

Sensitivity Analysis of Load Frequency and Number of Block Cycles on the Fatigue Limit Measurement by Thermographic Method

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Abstract: The fatigue limit is an important material property used for design purposes. The thermographic method was developed to evaluate this parameter in an optimized way, since the traditional techniques are expensive and time consuming. This technique correlates temperature increments with several loading amplitudes and estimates the fatigue limit as the stress below which no heat significant is generated. It depends on the number of cycles at each load increment and on the load frequency, to have a minimum heat generation for a good thermo-data acquisition. This work proposes a sensitivity analysis of both factors to determine their effects on fatigue limit measurements by the thermographic method. An experiment made using the full factorial design 2² has its data robustness checked with a variance analysis. The results show that fatigue limit measurements by the thermographic method have little dependence on both factors for the proposed experimental domain. In fact, it is more dependent on the incremental load cycles than on the applied frequency, since it delimits the loading application time and, consequently, the heat generation. The analysis of variance shows a good correlation coefficient between the experimental results and the design of experiment implemented.

Keywords: *fatigue limit, thermographic method, design of experiment.*

INTRODUCTION

The fatigue limit S_L is an important material property used for fatigue design. It is the parameter that defines the stress below which the material doesn't fail by fatigue, independently of the accumulated number of cycles. Although empirical estimations are available in the literature, its experimental evaluation is mandatory if a good accuracy is wanted. The traditional methods to obtain S_L require a large number of specimens and a long time to be completed, which led to the development of new techniques to obtain this property in a cheaper and faster way.

The thermography approach proposed by Risitano *et al.* (2000) uses only a few specimens that can be tested in a short time. Its main idea is to correlate blocks of incremental stress amplitude σ_a applied on standard fatigue specimens with the heat that they generate on their surfaces due to the crack initiation and early propagation processes. The relationship between number of block cycles N and the maximum temperature T_{max} that are induced for different stress amplitudes are used to evaluate the fatigue limit by checking from which σ_a there is a sudden heat increase.

As thermography measures heat generation on the specimen's surface, a minimum load frequency f is needed to ensure temperature increments clearly detectable by the measurement equipment. In addition, the number of cycles over which the different stress amplitudes are applied must be defined to ensure they reach the end of the so-called first material thermal phase N_1 . However, what is the real effect of the factors f and N_1 on fatigue limit measurements by the thermographic method? Is it necessary to define and control them accurately to obtain good S_L measurements?

To answer these questions, a design of experiment *DOE* is performed to obtain the fatigue limit response under f and N_1 variations. To accomplish it, a full factorial design 2² was used, considering a linear response for S_L , with three tests in the center of experimental domain. The results robustness is checked with an analysis of variance *ANOVA*.

EXPERIMENTAL METHODS

Traditional fatigue tests were performed on a rotating bending machine. The specimens were made of low carbon steel with average yield strength $S_Y = 576\text{MPa}$ and ultimate strength $S_{UT} = 666\text{MPa}$. Their geometry was defined to have the critical region in the middle of the specimens by a cross section reduction, as can be seen in Fig. 1.

Temperature variations were recorded by a thermographic camera FLIR A320, with resolution of 320×240 pixels, data acquiring frequency of 30Hz , and temperature sensibility of 50mK . In order to improve the camera performance, the specimens central region was coated with black paint to increase their emissivity, and a black cloth was used to cover camera and the testing machine to minimize the heat transfer to environment during the tests.

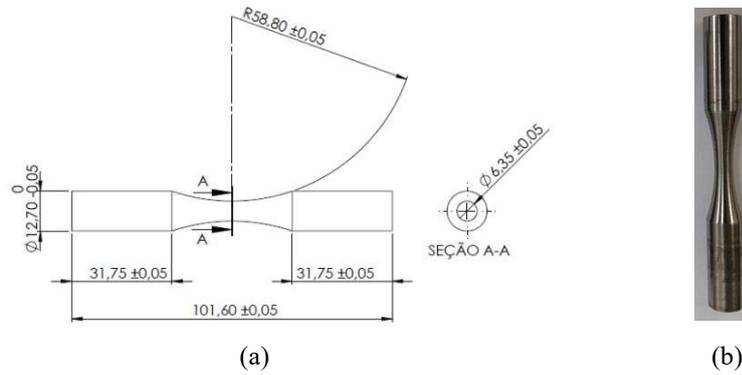


Figure 1 – Fatigue specimen: (a) geometry; (b) as manufactured.

Fatigue limits were obtained through incremental load tests, varying the stress amplitude σ_a every N_1 cycles, as shown in Fig. 2a. The stress amplitude increments $\sigma_a/S_{ut} = 0.35, 0.40, 0.44, 0.48, 0.50, 0.52, 0.54, 0.56, 0.60$ were previously defined in *Bandeira et al. (2017)*. Their temperature increase rate dT/dN_1 is then correlated with σ_a/S_{ut} , and a linear regression is fitted through their highest points to evaluate S_L , as the stress amplitude in which $dT/dN_1 = 0$, as illustrated in Fig. 2b.

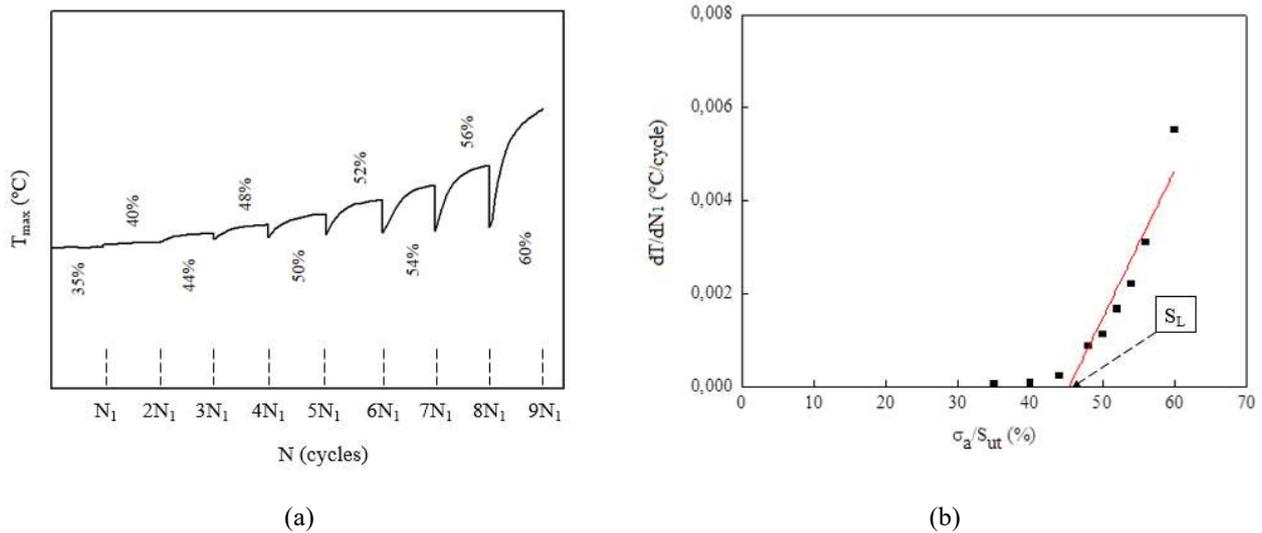


Figure 2 – Thermography method: (a) Incremental load test; (b) S_L evaluation.

The full factorial design 2^2 aims to obtain a linear response function $Y(f, N_1)$ defined by Eq. (1), through the execution of four different tests defined by a combination of the minimum (-1) and maximum (+1) values of each factor. The coefficients a_0, a_1, a_2 and a_{12} were obtained by the least squares method, whose matrix solution can be defined by Eq. (2), where X is the design matrix and y is the vector with experimental results, following *Goupy et al. (2007)*.

In addition to the four tests required by the full factorial design 2^2 , three tests were performed in the center of experimental domain to determine the *DOE* robustness via *ANOVA*. Table 1 shows the factors values (in real and reduced centralized variables *RCV*) for each test. The range of each factor was defined according to the previous experimental results presented in Bandeira *et al.* (2017).

$$Y(f, N_1) = a_0 + a_1f + a_2N_1 + a_{12}fN_1 \quad (1)$$

$$\mathbf{a} = (X^T X)^{-1} X \mathbf{y} \quad (2)$$

Table 1 – Full factorial design tests.

Test	f_{real} (rpm)	f_{RCV}	N_{1_real} (cycles)	N_{1_RCV}
1	7000	-1	2500	-1
2	10000	+1	2500	-1
3	7000	-1	7500	+1
4	10000	+1	7500	+1
5	8500	0	5000	0
6	8500	0	5000	0
7	8500	0	5000	0

The analysis of variance is a statistical approach to evaluate the robustness of the *DOE* response function. Its main objective is to find error sources for the possible differences between experimental results y and the response function Y . The *ANOVA* uses the sum of the squares of the residues SS_{res} between the experimental results and response function defined by Eq. (3), and the sum of the squares of the model SS_{mod} between the experimental results and their mean defined by Eq. (4), where i is an integer counter for the experimental tests and n is the total number of tests.

$$SS_{res} = \sum_{i=1}^n (y_i - Y_i)^2 \quad (3)$$

$$SS_{mod} = \sum_{i=1}^n \left(y_i - \frac{\sum_{i=1}^n y_i}{n} \right)^2 \quad (4)$$

SS_{res} and SS_{mod} are used to determine the correlation coefficient R^2 and the adjusted correlation coefficient R_a^2 , as defined by Eq. (5), where p is the number of coefficients of the response function, which are used to access the mean square of the residues MS_{res} and of the model MS_{mod} , as defined by Eq. (6). DF_{res} and DF_{mod} are the degrees of freedom of the residues and the model. These mean squares are used to calculate F_{ratio} as show in Eq. (7), which in turn used in Fisher's function to access the probability p_{value} of the response function not represent the experimental results.

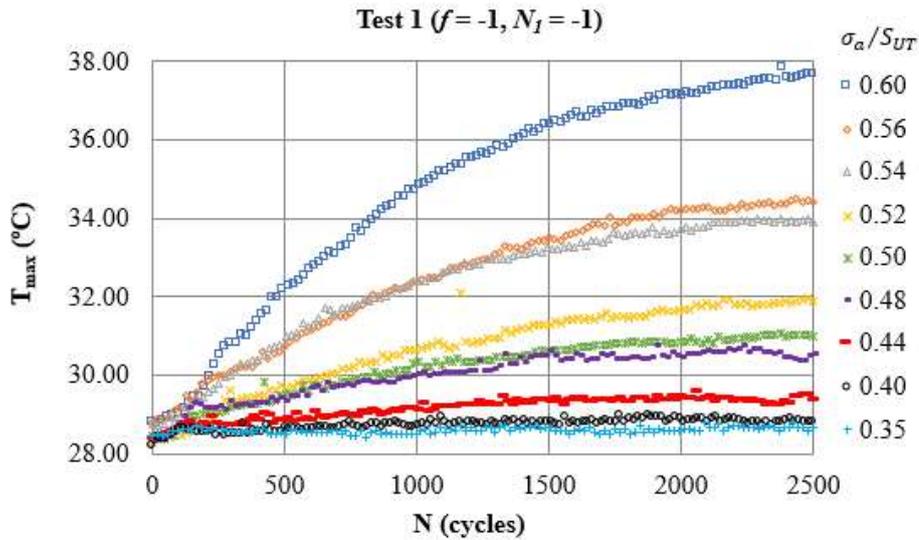
$$R^2 = \frac{SS_{mod}}{SS_{mod} + SS_{res}} \quad R_a^2 = 1 - \frac{(1-R^2)(n-1)}{(n-p+1)} \quad (5)$$

$$MS_{res} = \frac{SS_{res}}{DF_{res}} \quad MS_{mod} = \frac{SS_{mod}}{DF_{mod}} \quad (6)$$

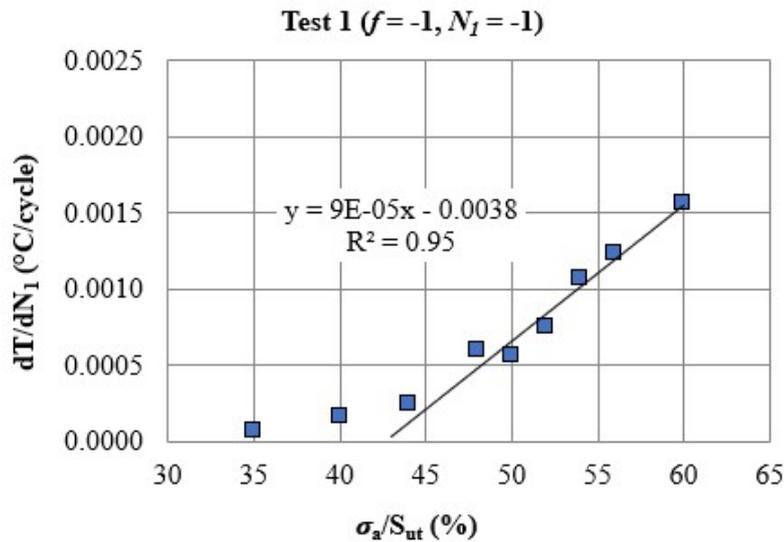
$$F_{ratio} = \frac{MS_{mod}}{MS_{res}} \quad (7)$$

RESULTS

Figure 3a shows the maximum temperature results obtained during test 1 ($f = -1$ and $N_1 = -1$) and Fig. 3b shows the correlation between their temperature increase rates and the stress amplitudes, as well as the linear regression fitted to evaluate S_L . Note that T_{max} increased by about 10°C since the first load step, and that the dT/dN_1 rates increased as the stress amplitude of each load block increased, particularly from $\sigma_a/S_{UT} > 44\%$.



(a)



(b)

Figure 3 – Thermography results for test 1: (a) T_{max} evolution; (b) S_L evaluation.

These thermography results were obtained for each test of Tab. (1), with different specimens. The S_L evaluation was used in Eq. (2) to obtain the coefficients of the response function $Y(f, N_1)$. Table 2 shows the fatigue limit results and the response function coefficients, also plotted in Fig. 4.

Table 2 – Fatigue limit results.

Test	f	N_1	$y = (S_L/S_{UT})100\%$	Response function coefficients
1	-1	-1	42.55	$a_0 = 46.45$
2	+1	-1	46.98	$a_1 = 1.277$
3	-1	+1	47.77	$a_2 = 1.672$
4	+1	+1	48.45	$a_{12} = -0.937$
5	0	0	46.90	
6	0	0	46.60	
7	0	0	45.90	

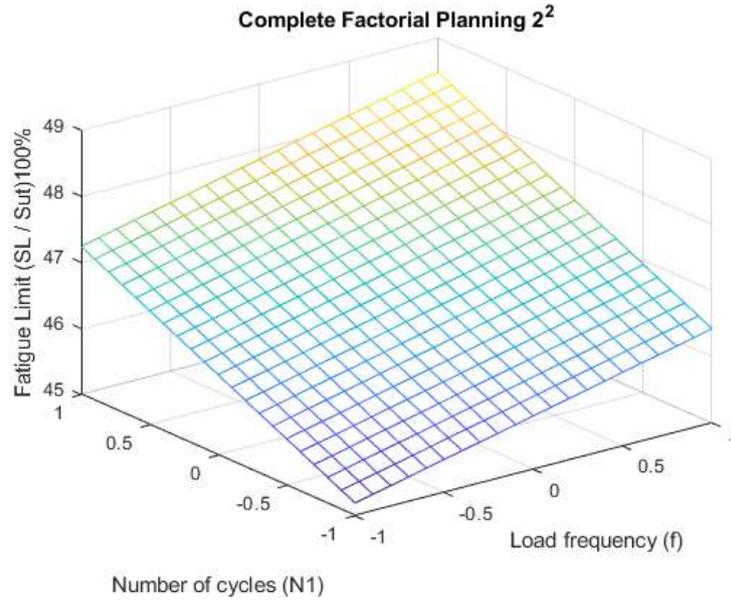


Figure 4 – Response function.

Note that the fatigue limits obtained by thermographic method have little dependence on both factors, with the largest difference below 5% with respect to the mean. In addition, coefficient a_2 is higher than a_1 , and a_{12} is the smallest one, which means that this coefficient is the least significant of all.

Table 3 shows the results obtained with *ANOVA* evaluation which confirms the response function adjusts well the experimental results due to the high F_{ratio} (and low p_{value}) in conjunction with R^2 and R_a^2 that were close to one.

Table 3 – *ANOVA* results.

Source	DF	SS	MS	F_{ratio}	R^2
Model	3	21.232	7.077	40.20	0.975
Residue	3	0.528	0.176	p_{value}	R_a^2
Total	6	21.76	7.253	<0.001	0.963

CONCLUSIONS

The results show that fatigue limit evaluations by the thermographic method has a small dependence on both: test frequency f and number of cycles of each load block N_1 ; for the material tested in the proposed experimental domain. The response function is well adjusted by the linear correlation approach proposed by the full factorial design 2^2 , since *ANOVA* evaluation shows low p_{value} and good correlation coefficients.

Tests 1, 2, 3 and 4 of Tab. 2 will be duplicated to increase the full factorial design robustness and to confirm the conclusions stated above. An additional *DOE* is also in underway, using a composite design with quadratic formulation to check if there is a nonlinear tendency of the fatigue limit evaluation by the thermographic method.

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