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Evaluation of Effective Electromechanical Coupling Coefficient of Piezoelectric Structures Considering Viscoelastic Properties of Adhesive Layer

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Abstract: This work intends to investigate the influences of the viscoelastic behavior of adhesive layer between host structure and surface-bonded piezoelectric patches on the performance of a smart multifunctional plate structure. A finite element model, developed for fully taking into account important properties of the adhesive layer combined with modal strain energy approach to derivate an approximate expression for modal loss factor are used to perform a comparative analysis of the estimated modal effective electromechanical coupling coefficient of the structure.

Keywords: Piezoelectric Materials, Finite Element Method, Viscoelasticity, Smart Structures.

INTRODUCTION

Versatility of structures containing a distributed network of piezoelectric patches has allowed researchers to consider them for multifunctional applications. Thus, surface-bonded piezoelectric patches onto flexible structures are widely employed in airplanes, automotive vehicles or aerospace applications, for active (Sunar and Rao, 1999), passive (Moheimani, 2003) and hybrid active-passive vibration control (Trindade and Benjeddou, 2002), energy harvesting (Sodano, Inman, and Park, 2004) and structural health monitoring applications, (Le et tal., 2015), amongst others. Whatever the specific application of the bonded piezoelectric elements on the flexible structure, it is import to estimate their performance whit a certain accuracy. It is well known however that this performance depends strongly on the effective electromechanical coupling coefficient (EMCCe) provided to host structure to which the piezoelectric patch is bonded, this represents the energy fraction that could be stored in the piezoelectric element when the structure vibrates in a given mode.

The EMCCe is an important parameter that has been used as an estimated indicator of performance for several applications of piezoelectric patches used as sensors and/or actuators, such as active control authority improvement, passive shunt damping, damage detection and piezoelectric energy harvesting. This has also been used as objective function to optimize design, positioning and sizing of piezoelectric transducers. The EMCCe depends on the electromechanical coupling coefficient of the material (EMCC ou k_{ij}^2) and on the mechanical coupling between the piezoelectric element and host structure, consequently it can be expected that the EMCCe will be smaller than the material EMCC (Trindade and Benjeddou, 2009).

On the performance of surface-bonded piezoelectric elements, a few works have been directed to investigate the effect of the adhesive layer present between host structure and piezoelectric patches, indicating a potential reduction on the mechanical coupling (De Faria, 2003; Trindade, Santos and Godoy, 2013; Tinoco et al., 2010). On the other hand, viscoelasticity has been widely studied looking for treatments with free or constrained layer of viscoelastic materials (VEM) for passive vibration control. This work intends to investigate the influence of the viscoelastic behavior of the adhesive layer between host structure and piezoelectric patches on the performance of a smart multifunctional plate-type structure with surface-bonded piezoelectric patches. A finite element model, developed for fully taking into account important properties of the adhesive layer combined with modal strain energy approach to obtain an approximate expression for modal loss factor are used to perform a comparative analysis of the estimated modal EMCCe when the adhesive layer is assumed slightly viscoelastic.

FINITE ELEMENT MODELING

A customized finite element model presented in (Velasquez and Trindade, 2015) is used to model a generic plate-like elastic structure containing a discrete distribution of surface-bonded piezoelectric patches. This mathematical model is thought to take into account important properties of the adhesive layer between the plate and piezoelectric patches such as thickness, Young's modulus and thickness non-uniformity. In this model a strain-voltage model is adopted considering linear and orthotropic piezoelectric materials, and thermal coupling is neglected. Thus, the equations of motion of the system, in matrix form, are given by

$$\begin{bmatrix} \boldsymbol{M} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} \end{bmatrix} \begin{Bmatrix} \ddot{\boldsymbol{u}} \\ \ddot{\boldsymbol{V}} \end{Bmatrix} + \begin{bmatrix} \boldsymbol{K}_u & \boldsymbol{K}_{uV} \\ \boldsymbol{K}_{uV}{}^T & -\boldsymbol{K}_V \end{bmatrix} \begin{Bmatrix} \boldsymbol{u} \\ \boldsymbol{V} \end{Bmatrix} = \begin{Bmatrix} \boldsymbol{F}_m \\ \boldsymbol{0} \end{Bmatrix} \tag{1}$$

Where the elementary matrices of mass, elastic stiffness (for constant electric field), piezoelectric stiffness and dielectric stiffness (for constant strain field) and the elementary vector of external applied forces are, respectively

$$\begin{gathered} \boldsymbol{M}^e = \int_\Omega \rho \bar{\boldsymbol{N}}_u^T \bar{\boldsymbol{N}}_u d\Omega_e\,; \quad \boldsymbol{K}_u{}^e = \int_\Omega \boldsymbol{B}_u{}^T \boldsymbol{c}^E \boldsymbol{B}_u d\Omega_e\,, \quad \boldsymbol{K}_{uV}{}^e = \int_\Omega \boldsymbol{B}_u{}^T \mathbf{e} \boldsymbol{B}_V d\Omega_e\,; \\ \boldsymbol{K}_V{}^e = \int_\Omega \boldsymbol{B}_V^T \boldsymbol{\epsilon}^\varepsilon \boldsymbol{B}_V d\Omega_e\,; \quad \boldsymbol{F}_m{}^e = \int_\Omega \bar{\boldsymbol{N}}_u^T \boldsymbol{f} d\Omega_e \end{gathered} \tag{2}$$

Where $\boldsymbol{c}^E$, $\mathbf{e}$ and $\boldsymbol{\epsilon}^\varepsilon$ are respectively the matrices of elasticity constants at constant electric field, piezoelectric constants and dielectric coefficients at constant strain field.

Electric boundary conditions

The mass matrix in the systems of equations in Eq. (1) lets to singularities that are easy eliminated when the electric boundary conditions are imposed. In this work, three types of electric boundary conditions will be considered for the piezoelectric patches, depending on the connection between top and bottom electrodes: short circuit (SC), open circuit (OC), and applied electric tension. Therefore, assuming that all electrodes on bottom surfaces of the piezoelectric patches are grounded, if the electrodes of the piezoelectric patch are in SC, induced electric potential at the top electrode is also null $\boldsymbol{V} = 0$, consequently, Eq. (1) is reduced to

$$\boldsymbol{M}\ddot{\boldsymbol{u}} + \boldsymbol{K}_u \boldsymbol{u} = \boldsymbol{F}_m \tag{3}$$

On the hand, if electrodes of the piezoelectric patches are in OC, the induced electric potential on top electrode can be calculate from the second row of Eq. (1) as

$$\widehat{\boldsymbol{V}} = \boldsymbol{K}_V{}^{-1} \boldsymbol{K}_{uV}{}^T \boldsymbol{u} \tag{4}$$

Substituting Eq. (4) in first row of Eq. (1), equations of motion similar to Eq. (3) are obtained

$$\boldsymbol{M}\ddot{\boldsymbol{u}} + \boldsymbol{K}_{eq} \boldsymbol{u} = \boldsymbol{F}_m \tag{5}$$

Where the equivalent stiffness K_{eq} is given by: $\boldsymbol{K}_{eq} = \boldsymbol{K}_u + \boldsymbol{K}_{uV} \boldsymbol{K}_V{}^{-1} \boldsymbol{K}_{uV}{}^T$. Solving Eq. (5) for the mechanical dof $\widehat{\boldsymbol{u}}$, the induced electric potential on top electrodes may be evaluated through post-processing from Eq.(4).

CALCULATION OF THE EFFECTIVE ELECTROMECHANICAL COUPLING COEFFICIENT (EMCCe)

For a generic flexible structure with a discrete distribution of piezoelectric patches, it is possible to evaluate the structure's EMCCe provided by each piezoelectric patch. A general procedure to calculate the EMCCe based on modeling has been proposed (Trindade and Benjeddou, 2009). The r^th modal parameters, mode $\boldsymbol{\phi}^r$ and natural frequency ω^r, of a structure with a discrete distribution of piezoelectric elements for SC and OC electric boundary conditions respectively can be found from Eq. (3) and Eq. (5) as

$$\left({\omega_{SC}^r}^2 \boldsymbol{M} + \boldsymbol{K}_u\right) \boldsymbol{\phi}_{SC}^r = 0; \quad \left({\omega_{OC}^r}^2 \boldsymbol{M} + \boldsymbol{K}_{\boldsymbol{eq}}\right) \boldsymbol{\phi}_{OC}^r = 0 \tag{6}$$

Thus, the EMCCe of the structure, provided by a piezoelectric element, when it is vibrating in the r^th mode shape is defined as

$$k_r^2 = \frac{{\omega_{OC}^r}^2 - {\omega_{SC}^r}^2}{{\omega_{OC}^r}^2} \tag{7}$$

MODAL STRAIN ENERGY APPROACH

Suppose the set of discretized equations of motion deduced via finite element method for a flexible structure containing piezoelectric elements whit SC or OC boundary conditions in Eq. (3) and (5), adding viscous damping to the structure, this takes the generic form:

$$\boldsymbol{M}\ddot{\boldsymbol{u}} + \boldsymbol{D}\dot{\boldsymbol{u}} + (\boldsymbol{K}_e + \boldsymbol{K}_v)\boldsymbol{u} = \boldsymbol{F}_m \tag{8}$$

Where **D** is the matrix of viscous damping, which contains other damping sources beside viscoelastic material, this must also be diagonalized by the same real modal matrix that diagonalizes **M** and $\boldsymbol{K}_e$. $\boldsymbol{K}_e$ is obtained from purely elastic elements (plate structure and piezoelectric elements), therefore, it is completely real and constant. $\boldsymbol{K}_v$ is obtained from the viscoelastic elements (adhesive layer), it will be complex and its real and imaginary parts will have ratio η_v; which represents the material loss factor of the adhesive. In the modal strain energy approach it is assumed that the damped system can be represented in terms of the undamped one, since appropriate damping terms are inserted into the uncoupled modal equations of motion:

$$\ddot{\alpha_r} + \eta^{(r)}\omega_r\dot{\alpha_r} + {\omega_r}^2\alpha_r = f_r \tag{9}$$

Where ω_r and $\eta^{(r)}$ are respectively the natural frequency and the loss factor of the rth mode and α_r is the rth modal coordinate, such that

$$u = \sum \boldsymbol{\phi}^{(r)}\alpha_r(t)\,; \quad r = 1, 2, 3 \ldots \tag{10}$$

$\boldsymbol{\phi}^{(r)}$ is the rth modal shape calculated from the undamped structure. For analyzed structure, the material loss factors of plate and piezoelectric elements are neglected, so that, the modal loss factor is found from:

$$\eta^{(r)} = \eta_v\left(U_v^{(r)}/U^{(r)}\right) \tag{11}$$

Where η_v is the material loss factor of adhesive layer at the rth resonant frequency, and $U_v^{(r)}/U^{(r)}$ represents the fraction of elastic strain energy of the adhesive layer when the structure is deformed at the rth modal shape. Thus, when the normal modes are obtained for a purely elastic problem, the total strain energy associated with a given modal shape of the structure and a portion attributable to strain in the adhesive layer, are respectively:

$$U_v^{(r)} = \boldsymbol{\phi}^{(r)T}\boldsymbol{K}_{vR}\boldsymbol{\phi}^{(r)}; \quad U^{(r)} = \boldsymbol{\phi}^{(r)T}\boldsymbol{K}_R\boldsymbol{\phi}^{(r)} \tag{12}$$

Once the stiffness matrix of the viscoelastic material (adhesive layer) is:

$$\boldsymbol{K}_v = \boldsymbol{K}_{vR} + i\boldsymbol{K}_{vi} = \boldsymbol{K}_{vR}(1 + i\eta_v) \tag{13}$$

Thus, dissipated energy due to the viscoelastic behavior of adhesive layer when the structure is deformed at the rth modal shape corresponds to imaginary part of the modal strain energy of the viscoelastic material, and it can be obtained from:

$${\omega_r}^2\eta^{(r)} = \frac{\boldsymbol{\phi}^{(r)T}\boldsymbol{K}_{vi}\boldsymbol{\phi}^{(r)}}{\boldsymbol{\phi}^{(r)T}\boldsymbol{M}\boldsymbol{\phi}^{(r)}} = \eta_v\frac{\boldsymbol{\phi}^{(r)T}\boldsymbol{K}_{vR}\boldsymbol{\phi}^{(r)}}{\boldsymbol{\phi}^{(r)T}\boldsymbol{M}\boldsymbol{\phi}^{(r)}} \tag{14}$$

Introducing the dissipated energy in the derivation process of the EMCCe formula in en Eq. (7), it leads to an expression for the update structure EMCCe taking into account for viscoelastic behavior of the adhesive layer:

$$k_r^2 = \frac{{\omega_{OC}^{(r)}}^2\left(1 - \eta_{OC}^{(r)}\right) - {\omega_{SC}^{(r)}}^2\left(1 - \eta_{SC}^{(r)}\right)}{{\omega_{OC}^{(r)}}^2} \tag{15}$$

DESCRIPTION OF THE PROPOSED ANALYSIS

In the proposed analysis, it is assumed that the performance of a surface-bonded piezoelectric element onto a flexible structure is strongly determined by the EMCCe it provides to host structure and consequently the global performance of the structure is also strongly influenced by this. The EMCCe depends on the electromechanical coupling coefficient of the material (EMCC or k_{ij}^2) and on the mechanical coupling between the piezoelectric element and the host structure. At the same time, the mechanical coupling between the patches and host structure depends on the mechanical properties of the adhesive layer; in present work, it is assumed that this can present a viscoelastic behavior, which would affect considerably the structure EMCCe and consequently its global performance.

For comparative analysis, a flexible plate with surface bonded piezoelectric patches is modeled, the adhesive layer between host structure and piezoelectric elements is assumed initially with completely elastic behavior and posteriorly a slightly viscoelastic behavior is introduced, them the modal loss factor for the first five mode shapes of vibration are calculated for both electric boundary conditions; short circuit and open circuit. The effect of the thickness of the adhesive layer and the segmentation of the piezoelectric material are also studied.

To determine the effect of the segmentation of the piezoelectric material on the modal loss factor, a successive segmentation procedure is performed to an initially unique piezoelectric patch. Simultaneously the thickness of adhesive layer is varied from a ten part to ten times its nominal value for each one configuration tested.

NUMERIC RESULTS

A rectangular aluminum plate with at all edges clamped and dimensions $420 \text{ x } 320 \text{ x } 2\ [\text{mm}^3]$ is considered. The aluminum properties are: Young modulus $E_{Al} = 70\ [\text{GPa}]$, Poisson ratio $\nu_{Al} = 0.34$ and mass density $\rho_{Al} =$

2700 [Kg/m^3]. The adhesive layer has nominal thickness $e_{Ad} = 0.1$ [mm] and nominal properties: Young modulus $E_{Ad} = 2.7$ [GPa], Poisson ratio $\nu_{Ad} = 0.4$ and mass density $\rho_{Ad} = 1140$ [Kg/m^3]. The material used for the piezoelectric patches is a piezoceramic PZT-5H with thickness $e_{pz} = 0.5$ [mm], and electromechanical properties listed in Table 1. A uniform mesh of 42×32 elements has been used to discretize the aluminum plate and consequently adhesive layer and piezoelectric patches have also been discretized by a uniform mesh. The material loss factor of the adhesive layer has been assumed constant and very slight $\eta_v = 0.1$.

Table 1. Electromechanical properties of the PZT -5H

Elastic properties (at constant electric field) [GPa]			**Piezoelectric properties** [C/m^2]		
$c_{11} = c_{22} = 127.2$	$c_{12} = 80.2$	$c_{13} = c_{23} = 84.7$	$e_{31} = e_{32} = -6.6$	$e_{33} = 23.2$	$e_{15} = e_{24} = 17.0$
$c_{33} = 117.4$	$c_{44} = 23.5$	$c_{55} = c_{66} = 23$	**Dielectric properties (at constant strain field)** [nF/m]		
Mass density [Kg/m^3]		$\rho = 7500$	$\epsilon_{11} = \epsilon_{22} = 15.09$	$\epsilon_{33} = 12.69$	

Results have been obtained for two different configurations: a plate with 57.14 % of covered area for piezoelectric material and for a plate with 28.57 % of covered area. Modal loss factor for short circuit and open circuit are showed in figures 1 for the first configuration with 57.14 % of covered area and in figure 2 for configuration with 28.57% of covered area. It can see figures 1 and 2 that modal loss factor increases with the segmentation level and whit the thickness of adhesive layer. The modal loss factors are slightly bigger for open circuit boundary conditions.

Results for calculated values of EMCCe and modified EMCCe due to the loss for several configurations are also shown in figures 3 and 4. The effect of the viscoelastic behavior of the adhesive layer on the modal EMCCe has been very slightly. But, bigger effects occur for biggest values of the thickness of adhesive layer.

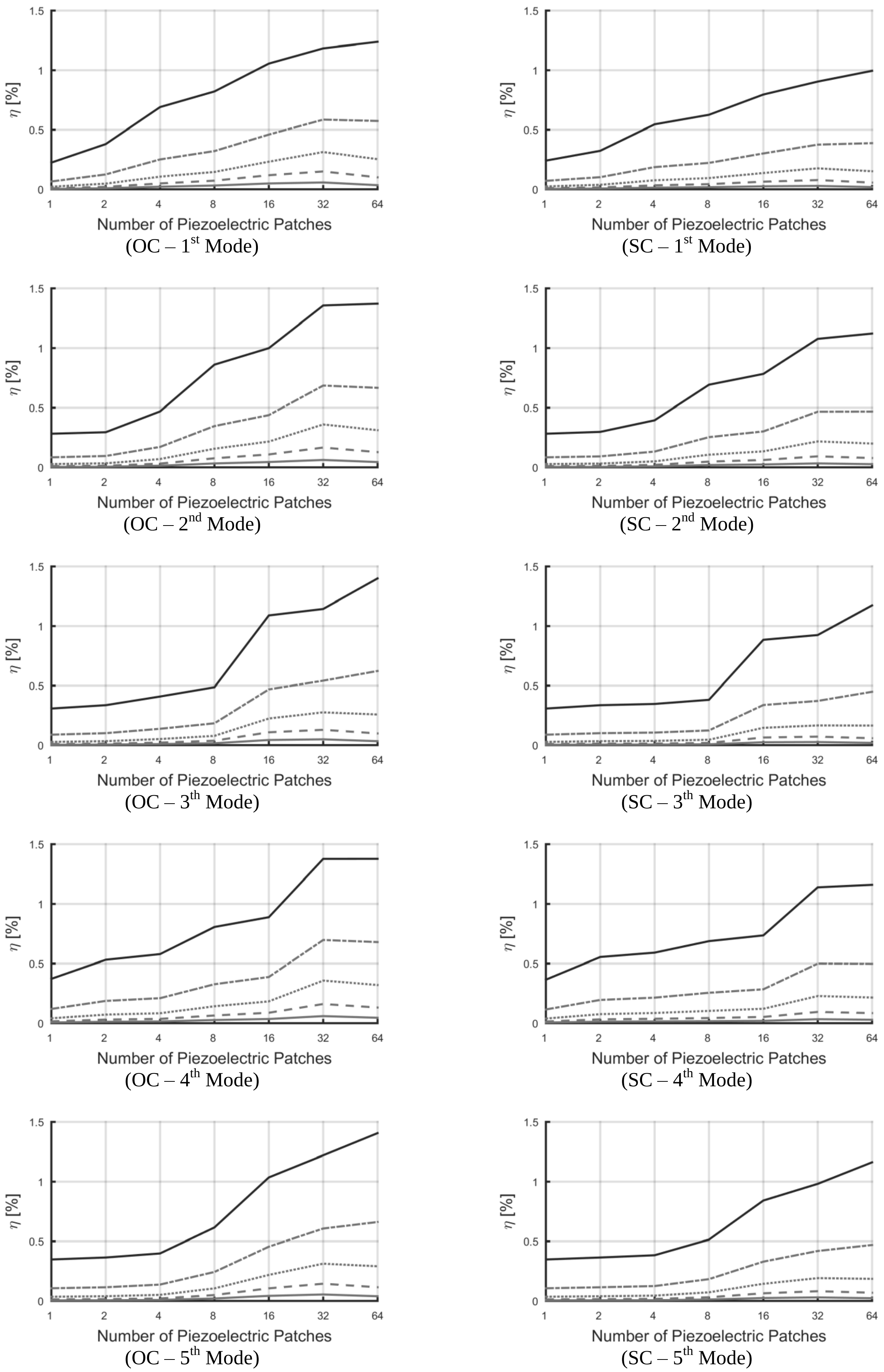


Figure 1. Modal loss factor for OC and SC electric boundaries conditions as a function of the segmentation level and the thickness of the adhesive layer for a plate with 57.14% of covered area. (-) 0.01 mm; (--) 0.03 mm; (:) 0.10 mm; (.-) 0.32 mm; (-) 1.0 mm.

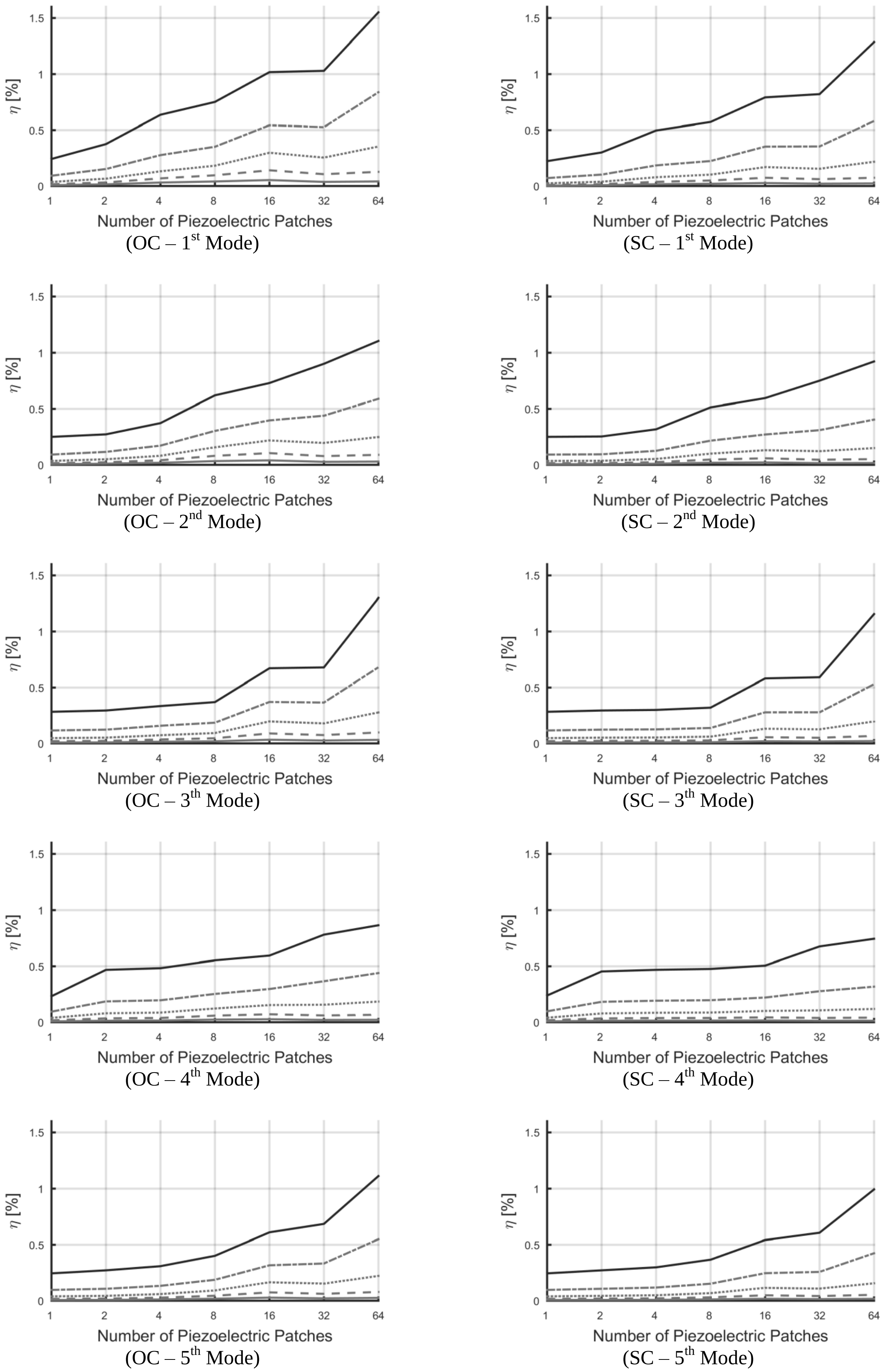
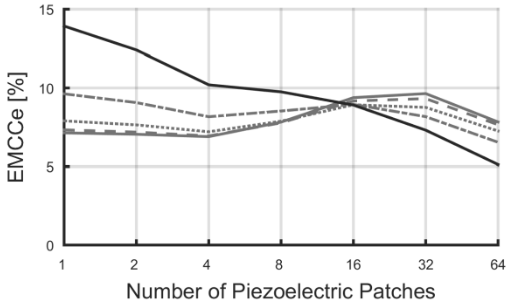
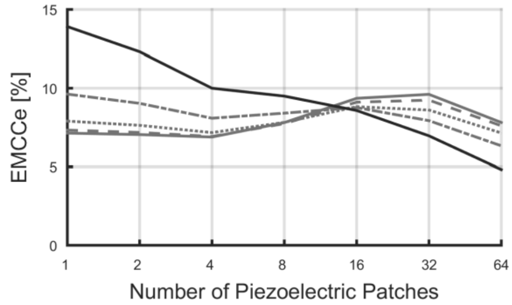


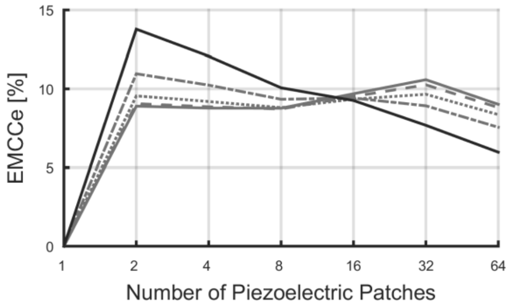
Figure 2. Modal loss factor for OC and SC electric boundaries conditions as a function of the segmentation level and the thickness of the adhesive layer for a plate with 28.57% of covered area. (-) 0.01 mm; (--) 0.03 mm; (:) 0.10 mm; (.-) 0.32 mm; (-) 1.0 mm.



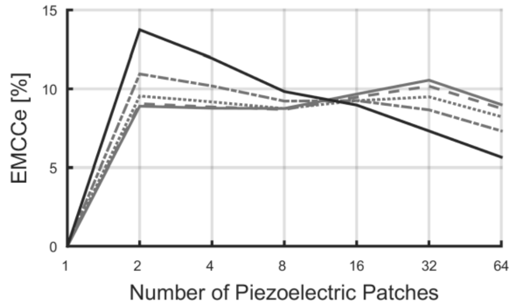
(Elastic - 1st Mode)



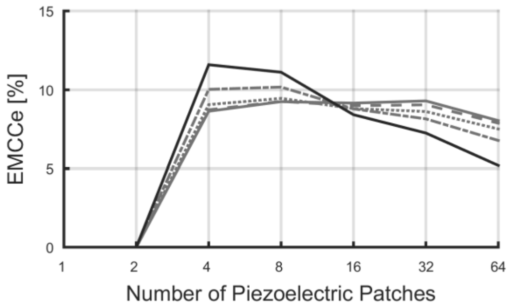
(Viscoelastic – 1st Mode)



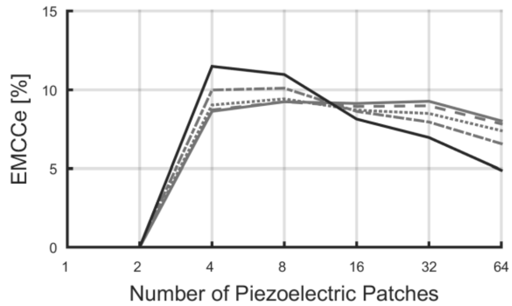
(Elastic – 2nd Mode)



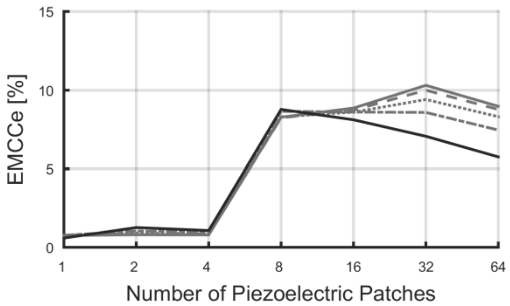
(Viscoelastic – 2nd Mode)



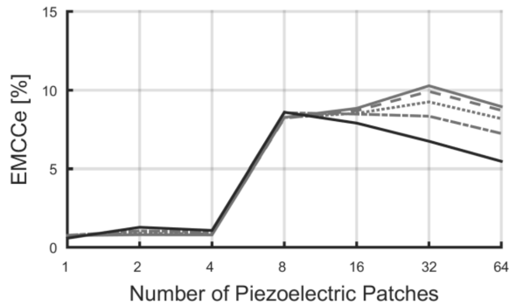
(Elastic – 3th Mode)



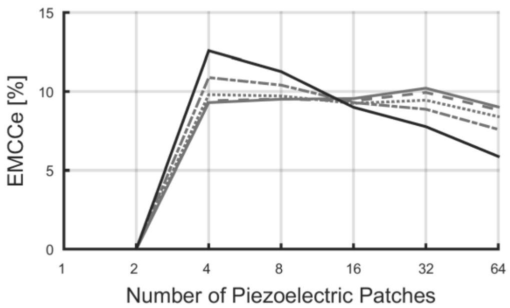
(Viscoelastic – 3th Mode)



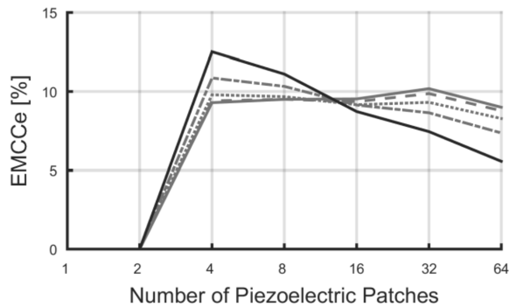
(Elastic – 4th Mode)



(Viscoelastic – 4th Mode)



(Elastic – 5th Mode)



(Viscoelastic – 5th Mode)

Figure 3. Modal EMCCe for elastic and viscoelastic behavoir of adhesive layer as a function of the segmentation level and the thickness of the adhesive layer for a plate with 57.14% of covered area. (-) 0.01 mm; (--) 0.03 mm; (:) 0.10 mm; (.-) 0.32 mm; (-) 1.0 mm.

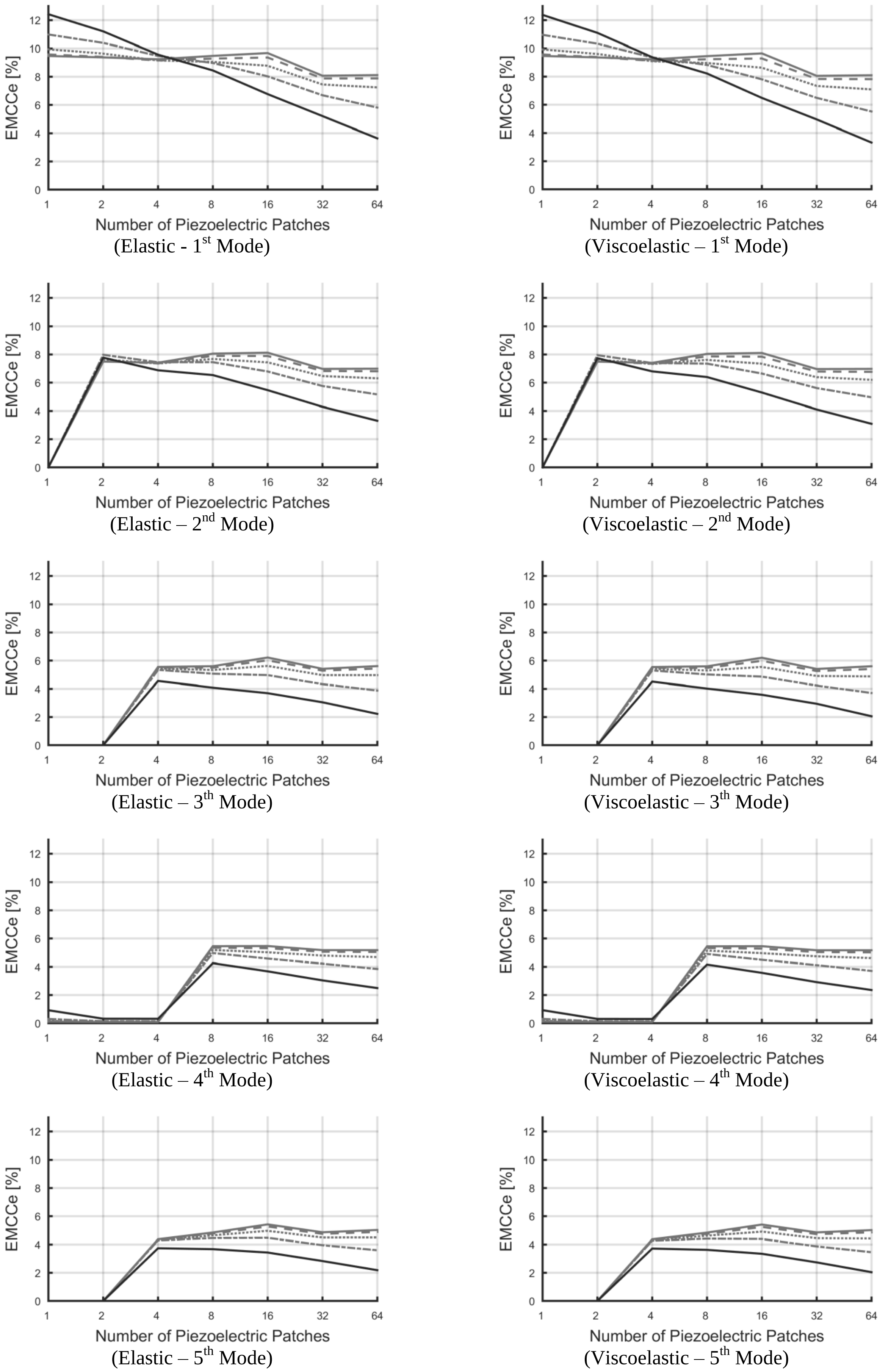


Figure 4. Modal EMCCe for elastic and viscoelastic behavoir of adhesive layer as a function of the segmentation level and the thickness of the adhesive layer for a plate with 28.57% of covered area. (-) 0.01 mm; (--) 0.03 mm; (:) 0.10 mm; (.-) 0.32 mm; (-) 1.0 mm.

CONCLUDING REMARKS AND FUTURE WORKS

Partial results indicate, as expected, a slight decrease of the EMCCe due to the viscoelastic behavior of the adhesive layer, this can be understood as a decrease of the total strain energy, on the piezoelectric element, available to be converted into electric energy. Although this difference may be very small due to small volume of the adhesive layer, this could become significant for piezoelectric patches which are composed for piezoelectric fibers embedded in an epoxy matrix. Futures works are being directed to extend the study to understand how the properties of the adhesive can affect the EMCCe when it is assumed to present a viscoelastic behavior and to evaluate a potential viscoelastic behavior of the piezoelectric patches.

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REFERENCES

De Faria, A. R., 2003, "The Impact Of Finite-Stiffness Bonding On The Sensing Effectiveness Of Piezoelectric Patches", Smart Materials & Structures, V. 12, N. 4, pp. N5-N8.

Le, M. Q.; Capsal, J. F.; Lallart, M.; Hebrard, Y.; Van Der Ham, A.; Reffe, N.; Geynet, L.; and Cottinet, P. J. , 2015 "Review on energy harvesting for structural health monitoring in aeronautical applications," Progress in Aerospace Sciences, 79, pp. 147-157.

Moheimani, S. O. R., 2003"A Survey Of Recent Innovations In Vibration Damping And Control Using Shunted Piezoelectric Transducers", Ieee Transactions On Control Systems Technology, V. 11, N. 4, pp. 482-494.

Sodano, H. A.; Inman, D. J.; Park G., 2004 "A review of power harvesting from vibration using piezoelectric materials," Shock and Vibration Digest, vol. 36, pp. 197-206.

Sunar, M.; Rao, S. S., 1999, "Recent Advances In Sensing And Control Of Flexible Structures Via Piezoelectric Materials Technology", Applied Mechanics Reviews, V. 52, pp. 1-16.

Tinoco, H. A.; Serpa, A. L.; Ramos, A. M., 2010, "Numerical Study Of The Effects Of Bonding Layer Properties On Electrical Signatures Of Piezoelectric Sensors". Mecánica Computacional, V. 29, N. 86, pp. 8391-8409.

Trindade, M. A.; Benjeddou, A., 2002, "Hybrid Active-Passive Damping Treatments Using Viscoelastic And Piezoelectric Materials: Review And Assessment", Journal Of Vibration And Control, V. 8, N. 6, pp. 699-745.

______, 2009, "Effective Electromechanical Coupling Coefficients Of Piezoelectric Adaptive Structures: Critical Evaluation And Optimization", Mechanics Of Advanced Materials And Structures, V. 16, N. 3, pp. 210-223.

Trindade, M.A.; Santos, H.F.L. and Godoy, T.C., 2013, "Effect of bonding layer uncertainties on the performance of surface-mounted piezoelectric sensors and actuators". In Procedings of the XIV Internacional Symposium on Dynamic Problems of Mechanics (DINAME). Rio de Janeiro: ABCM.

Velasquez, J. Q.; Trindade, M. A., 2015, "Effect Of Adhesive Layer Properties On The Performance Of Surface-Mounted Piezoelectric Sensors And Actuators", In Proceedings of the 1th Meeting on Aeronautical Composite Materials and Structures (MACMS), São Carlos, 2015

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