

A Comparative Analysis of the Longitudinal Compressive Strength Models of Composite Materials

Lucas L. Vignoli ^{1,2}, Marcelo A. Savi ¹, Ranulfo M.C. Neto ²

¹ Center for Nonlinear Mechanics, COPPE, Department of Mechanical Engineering, Universidade Federal do Rio de Janeiro, Brazil

² Department of Mechanical Engineering, Universidade Federal do Rio de Janeiro - Campus Macaé, Brazil

Abstract: Unidirectional composite laminates usually present distinct behavior in tension and compression, since compressive loads parallel to fibers may induce a wide range of failure mechanisms. There are several micromechanical models to estimate the longitudinal compressive strength of unidirectional laminae. However, there is no agreement about which one is able to obtain the best estimation. In this paper, a comparative analysis is carried out evaluating seven analytical models and checking their predictions with experimental data. Results indicate that even the most accurate models have an average error higher than 20%, indicating the need of theories refinement.

Keywords: micromechanics, compressive strength, failure, analytical models

INTRODUCTION

Unidirectional composite laminates have been widely investigated on the last decades and a considerable advance in failure modelling is realized due to the World Wide Failure Exercise (WWFE) (Soden *et al.*, 2004; Kaddour & Hinton, 2013; Kaddour *et al.*, 2013b). The WWFE is an international effort to compare different failure criteria where fibers, matrices, laminae, lay-up and load are provided by the organizers and the participants have to estimate the failure characteristics. Among the participants, just two groups use an analytical micromechanical approach to compute the properties of the homogenized laminae, using the Bridging (Huang & Liu, 2014) and Chamis (Chamis *et al.*, 2013) models. Some others use numerical homogenization procedures or just the equivalent properties of the laminae. Nevertheless, lamina equivalent properties are valid only for the specific fiber volume fraction tested. For practical applications, new test for each value of fiber volume fraction is prohibitive. On the other hand, the computational cost for numerical modelling also is a big issue (Andrianov *et al.*, 2018). Hence, the analytical formulations become a useful tool for parametric and optimization analyses. For unidirectional laminae, the compressive strength in longitudinal directional is one of the most challenging issues in micromechanics due to the broad of failure mechanisms including fiber breaking, microbuckling and kinking.

This paper deals with a comparative analysis of micromechanical models to estimate longitudinal compressive strength and the experimental values reported by the WWFE organizers. Seven different models are of concern. Failure predictions and parametric analysis are carried out.

MICROMECHANICAL MODELS

The first micromechanical model to estimate the longitudinal compressive strength of unidirectional laminae was proposed by Rosen (1965). A detailed discussion about this model is presented by Jones (1999). Microbuckling is assumed to be the failure mechanism and it is classified in two modes: extensions and shear. Figure 1 presents these two modes. On the left, the initial fiber distribution is represented; on the middle, there is a distortion on matrix, namely shear mode; and on the right, there is an extension/compression on the matrix, namely extension mode.

Modelling the fibers as Euler-Bernoulli beams in an elastic foundation, the compressive strength is estimated by

$$S_{11}^c = \min \left(2 \left[V_f + (1 - V_f) \frac{E^m}{E_1^f} \right] \sqrt{\frac{V_f E^m E_1^f}{3(1 - V_f)}}, \frac{G^m}{1 - V_f} \right) \quad (1)$$

where V_f is the fiber volume fraction, E^m and G^m are the elastic and shear modulus of the matrix and E_1^f is the longitudinal elastic modulus of the fiber.

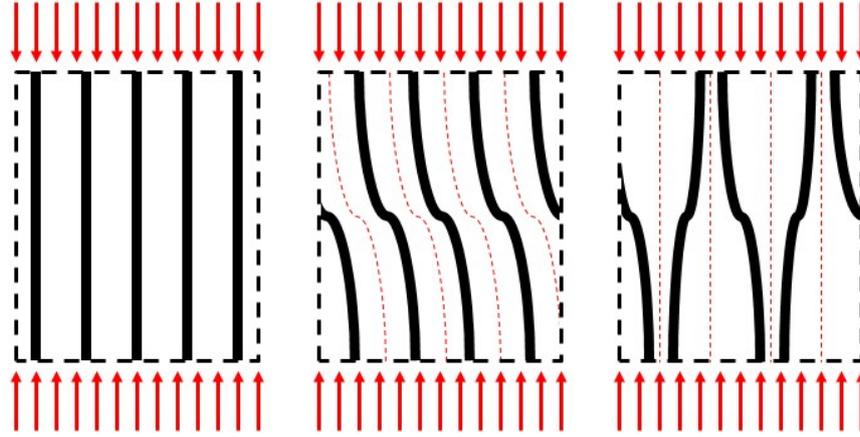


Figure 1 – Failure modes proposed by Rosen (1961).

Despite these two failure modes reported, the majority of models does not consider the extension mode because it just takes place for very low fiber volume fractions.

Argon (1972) and Budiansky (1983) considered the fiber misalignment and matrix yield effect in kinking failure. Budiansky & Fleck (1993) presented an extension of these studies considering other plasticity models for matrix, including hardening effects. According to the authors, a satisfactory estimative is obtained assuming that the matrix has elastoplastic behavior without hardening. The longitudinal compressive strength is computed by

$$S_{11}^c = \frac{G_{12}}{1 - (\bar{\phi} / \gamma_Y)} \quad (2)$$

where $\bar{\phi}$ is the misalignment angle and γ_Y is the yield shear strain. The ratio $\bar{\phi} / \gamma_Y$ ranges between 0 and 8. For the present study, this ratio is calibrated according to the experimental data to decrease the average error.

Lo & Chim (1992) modelled the fibers using the Timoshenko beam model and using the minimal potential energy theory proposed the following equation

$$S_{11}^c = \frac{G_{12}}{1.5 + 12(6 / \pi)^2 (G_{12} / E_1)} \quad (3)$$

where E_1 and G_{12} are the longitudinal elastic modulus and the shear modulus of the lamina and are estimated by $E_1 = E_1^f V_f + (1 - V_f) E^m$ and $G_{12} = G^m [G_{12}^f (1 + V_f) + G^m (1 - V_f)] / [G_{12}^f (1 - V_f) + G^m (1 + V_f)]$, respectively, and G_{12}^f is the fiber in-plane shear modulus.

Barbero (1998) used a hyperbolic relation for shear strain-stress curve of the lamina to model the failure where fiber initial misalignment induce shear. The fiber imperfections are obtained by a Gaussian function. The simplified equation suggested is

$$S_{11}^c = G_{12} \left(1 + 4.76 \frac{G_{12} \bar{\phi}}{S_{12}^s} \right)^{-0.69} \quad (4)$$

where lamina shear strength is computed by $S_{12}^s = \{1 - (\sqrt{V_f} - V_f)[1 - (G^m / G_{12}^f)]\} S_s^m$ and S_s^m is the matrix shear modulus (Barbero, 2018). Barbero (1998) highlighted that Eq. (4) is not a semi-empirical based, it is based on the principle of potential energy and continuum damage mechanics with some parameters calibrated to obtain a simple equation form, For the complete equation, see Barbero (1998).

Pimenta *et al.* (2009a) reported a detailed discussion about the steps of longitudinal compressive failure, from damage initiation up to final failure. Based in experimental observations, a numerical procedure is proposed to investigate failure progression. The 2D numerical model includes matrix elastoplastic constitutive relation, cohesive interface zones between fibers and matrix and fiber misalignment. Based in these results, Pimenta *et al.* (2009b) proposed the following analytical formulation to estimate the compressive strength.

$$S_{11}^c = S_s^m \left[\frac{G_{2D}^m d_f + (\pi / L)^2 E_1^f I_f}{S_s^m + \pi(\bar{y}_0 / L) G_{2D}^m} \right] \frac{V_f^{2D}}{A_f} \quad (5)$$

where $A_f = d_f$ and $I_f = d_f^3 / 12$ are the 2D area and second moment of inertia of the fibers, \bar{y}_0 and L are parameters used to modelling the fiber misalignment with a sinoidal shape, with $21\mu m < \bar{y}_0 < 70\mu m$ and $1050\mu m < L < 2800\mu m$, $t_m = d_f [(\pi / 2\sqrt{3}V_f)^{0.5} - 1]$, $V_f^{2D} = d_f / (d_f + t_m)$ and $G_{2D}^m = G^m / (1 - V_f^{2D})$. This model assumes a uniform stress distribution along the matrix, which is a better approximation for higher values of fiber volume fractions.

Assuming that the fiber fails and based on the load distribution proposed by the rule of mixture, the Bridging model use the following equation for the compressive strength (Huang & Liu, 2014)

$$S_{11}^c = \left[V_f + (1 - V_f) \left(\frac{E^m}{E_1^f} \right) \right] S_c^f \quad (6)$$

Chamis (Chamis *et al.*, 2013) proposed that compression strength is based on the fiber crushing, intraply delamination and fiber microbuckling. Mathematically, the strength is computed by

$$S_{11}^c = \min \left\{ \left[V_f + (1 - V_f) \left(\frac{E^m}{E_1^f} \right) \right] (1 - V_v) S_c^f, 10 S_{12}^s + 2.5 S_s^m \left[1 - \sqrt{\frac{4V_v}{\pi(1 - V_f)}} \right], \frac{G^m \left[1 - \sqrt{\frac{4V_v}{\pi(1 - V_f)}} \right]}{1 - V_f(1 - V_v) \left[1 - \frac{G^m}{G_{12}^f} \left[1 - \sqrt{\frac{4V_v}{\pi(1 - V_f)}} \right] \right]} \right\} \quad (7)$$

where V_v is the void volume fraction. Note that the first failure model of the Chamis model is equivalent to the Bridging model.

RESULTS AND DISCUSSION

Compression strength is analyzed based on experimental data from the WWFE (Soden *et al.*, 1998; Kaddour & Hinton, 2012; Kaddour *et al.*, 2013a). Thirteen experimental tests are employed to verify analytical model performance. The calibration parameters are the following, considering the smallest error prediction: $\bar{\phi} / \gamma_Y = 6.1$ (Budiansky), $\bar{\phi} / \gamma_Y = 1.2^\circ$ (Barbero), $\bar{y}_0 = 39\mu m$, $L = 2800\mu m$ (Pimenta) and $V_v = 0$ (Chamis).

Two different approaches are employed evaluating the average error considering all the data available and the ranges of error considering six cases: smaller than 10%, between 10% and 20%; between 20% and 30%, between 30% and 40%, between 40% and 50% and higher than 50%. The first one gives a value easier to compare, while the second one offers a useful measure that is able to check the representative average error. For instance, if a data has a discrepant error, the average value is highly influenced, while the ranges allow a better representation. Results with both methodologies are presented in Figure 2.

Results of the average error (left) indicate that Bridging and Lo & Chim models obtained the best prediction, both around 20%. However, considering the ranges of error (right) the model proposed by Lo & Chim obtained a considerable improvement when compared with the others. Note that the two highlighted models, namely Bridging and Lo & Chim, assume different failure mechanisms of the laminae, indicating the complex behavior of these structures under compressive load.

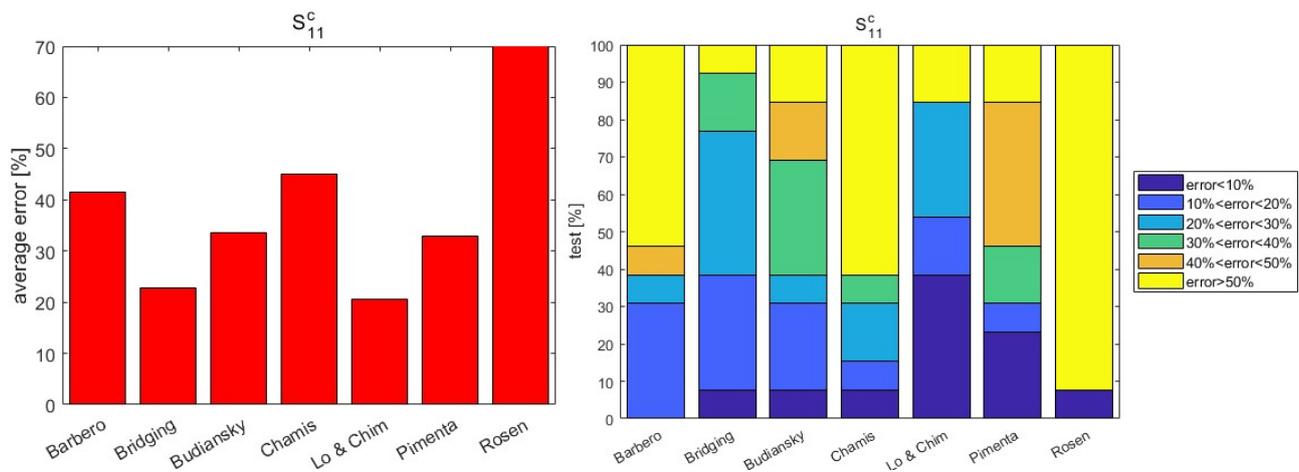


Figure 2 – Average error of the models estimations (left) and ranges of error of the models estimations (right).

Considering these results, the following points are highlighted:

- i) for the ranges of error, Lo & Chim model obtained the highest amount of predictions with error smaller than 10%, around 40%, and more than 80% of the cases with errors smaller than 30%;
- ii) the average error of Lo & Chim and Bridging models is around 20%;
- iii) the Rosen model resulted in worst prediction, nevertheless its contribution indicating buckling failure is fundamental for almost all of the most recent models;
- iv) although the Pimenta model does not obtain the best prediction, it has also a great contribution indicating phenomenologically the failure propagation with a broad range of mechanisms;
- v) Chamis model indicates three different failure modes, what may become an advantage, however it underestimates the buckling and delamination strengths.

CONCLUSIONS

This paper deals with the micromechanical modelling of the longitudinal compressive strength of composite unidirectional laminae using analytical tools. It should be pointed out that, even the best models present average errors higher than 20%, indicating the need of improvements. Among the evaluated models, the one proposed by Lo & Chim obtained the best predictions when compared with the experimental data from the WWFE. Additionally, the estimation obtained by the Bridging model also had a considerable amount of cases with error smaller than 30%. Based in these results, the most recommended criterion to estimate the longitudinal compressive strength is use the smallest value between Lo & Chim and Bridging models' prediction, since these criteria are able to model different failure mechanism.

With this approach, the failure by longitudinal compression is understood as a competition between two mechanisms that is influenced by constituent's properties and volume fractions.

ACKNOWLEDGMENTS

The authors would like to acknowledge the support of the Brazilian Research Agencies CNPq, CAPES and FAPERJ. The Air Force Office of Scientific Research (AFOSR) is also acknowledged.

REFERENCES

- Andrianov, I.V., Awrejcewicz, J., Danishv'sky, V.V., 2018, "Asymptotical Mechanics of Composites - Modelling Composites without FEM", Springer.
- Argon, A.S., 1972, "Fracture of composites", Treatise on materials science and technology, New York, Academy Press.
- Barbero, E.J., 1998, "Prediction of Compression Strength of Unidirectional Polymer Matrix Composites", *Journal of Composite Materials*, Vol. 32, pp. 483-502.
- Barbero, E.J., 2018, "Introduction to Composite Materials Design", 3 ed., CRC Press.
- Budiansky, B., 1983, "Micromechanics", *Computers & Structures*, Vol.16, pp. 3-12.
- Budiansky, B., Fleck, N.A., 1993, "Compressive Failure of Fibre Composite", *Journal of the Mechanics and Physics of Solids*, Vol.41, pp. 183-211.
- Chamis, C.C., Abdi, F., Garg, M., Minnetyan, L., Baid, H., Huang, D., Housner, J., Talagani, F., 2013, "Micromechanics-based progressive failure analysis prediction for WWFE-III composite coupon test cases", *Journal of Composite Materials*, Vol. 47, pp. 2695-2712.
- Huang, Z.M., Liu, L., 2014, "Predicting strength of fibrous laminates under triaxial loads only upon independently measured constituent properties", *International Journal of Mechanical Sciences*, Vol. 79, pp. 105-129.
- Huang, Z.M., Xin, L.M., 2017, "In situ strengths of matrix in a composite", *Acta Mech. Sin.*, Vol. 33, pp. 120-131.
- Jones, R. M., 1999, *Mechanics of Composite Materials*, 2 ed., Taylor & Francis Editions.
- Kaddour, A.S., Hinton, M.J., 2012, "Input data for test cases used in benchmarking triaxial failure theories of composites", *Journal of Composite Materials*, Vol. 46, pp. 2295-2312.
- Kaddour, A.S., Hinton, M.J., 2013, "Maturity of 3D failure criteria for fibre reinforced composites: Comparison between theories and experiments: Part B of WWFE-II", *J. Compos. Mater.*, Vol. 47, pp. 925-966.
- Kaddour, A.S., Hinton, M.J., Smith, P.A., Li, S., 2013a "Mechanical properties and details of composite laminates for the test cases used in the third world-wide failure exercise", *Journal of Composite Materials*, Vol. 47, pp. 2427-2442.
- Kaddour, A.S., Hinton, M.J., Smith, P.A., Li, S., 2013b, "The background to the third world-wide failure exercise", *Journal of Composite Materials*, Vol. 47, pp. 2417-2426.
- Lo, K.H., Chim, E.S.M., 1992, "Compressive Strength of Unidirectional Composites", *Journal of Reinforced Plastics and Composites*, Vol. 11, pp. 838-896.
- Pimenta, S., Gutkin, R., Pinho, S.T., Robinson, P., 2009b, "A micromechanical model for kink-band formation: Part I - Experimental study and numerical modelling", *Composites Science and Technology*, Vol. 69, pp. 956-964
- Pimenta, S., Gutkin, R., Pinho, S.T., Robinson, P., 2009b, "A micromechanical model for kink band formation: Part II - Analytical modelling", *Composites Science and Technology*, Vol. 69, pp. 956-964
- Rosen, V.W., 1965, *Mechanics of composite strengthening. Fibre composite materials*. Metals Park, Ohio: American Society of Materials.
- Soden, P.D., Hinton, M.J., Kaddour, A.S., 1998, "Lamina Properties, Lay-up Configurations and Loading Conditions for a Range of Fibre-Reinforced Composite Laminates", *Composites Science and Technology*, Vol. 58, pp. 1011-1022.
- Soden, P.D., Kaddour, A.S., Hinton, M.J., 2004, "Recommendations for designers and researchers resulting from the world-wide failure exercise", *Compos. Sci. Technol.*, Vol. 64, pp. 589-604.

RESPONSIBILITY NOTICE

The authors are the only responsible for the printed material included in this paper.