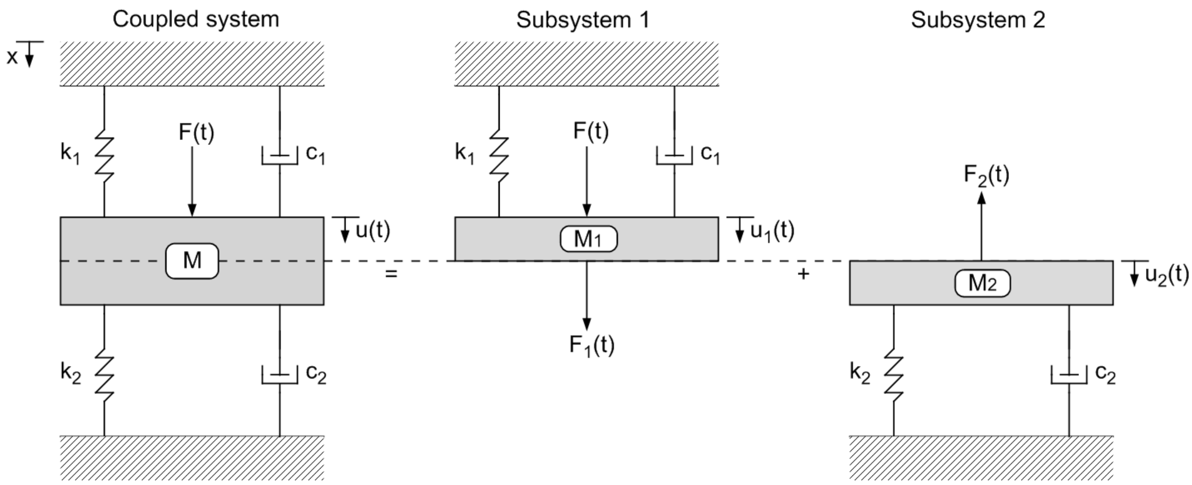


Figure 4 – Coupled one degree of freedom system divided in subsystems 1, 2 (Damasceno, 2013).

During the iterative coupling procedure, the soil and foundation (structure) are treated by distinct methodologies and at every time step the coupling technique tries to determine kinematic compatibility and force equilibrium at the soil-foundation interface. In this paper, it was considered the coupled system as one-degree-of-freedom system excited by a cosine force. This case, in particular, has the analytical solution and it will be compared with subsystems 1,2 evaluated by numerical methods.

Figure 5 bellow presents the coupling iterative algorithm. In the first loop, the coupling forces f_1 , f_2 (subsystems 1,2) are arbitered. The second loop is related to iterative process. The convergence criterion of this technique is based on the difference between the response of subsystems 1,2. In this work, it was applied an error about 10^{-12}. The objective of the iterative coupling procedure is to obtain $u = u_1 = u_2$. In the next section, this relation will be analyzed for the proposed methods.

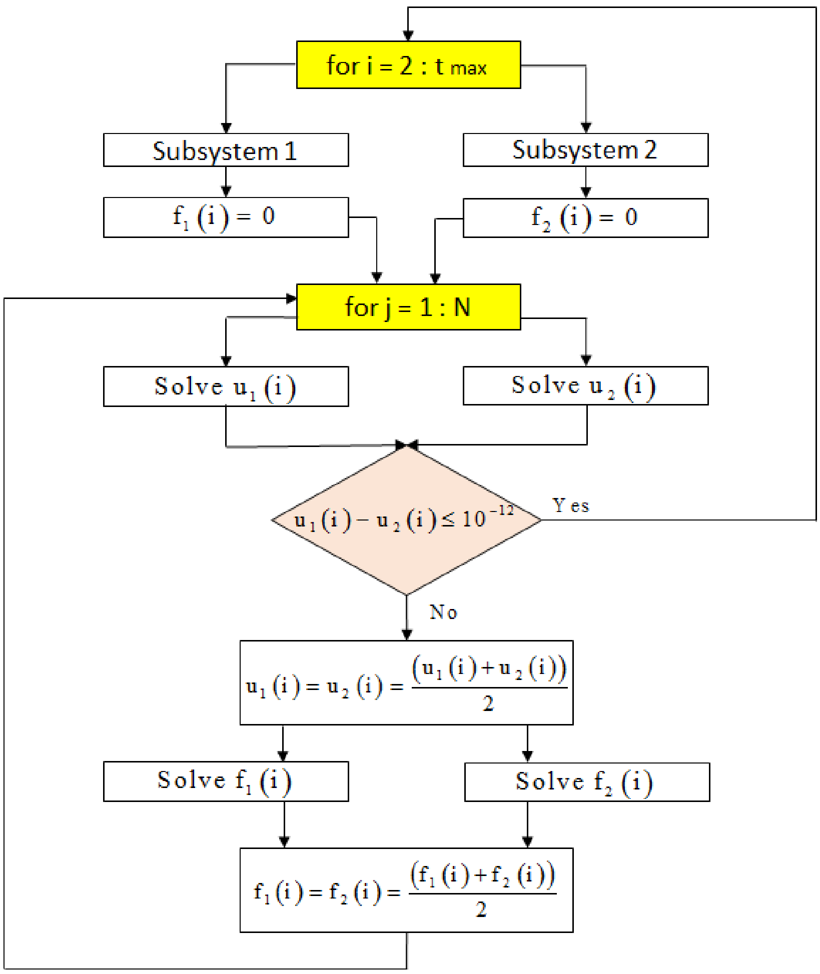


Figure 5 – Coupling iterative algorithm (Tovo, 2018).

Coupling force for Newmark algorithm

The calculation of the coupling force of the subsystems 1,2 are obtained manipulating the expressions used to calculate the displacement. In the Newmark algorithm, the displacement is given by the Eq. (1) bellow:

$$M_1.\ddot{u}^{(1)}_{i+1} + c_1.\dot{u}^{(1)}_{i+1} + k_1.u^{(1)}_{i+1} = F_{i+1} + F^{(1)}_{i+1} \tag{1}$$

$$F^{(1)}_{i+1} = M_1.\ddot{u}^{(1)}_{i+1} + c_1.\dot{u}^{(1)}_{i+1} + k_1.u^{(1)}_{i+1} - F_{i+1} \tag{2}$$

$$M_2.\ddot{u}^{(2)}_{i+1} + c_2.\dot{u}^{(2)}_{i+1} + k_2.u^{(2)}_{i+1} = -F^{(2)}_{i+1} \tag{3}$$

$$F^{(2)}_{i+1} = -\left[M_2.\ddot{u}^{(2)}_{i+1} + c_2.\dot{u}^{(2)}_{i+1} + k_2.u^{(2)}_{i+1}\right] \tag{4}$$

In Eqs. (1), (2), (3), (4), the top index represents the subsystem and the index (i+1) the step of time.

Coupling force for convolution approximated by constant element

According to Tovo (2018), the displacement obtained through the convolution integral approximated by element constant is given by the Eq. (5):

$$u_{total}(t_{máx} = n\Delta t) = \sum_{i=1}^{n} f_{(i+1)}.h_{(n-(i-1))}.\Delta t \tag{5}$$

Manipulating the Eq. (5), it´s possible to obtain the coupling force equation of the subsystems 1,2, yields:

$$u_{total} = u_{partial} + \underbrace{f_{(n+1)}.h_{(1)}.\Delta t}_{i=n} \tag{6}$$

$$f_{(n+1)} = \frac{\left(u_{total} - u_{partial}\right)}{h_{(1)}.\Delta t} \tag{7}$$

Coupling force for convolution approximated by first order shape functions

The displacement obtained through the convolution integral approximated by first order shape functions is given by the Eq. (8) (Tovo, 2018):

$$u_{total}(t_{max} = n\Delta t) = \sum_{i=1}^{n} \left\{ \begin{bmatrix} f_{(i)} & f_{(i+1)} \end{bmatrix}_{1x2} . \begin{bmatrix} \left(\frac{1}{6}\right) & \left(\frac{1}{3}\right) \\ \left(\frac{1}{3}\right) & \left(\frac{1}{6}\right) \end{bmatrix}_{2x2} . \begin{bmatrix} h_{((n-i)+1)} \\ h_{((n-i)+2)} \end{bmatrix}_{2x1} \right\} .\Delta t \tag{8}$$

Analogously, manipulating the Eq. (8), it´s possible to determine the coupling force equation of the subsystems 1,2:

$$u_{total} = u_{partial} + \underbrace{\left\{ f_{(n)} . \left[h_{(1)} . \left(\frac{1}{6}\right) + h_{(2)} . \left(\frac{1}{3}\right) \right] + f_{(n+1)} . \left[h_{(1)} . \left(\frac{1}{3}\right) + h_{(2)} . \left(\frac{1}{6}\right) \right] \right\} .\Delta t}_{i=n} \tag{9}$$

$$f_{(n+1)} = \frac{\left(u_{total} - u_{partial}\right) - \left\{ f_{(n)} . \left[h_{(1)} . \left(\frac{1}{6}\right) + h_{(2)} . \left(\frac{1}{3}\right) \right] .\Delta t \right\}}{\left[h_{(1)} . \left(\frac{1}{3}\right) + h_{(2)} . \left(\frac{1}{6}\right) \right] .\Delta t} \tag{10}$$

NUMERICAL RESULTS

In this section, the numerical results of subsystems 1,2 are compared with the analytical solution of coupled system. In this paper, the coupled system represents a one degree of freedom system excited by a cosine force. In Eq. (11), F_0 is the amplitude of the cosine force and ω is the excitation frequency of the system.

$$F(t) = F_0 \cdot \cos(\omega t) \tag{11}$$

According to Inman (2014), the analytical response of a one-degree-of-freedom (1 DOF) mass-spring-damping system excited by a cosine force is given by the sum of particular and homogeneous solution, as shown in Eq. (12), and its parameters are: ωn = (k/m)1/2 is the natural frequency of the system, ξ = c/(2.(k.m)1/2) is the damping factor, and ωd = ωn.(1- ξ2)1/2 is the cushioned frequency of the system.

$$u(t)_{analytical} = A_m e^{(-\xi\omega_n t)} \sin(\omega_d t + \varphi) + X_m \cos(\omega t - \theta) \tag{12}$$

According to Inman (2014), the analytical impulse response of a one-degree-of-freedom (1 DOF) mass-spring-damping system is given by Eq. (13), and $\hat{F}$ is the amplitude of the impulse force.

$$u(t)_{impulse} = \left(\frac{\hat{F} e^{-\xi\omega_n t}}{m\,\omega_d} \right) . \sin(\omega_d t) \tag{13}$$

Thus, according to theorem of convolution mentioned in this paper, it´s possible to determine the dynamic response of the 1 DOF system to an arbitrary force through the convolution integral, because the impulse response of the linear system is known, as can be seen in Eq. (13). For example, the dynamic response of the 1 DOF system to a cosine force excitation can be determined by convolution integral represented by Eq. (14) bellow:

$$u(t_{max}) = \int_{\tau=0}^{\tau=t_{max}} \underbrace{\left[F_0 \cos(\omega\tau) \right]}_{\text{External Load}} . \underbrace{\left[\left(\frac{\hat{F} e^{-\xi\omega_n (t_{max} - \tau)}}{m\,\omega_d} \right) . \sin\left(\omega_d (t_{max} - \tau)\right) \right]}_{\text{Impulse Response}} d\tau \tag{14}$$

In this work, the iterative coupling procedure is studied with distinct combinations of methods: Newmark-Convolution and Convolution-Convolution. In each combination, the first method is related to subsystem 1 (structure) and the second method is related to subsystem 2 (soil). For example, in the Newmark-Convolution, the structure is calculated using Newmark algorithm and the soil is calculated using numerical convolution. It is important to know that in this paper, the coupled system, subsystems 1,2 are considered one DOF systems in the calculus of numerical results. The intention in the future is to apply the iterative coupling procedure to real soil-structure problems.

Table 1 presents the values used in the iterative coupling procedure. In this case, it was considered that $M_1=M_2=M/2$, $k_1=k_2=k/2$ and $c_1=c_2=c/2$.

Table 1 – Parameters of dynamic system [IS].

	Coupled System	Subsystem 1	Subsystem 2
Mass [kg]	2	1	1
Damping constant [kg.N/s]	0.2	0.1	0.1
Spring constant [N/m]	2	1	1

Newmark – Convolution approximated by constant element

The first combination used in iterative coupling is the Newmark-Convolution approximated by constant element. Figure 6 presents a comparison between analytical (coupled system) and numerical (subsystems 1,2) solutions. According to Fig. 6, the numerical solutions are close to the analytical solution.

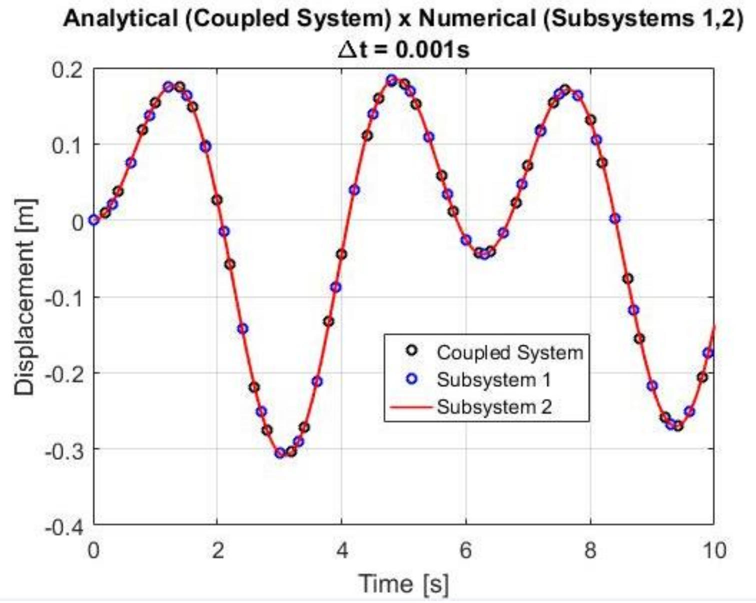


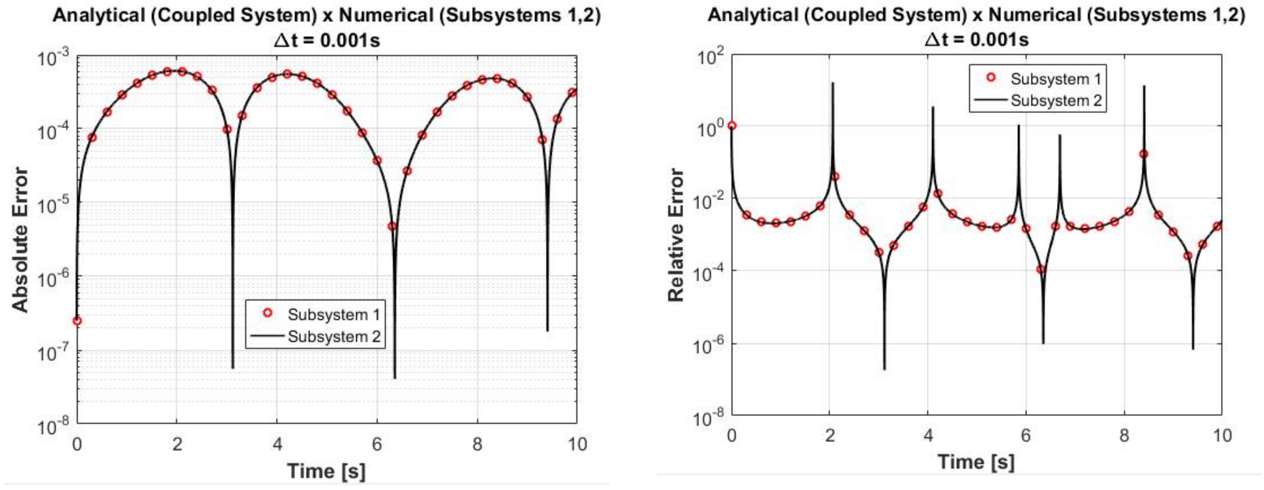
Figure 6 – Dynamic response, in terms of displacement, of coupled system and subsystems 1,2.

Figure 7 presents the absolute and relative errors between analytical (coupled system) and numerical (subsystems 1,2) solutions.

The absolute and relative errors are defined by Eqs. (15) and (16), respectively. The terms $u_{analytical}$ and $u_{numerical}$ are the displacements of the system obtained through analytical and numerical method. The absolute error for this combination consists in 10^{-3} approximately (Fig. 7a).

$$error_{absolute} = \left| u_{analytical} - u_{numerical} \right| \tag{15}$$

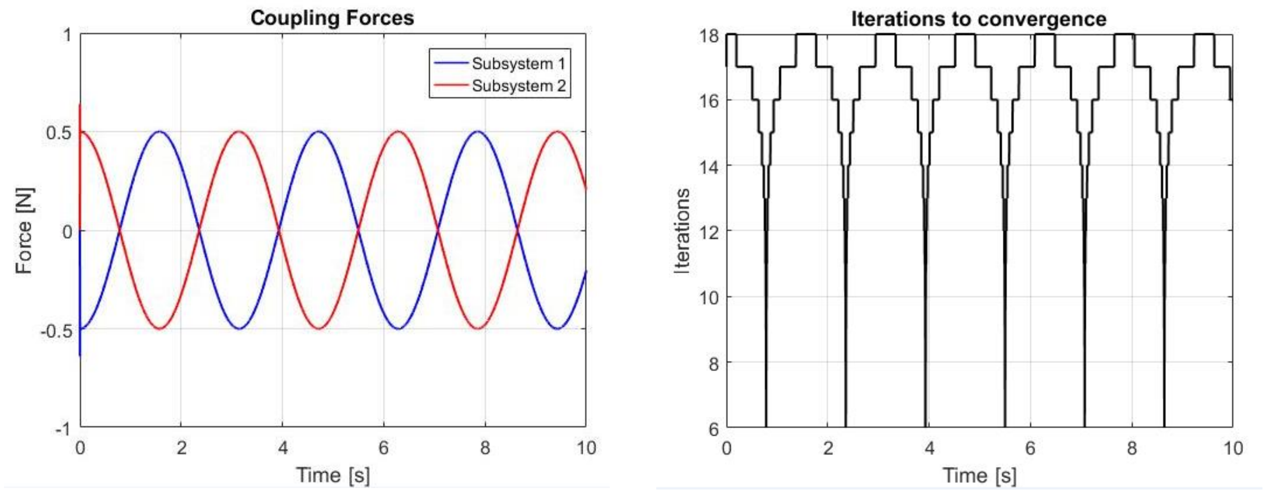
$$error_{relative} = \left| \frac{u_{analytical} - u_{numerical}}{u_{analytical}} \right| \tag{16}$$



(a) **(b)**

Figure 7 – Analytical x Numerical. (a) Absolute error. (b) Relative error.

Figure 8a presents the force coupling of subsystems 1,2 after the convergence in each step time. The amplitude of coupling forces is 0.5 [N] and represents $F_0/2$. Figure 8b presents the number of iterations necessaries to process convergence in each step time.



(a) **(b)**

Figure 8 – (a) Coupling forces. (b) Iterations to convergence.

Newmark – Convolution approximated by first order shape functions

The second combination used in iterative coupling is the Newmark-Convolution approximated by first order shape functions. Figure 9 presents a comparison between analytical (coupled system) and numerical (subsystems 1,2) solutions. According to Fig. 9, the numerical solutions are close to the analytical solution.

Figure 10 presents the absolute and relative errors between analytical (coupled system) and numerical (subsystems 1,2) solutions. The absolute error for this combination consists in 10^{-4} approximately (Fig. 10a).

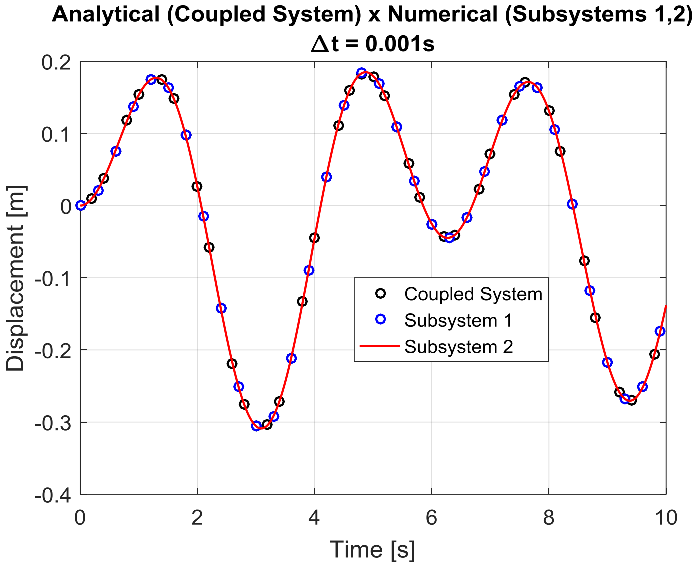
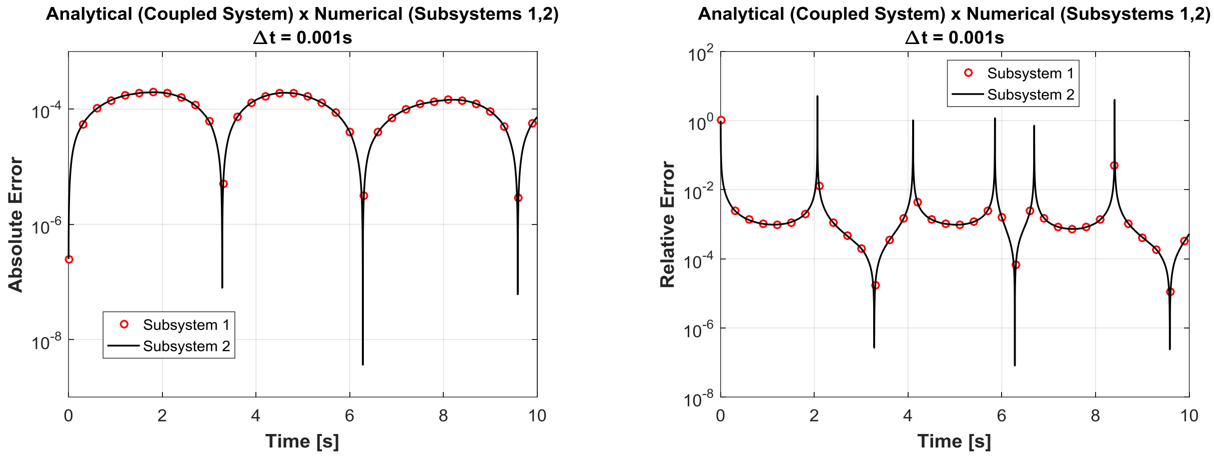


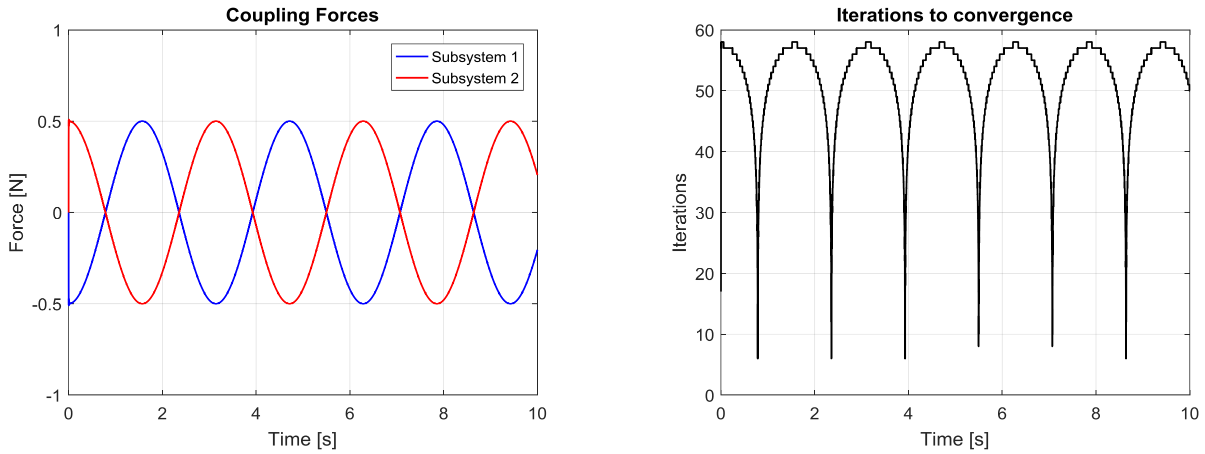
Figure 9 – Dynamic response, in terms of displacement, of coupled system and subsystems 1,2.



(a) **(b)**

Figure 10 – Analytical x Numerical. (a) Absolute error. (b) Relative error.

Figure 11a presents the force coupling of subsystems 1,2 after the convergence in each step time. The amplitude of coupling forces is 0.5 [N] and represents $F_0/2$. Figure 11b presents the number of iterations necessaries to process convergence in each step time.

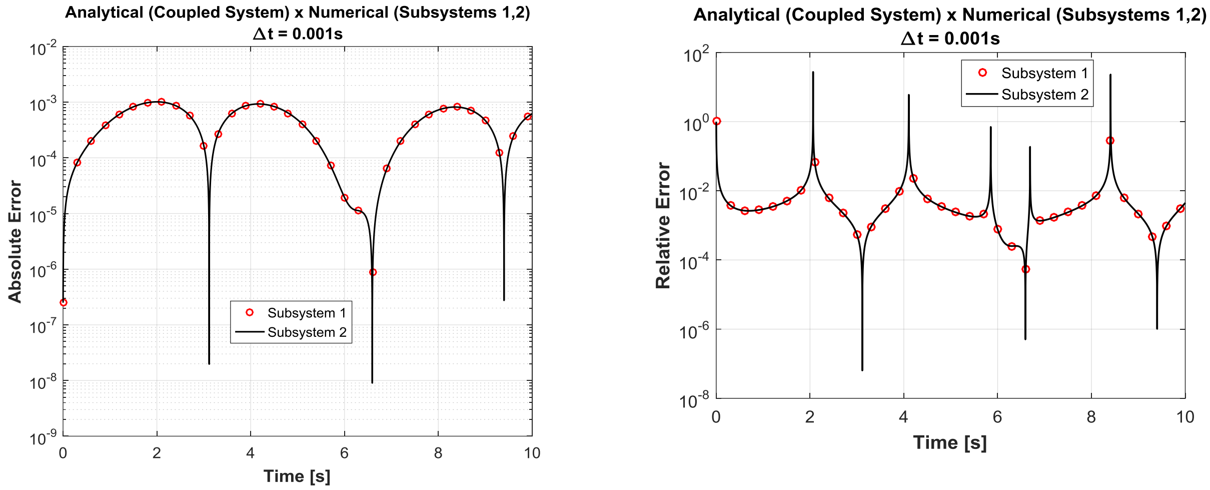


(a) **(b)**

Figure 11 – (a) Coupling forces. (b) Iterations to convergence.

Convolution – Convolution : both approximated by constant element

The third combination used in iterative coupling is the Convolution-Convolution, both approximated by constant element. Figure 12 presents the absolute and relative errors between analytical (coupled system) and numerical (subsystems 1,2) solutions. The absolute error for this combination consists in 10^{-3} approximately (Fig. 12a).

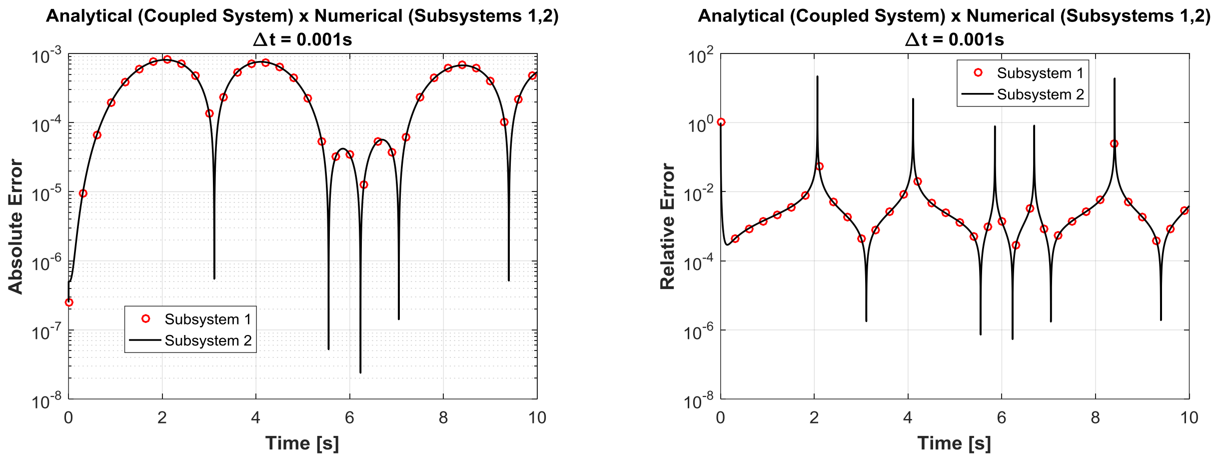


(a) **(b)**

Figure 12 – Analytical x Numerical. (a) Absolute error. (b) Relative error.

Convolution – Convolution : both approximated by first order shape functions

The last combination used in iterative coupling is the Convolution-Convolution, both approximated by first order shape functions. Figure 13 presents the absolute and relative errors between analytical (coupled system) and numerical (subsystems 1,2) solutions. The absolute error for this combination consists in 10^{-3} approximately (Fig. 13a).

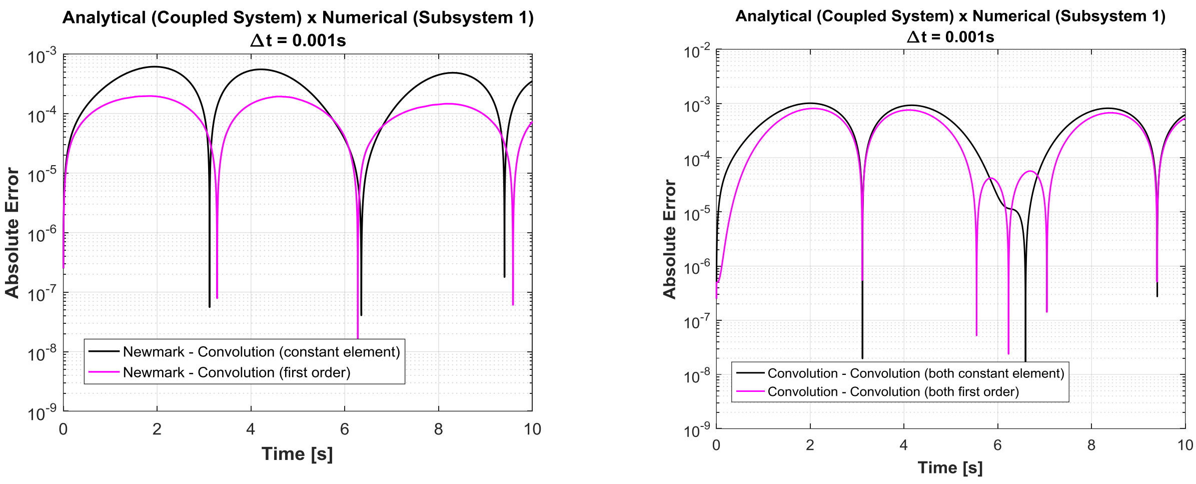


(a) **(b)**

Figure 13 – Analytical x Numerical. (a) Absolute error. (b) Relative error.

Comparison between different methods

Figure 14 presents a comparison between the different combinations applied in the iterative coupling procedure. Figure 14a presents the absolute error between analytical (coupled system) and numerical (subsystem 1) solutions, considering two combinations Newmark-Convolution. Figure 14b presents the absolute error between analytical (coupled system) and numerical (subsystem 1) solutions, considering two combinations Convolution-Convolution.



(a) **(b)**

Figure 14 – Absolute error between analytical and numerical solutions. (a) Newmark - Convolution. (b) Convolution - Convolution.

CONCLUSIONS

This article presented the iterative coupling using Newmark algorithm and numerical convolution approximated by constant and linear functions. The combination Newmark-Convolution approximated by first order shape functions is better than Newmark-Convolution approximated by constant element. In the other hand, the both combinations Convolution-Convolution don't present a significant difference between them. According to the results presented, the Newmark-Convolution can be applied in the study of soil-structure problem. Furthermore, the possibility of extending the procedure to systems with larger number of degrees of freedom can be considered in future studies.

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