

# Reinforced Concrete Time-Dependent Reliability Considering Penetration of Chlorides via Boundary Element Method

Henrique Machado Kroetz <sup>1</sup>, Giovanni Pais Pellizzer <sup>1,2</sup>, Edson Denner Leonel <sup>1</sup> and André Teófilo Beck <sup>1</sup>

<sup>1</sup> University of São Paulo, Structural Engineering Department – Av. Trabalhador São Carlense, 400, 13566-590, São Carlos-SP, Brazil

<sup>2</sup> Federal University of Mato Grosso do Sul, Faculty of Engineering, Architecture and Urbanism and Geography – Av. Costa e Silva, no number, Cidade Universitária, 79070-900, Campo Grande - MS, Brazil.

*The degradation of structural materials is an inevitable process, since they are over time subjected to several deleterious physical-chemical phenomena, interfering with the structural performance. Especially in reinforced concrete structures, corrosion of reinforcing bars caused by chloride ions is one of the main pathological manifestations. Chloride ions diffuse through the microstructure of the concrete and when they reach a significant concentration at the concrete / steel interface, the corrosive process is initiated. For a complete analysis of the structural life, it is fundamental that the approach used to predict the behavior of the structure encompasses both the diffusion mechanism and the mechanical degradation of the steel reinforcement in the concrete. Since the process involves a significant amount of uncertainties a reliability analysis is necessary in this context. The present work presents a model that is capable of providing responses in terms of failure probabilities for reinforced concrete structures submitted to mechanical loads and penetration of chloride ions. The diffusion of chloride ions is modeled using the boundary element method while the mechanical loads are generated by means of stochastic processes. An application example is presented, demonstrating the capacity of the proposed model.*

**Keywords:** Reinforced Concrete, Corrosion, Time-dependent Reliability, Boundary Element Method

## 1. INTRODUCTION

Despite idealizations adopted in design phase, real structures are unavoidably subjected to a probability of failure. To worsen that, the occurrence of construction pathologies is not uncommon as the structure ages, leading to aesthetical and financial losses or even to lower structural safety. Thus, a thorough structural analysis methodology should encompass both structural reliability and the effect of time-dependent degradation. Particularly in reinforced concrete structures, the main degradation issue is known to be corrosion induced by chlorides penetration, a well studied phenomenon (CEB, 1992; Gonzalez et al., 1995; Cascudo 1997; Val and Melchers, 1997; Val and Stewart, 2003; Apostolopoulos and Papadakis, 2008; Suo and Stewart, 2009; Zhang et al., 2010; Bastidas-Arteaga et al. 2011). On the other hand, to the best knowledge of the authors, there is no work coupling the study of reinforcement depassivation time, consequent time-dependent corrosion, loss of structural resistance, and the effects of these phenomena in a time-dependent reliability framework. Solving a structural reliability problem can be a complex quest, since multidimensional integrals have to be solved over a potentially irregular domain. Approximate solutions have been developed, such as the First Order Reliability Method (FORM) (Hasofer and Lind, 1974). However, for highly non-linear problems, for problems with multiple failure modes, or for system reliability, FORM is not accurate enough. It is obtained by Monte Carlo Simulation (MCS). In this technique, the failure probability is interpreted as the mean value of a stochastic experiment where a large number of random variable samples are generated (Dietlevsen and Madsen, 2007)

This paper aims to provide a framework for the time-variant reliability analysis of reinforced concrete structures subjected to corrosion caused by the penetration of chlorides. A Monte Carlo based time-dependent reliability is performed throughout the time span of the analysis, in order to identify the way probability of structural failure evolves in time. A sound model based on the Boundary Element Method (BEM) is built to simulate the evolution of the corrosion process until the depassivation of the concrete reinforcement bars, and the evolving loss of resistance is considered from this point on. Details about the adopted BEM formulation are discussed in the next session.

## 2. BOUNDARY ELEMENT METHOD FORMULATION APPLIED TO DIFFUSION PROBLEMS

The Poisson's equation, which derives its name from the French mathematician and physicist Siméon Denis Poisson (1781-1840), has important applications in various fields of science. In engineering problems, this equation allows the

modeling of phenomena related to thermal conductivity, torsion, conduction of substances, among others. This equation can be written as:

$$\nabla^2 u + b = 0 \quad (1)$$

Where  $u$  is the potential and  $b$  is a domain term. Considering that the domain term is null ( $b = 0$ ), the Poisson's equation is written as:

$$\nabla^2 u = 0 \quad (2)$$

This equation is called Laplace's equation. The boundary conditions of the problem are:

- Essential:  $u = \bar{u}$  in  $\Gamma_1$
- Natural:  $q = \bar{q} = \frac{du}{d\eta}$  in  $\Gamma_2$

Where  $\bar{u}$  is the prescribed potential value,  $\bar{q}$  is the prescribed flow value,  $\Gamma_1$  and  $\Gamma_2$  the correspond boundaries and  $\frac{du}{d\eta}$  is the directional derivative of  $u$  in relation to the normal vector  $\eta$ , as shows in Figure 1;

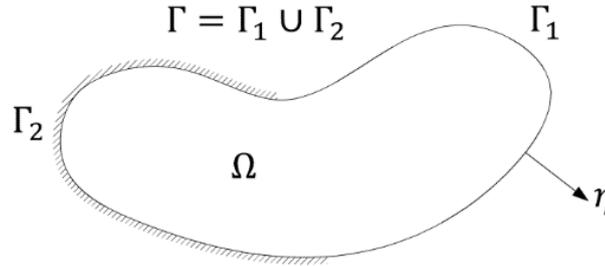


Figure 1 – Domain, boundaries and normal vector to an arbitrary body

It should be emphasized that the last two equations are used to describe time independent problems. The equation describing the diffusion problem is as follows:

$$\nabla^2 u - \frac{1}{\kappa} \frac{\partial u}{\partial t} = 0 \quad (3)$$

Where  $\kappa$  is a domain-related parameter, such as thermal diffusivity or diffusion coefficient, and  $t$  is time. The boundary conditions are analogous to those of the Laplace's equation, that is, one must first know the potential  $u$  or the flow  $q$  in the entire boundary  $\Gamma$  at each time instant.

In this study the domain term is considered null. Thus, for the problem studied, it is assumed that the initial chloride concentration in the domain is equal to zero, that is, it is assumed that the concrete does not contain chlorides incorporated in the mass previously. Consideration of nonzero initial conditions could be done by subdividing the domain into cells and integrating known functions into these cells.

The partial differential Equation (3) can be converted into an integral contour equation using the Laplace transform, finite differences or using fundamental time dependent solutions, as can be seen in Wrobel (2002). In this work, the last approach is used. Applying the technique of weighted residues, integrating by parts and employing a limit process, the following integral equation of contour is obtained:

$$(\underline{\xi}) u(\underline{\xi}, t_F) = \kappa \int_{t_0}^{t_F} \int_{\Gamma} q(\underline{x}, t) u^*(\underline{\xi}, \underline{x}, t_F, t) d\Gamma(\underline{x}) dt - \kappa \int_{t_0}^{t_F} \int_{\Gamma} u(\underline{x}, t) q^*(\underline{\xi}, \underline{x}, t_F, t) d\Gamma(\underline{x}) dt \quad (4)$$

Where  $\underline{\xi}$  refers to source points,  $\underline{x}$  refers to field points,  $t_0$  is the analysis initial time,  $t_F$  is the observation time,  $u^*$  and  $q^*$  are the fundamental time-dependent solutions and  $c$  is a parameter in function of the internal angle of the source point. The parameter  $c$  is equal to 1 whether the source point is located in the domain and equal to 0 whether is located outside the domain. This parameter is equal to 0.5 whether the source point is located in the boundary, since the boundary has a smooth geometry. In 2D case, the fundamental solutions are:

$$u^*(\underline{\xi}, \underline{x}, t_F, t) = \frac{1}{4\pi\kappa t} \exp\left(-\frac{r^2}{4\kappa t}\right) \quad (5)$$

$$q^*(\underline{\xi}, \underline{x}, t_F, t) = -\frac{r}{8\pi\kappa^2 t^2} \frac{\partial r}{\partial \eta} \exp\left(-\frac{r^2}{4\kappa t}\right) \quad (6)$$

Where  $\tau = t_F - t$ ,  $r$  is the distance between the source point  $\underline{\xi}$  and the field point  $\underline{x}$  and  $r \frac{\partial r}{\partial \eta}$  is the projection of distance  $r$  over the boundary normal vector.

Dividing boundary  $\Gamma$  in  $Ne$  boundary elements, time interval  $t_F - t_0$  in  $Nt$  time steps and inverting the integration order, the following discretized equation is obtained:

$$c_i u_i^{Nt} = \kappa \sum_{j=1}^{Ne} \sum_{k=1}^{Nt} \int_{\Gamma_j} \int_{t_0^k}^{t_f^k} q^k u^* dt d\Gamma_j - \kappa \sum_{j=1}^{Ne} \sum_{k=1}^{Nt} \int_{\Gamma_j} \int_{t_0^k}^{t_f^k} u^k q^* dt d\Gamma_j \quad (7)$$

Where  $u_i^{Nt}$  is the value of the potential in time  $t^{Nt}$  and in the source point  $i$ . Assuming that the functions  $u$  and  $q$  are constant in each time step and considering that the source points are located in a boundary with a smooth geometry, last equation is rewritten as:

$$0.5 u_i^{Nt} = \kappa \sum_{j=1}^{Ne} \int_{\Gamma_j} \sum_{k=1}^{Nt} q^k \int_{t_0^k}^{t_f^k} u^* dt d\Gamma_j - \kappa \sum_{j=1}^{Ne} \int_{\Gamma_j} \sum_{k=1}^{Nt} u^k \int_{t_0^k}^{t_f^k} q^* dt d\Gamma_j \quad (8)$$

Inverting the summation order, assuming that  $u^k$  and  $q^k$  are constants in each node and element, we obtain:

$$0.5 u_i^{Nt} = \kappa \sum_{k=1}^{Nt} \sum_{j=1}^{Ne} q_j^k \int_{\Gamma_j} \int_{t_0^k}^{t_f^k} u^* dt d\Gamma_j - \kappa \sum_{k=1}^{Nt} \sum_{j=1}^{Ne} u_j^k \int_{\Gamma_j} \int_{t_0^k}^{t_f^k} q^* dt d\Gamma_j \quad (9)$$

It should be noted that in this last equation it is necessary to know the values of  $u$  and  $q$  in each node  $i$ , in each element  $j$ , in each discrete time instant  $k$ . The last equation is then rewritten as follows:

$$0.5 u_i^{Nt} = \kappa \sum_{k=1}^{Nt} \left( \sum_{j=1}^{Ne} q_j^k \int_{\Gamma_j} U_k^* d\Gamma_j - \sum_{j=1}^{Ne} u_j^k \int_{\Gamma_j} Q_k^* d\Gamma_j \right) \quad (10)$$

The fundamental solutions integrated in time are calculated analytically, resulting in:

$$U_k^* = \int_{t_0^k}^{t_f^k} u^* dt = \frac{1}{4\pi\kappa} \left\{ E_1 \left[ \frac{r^2}{4\kappa(t_F - t_0^k)} \right] - E_1 \left[ \frac{r^2}{4\kappa(t_F - t_f^k)} \right] \right\} \quad (11)$$

$$Q_k^* = \int_{t_0^k}^{t_f^k} q^* dt = -\frac{1}{2\pi r \kappa} \left\{ \exp \left[ -\frac{r^2}{4\kappa(t_F - t_0^k)} \right] - \exp \left[ -\frac{r^2}{4\kappa(t_F - t_f^k)} \right] \right\} \quad (12)$$

Where  $E_1$  is the exponential-integral function. Equation (10) can be written in terms of the influence coefficients  $H$  and  $G$  for each source point as:

$$0.5 u_i^{Nt} = \sum_{k=1}^{Nt} \left( \sum_{j=1}^{Ne} G_{ij}^k q_j^k - \sum_{j=1}^{Ne} \widehat{H}_{ij}^k u_j^k \right) \quad (13)$$

Where  $H$  and  $G$  are the matrices that contain the integration kernels:

$$G_{ij}^k = \int_{\Gamma_j} U_k^* d\Gamma_j \quad (14)$$

$$\widehat{H}_{ij}^k = \int_{\Gamma_j} Q_k^* d\Gamma_j \quad (15)$$

$$H_{ij}^k = \begin{cases} \widehat{H}_{ij}^k + 0.5 & \text{if } k = 1 \text{ and } i = j \\ \widehat{H}_{ij}^k & \text{in other cases} \end{cases} \quad (16)$$

There is a singular integration when the source point belongs to the field element in the first time step. Thus, the singularity subtraction technique is used to regularize the integral calculation. The kernels are numerically integrated using the Gauss' quadrature for each boundary element. The geometry of the boundary elements is approximated using polynomial shape functions. By employing a time-march system it is possible to calculate the potential and flux value at the points of the contour at a given time  $k$  by knowing the value of these parameters at times prior to  $k$ , starting from

$k = 1$  to  $k = Nt$ . The potential at internal points is obtained analogously to what is represented in Equation (13), once the flux and potential values in the contour are known and  $c(\xi) = 1$  is adopted.

This framework is suitable for a large number of applications. In this work, we are particularly interested in the study of the evolution of chloride penetration in reinforced concrete, as will be detailed in the next section.

The computational implementation of the BEM formulation described herein was validated through a comparative study with other formulations, as can be seen in Pellizzer and Leonel (2018). The accuracy, stability and convergence of the numerical method were also verified in the work mentioned. One of the main advantages of BEM is the reduction of the processing time. Among other reasons, this reduction occurs because there is a reduction of the mesh dimension, resulting in a system with a smaller number of equations to be solved.

### 3. CORROSION PROCESS

One of the first qualitative idealizations of the useful life of reinforced concrete structures subjected to chloride ions penetration was proposed by Tuutti (1982). In this model, structural useful life is divided in two phases: initiation and propagation. In the initiation phase the chloride ions present on the outer surface of the structure move through the micro pores of the concrete. When the concentration of chlorides at the concrete/reinforcement interface reaches a threshold value, reinforcement depassivation occurs, ending the initiation phase. Propagation phase then begins, where the corrosion chemical reactions that occur result in deleterious mechanical effects on steel and concrete. Initiation and propagation phases occur in parallel within the same reinforced concrete element, i.e., some rebar may have already undergone depassivation while others have not. This is due to the high randomness present in the variables involved in the corrosive process.

In the initiation phase, the main transport mechanism of chloride ions through the concrete's micro pores is the diffusion. The diffusion of chloride ions in concrete is often modeled using Fick's laws (Vu and Stewart, 2000; Samson et al., 2003; Val et al., 2009; Mangat and Molloy, 2010; Guzmán et al., 2011). Some disadvantages of most of the solutions presented for the differential equations that describe Fick's laws are that they are solutions obtained from semi-infinite domains and that have constant boundary conditions over time. Considering that it is extremely important to adequately predict when the corrosion propagation period will begin, in the present work is presented an alternative approach based on the Boundary Element Method (BEM) for this purpose. Among the advantages of BEM model is the possibility of varying boundary conditions over time and the possibility of analyzing two-dimensional domains with any contours of geometry. Therefore, the formulation presented here seeks to be a guide for a more reliable modeling of the reality of the phenomenon.

In the propagation phase, one of the main effects of corrosion is the reduction of reinforcement steel area. Chemical reactions that consume the constituent metal of the rebars produce two basic types of corrosion: uniform and pitting. In this work it is considered uniform corrosion in the reinforcement. The model adopted is presented in the work of Thoft-Christensen and Hansen (1994) and Valand Melchers (1997). Based on Faraday's laws, the authors propose an equation to predict the residual diameter of the corroded rebar as follows:

$$d(t_c) = d_0 - 0.0232 i_{corr} t_c \quad (17)$$

Where  $d_0$  is the original diameter of the non-corroded rebar in  $mm$ ,  $i_{corr}$  is the corrosion rate in  $\mu A/cm^2$  (microampere per square centimeter) and  $t_c$  is the elapsed time after the depassivation of the rebar in years. Once the diameter of the corroded bar  $d(t_c)$  is known, it is possible to calculate the steel area of each of the steel rebar and then the total steel area of a given reinforced concrete cross section. The corrosion rate  $i_{corr}$  in this work is calculated using the empirical equation presented by Vu and Stewart (2000). This equation is based on measurements available in the literature and on the conversion of the oxygen diffusion rate to the corrosion rate by considering the percentage of corrosion products and the molecular equations of corrosion in the cathodic zone. The obtained expression that relates the rate of corrosion with the elapsed time after the depassivation of the reinforcements is:

$$i_{corr}(t_c) = \left[ \frac{37.8(1 - w/c)^{-1.64}}{cov} \right] 0.85 t_c^{-0.29} \quad (18)$$

Where  $i_{corr}(t_c)$  is the corrosion rate in  $\mu A/cm^2$  (microampere per square centimeter),  $w/c$  is the water/cement ratio,  $cov$  is the reinforcement concrete cover thickness in  $mm$  and  $t_c$  is the elapsed time after the depassivation of the rebar in years. In this study, other deleterious mechanisms of the propagation phase such as reinforcement stress reduction, modification in the stress transfer mechanism between steel and concrete and the appearance of internal cracks are not taken into account.

### 4. TIME-VARIANT RELIABILITY

Let  $X(t, \omega)$  be a set of  $M = p + q$  elements that represents the randomness of a mechanical problem. Material properties and geometric characteristics are typically described as random variables, Adaptive Kriging Strategy for Risk

Optimization with Time-Dependent Reliability represented by  $X_j(\omega)$   $j = 1, \dots, p$ . Loads can be modeled as random processes of time  $X_k(\omega, t)$   $j = p + 1, \dots, p + q$ . In this notation,  $\omega$  stands for the outcome in the space of outcomes  $\Omega$ . Furthermore, for optimization problems let  $\mathbf{d}$  be a vector that gathers together all the system's design parameter. This vector may include parameters that describe moments of random variables, should tolerances on design dimensions be included in the analysis (Moustapha, 2016). Assume a structural limit state function that denotes safe states if it is greater than zero and failure if it is smaller than zero. Thus the boundary between desirable and undesirable structure responses is given by  $g(\mathbf{d}, \mathbf{t}, \mathbf{X}(t, \omega))$  such that:

$$\begin{aligned} D_f(t, \mathbf{d}) &= \{\mathbf{d}, \mathbf{X}(t, \omega): g(\mathbf{d}, \mathbf{t}, \mathbf{X}(t, \omega)) \leq 0\} \text{ is the failure domain} \\ D_s(t, \mathbf{d}) &= \{\mathbf{d}, \mathbf{X}(t, \omega): g(\mathbf{d}, \mathbf{t}, \mathbf{X}(t, \omega)) > 0\} \text{ is the safe domain} \end{aligned} \quad (19)$$

For a given limit state, the instantaneous probability of failure  $P_{fi}$  at a time  $t = \tau$  is given by:

$$P_{fi}(\mathbf{d}, \tau) = \mathbb{P}(g(\mathbf{d}, \tau, \mathbf{X}(\tau, \omega)) \leq 0) = \int_{D_f(\mathbf{d}, \tau)} f(\mathbf{x}) d\mathbf{x} \quad (20)$$

Where  $\mathbb{P}$  denotes the probability of the event and  $f(\mathbf{x})$  is the joint probability density function of all the random variables for a configuration  $\mathbf{d}$  at a time  $\tau$ . In this work, we are rather interested in the so-called cumulative probability of failure  $P_{fc}(t_1, t_2)$  which is defined for a given  $\mathbf{d}$  as the probability of occurrence of a structural failure within the time interval  $[t_1, t_2]$ :

$$P_{fc}(\mathbf{d}, t_1, t_2) = \mathbb{P}[\exists \tau \in [t_1, t_2]: g(\mathbf{d}, \mathbf{t}, \mathbf{X}(t, \omega)) \leq 0] \quad (21)$$

Different approaches have been suggested to compute  $P_{fc}$ . The so-called out-crossing approach has been widely used. Examples of well-know methods include the PHI2 approach (Andrieu-Renaud et al., 2004) and the asymptotic PHI2 method (Sudret, 2008). The accuracy of such methods are however impeded by the introduction of approximation methods such as first-order reliability method (FORM). FORM is indeed known to lead to spurious results in presence of highly non-linear limit states and multiple design points. Henceforth, an approach based on direct simulation is instead considered in this paper.

#### 4.1 Monte Carlo-Based Estimation Of The Cumulative Failure Probability

The basic idea is to sample trajectories of the limit-state function over a given time interval and then count the number of such trajectories for which failure occurs. To achieve this, the random process of interest is first discretized, i.e. represented by a finite set of random variables (Sudret and Der Kiureghian, 2000). The discretization method employed in this work is the *expansion optimal linear estimation* (EOLE), as presented by Li and Der Kiureghian(1993).

Let us consider a scalar Gaussian random process  $\mathbf{X}(t, \omega)$ , with mean  $\mu$ , standard deviation  $\sigma$  and auto-correlation coefficient function  $\rho_X(t_1, t_2)$ .  $\mathcal{P}$  time points are selected in the interval  $[0: T]$ , so that  $t_1 = 0$  and  $t_p = T$ . The EOLE expansion is then given by:

$$\mathbf{X}(t, \omega) = \mu + \sigma \sum_{i=1}^r \frac{\xi_i(\omega)}{\sqrt{\lambda_i}} \phi_i^T \mathbf{C}_{t, t_i}(t) \quad (22)$$

Where  $\{\xi_i(\omega), i = 1, \dots, \mathcal{P}\}$  are independent standard normal variables,  $\{\phi_i, \lambda_i, i = 1, \dots, r\}$  are the eigenvectors and eigenvalues of the correlation matrix  $\mathbf{C}$  sorted in decreasing order, with  $C_{ij} = \rho_X(t_i, t_j)$ ,  $i, j = \{1, \dots, \mathcal{P}\}$ . Note that here the expansion is truncated to  $r \leq \mathcal{P}$  terms, the value of which defines the so-called order of expansion. The random process discretization is hereby performed using the software UQlab (Marelli and Sudret, 2014);

Once a proper discretization is obtained one is able to draw trajectories of the limit state function  $g(\mathbf{d}, \mathbf{t}, \mathbf{X}(t, \omega))$  in the time interval  $[0: T]$ , for a given  $\mathbf{d}$ . This is done by considering both the EOLE expansions of the random and realizations of the time independent random variables. In practice, the simulated values are stored in an array  $G$  of dimension  $1 \times N$ , where  $N$  is the number of time instants in which the limit state equation is discretized. Each position  $i$  of this array corresponds to a time  $t_i = (i - 1)\Delta t$  where  $\Delta t = \frac{T}{N-1}$  is the sampling step, assuming a uniform discretization. For each  $t_i$ , consider a counter  $k_j$  that is increased every time  $g$  presents the first outcrossing in the interval  $[t_i; t_{i+1}]$ . A brute Monte Carlo estimation for the cumulative probability of failure, considering  $N_{MC}$  Monte Carlo samples is then given by:

$$P_{fc}(0, t_i) = \frac{1}{N_{MC}} k_j \quad (23)$$

## 5. PROPOSED METHODOLOGY

In order to address the safety of a concrete structure throughout its whole life span, the first adopted step is to study the time until the reinforcement bars are depassivated, starting the corrosion process. The analysis is divided in two rather independent steps (Problem 1 and Problem 2). In the first one, uncertainties regarding the chloride penetration problem are considered as input to the BEM based analysis: chloride threshold value  $C_{limit}$ , surface chloride concentration  $C_s$ , diffusion coefficient in concrete  $\kappa$  and cover thickness  $CT$ . This problem is solved  $10^6$  times, in order to provide a million depassivation times, which are considered as inputs to the second problem, that consists in the Monte Carlo based time-dependent reliability analysis. This step also considers uncertainties in the structural parameters: concrete strength  $f_{ck}$ , steel strength  $f_{yk}$  and the loads of the problem, hereby generically represented by  $q$ . The way the two steps relate is illustrated by Figure 2:

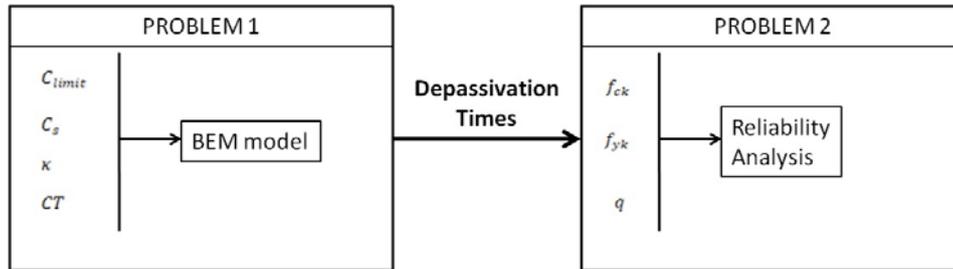


Figure 2 – Proposed Methodology

## 6. APPLICATION EXAMPLE

Consider the 4 meter span beam represented in Figure 3. The beam has rectangular cross-section of 40cmX20cm, and 3 longitudinal reinforcement bars with diameter 12.5mm. It is also subject to a time-variant load  $q$ , described by a random process with Gaussian autocorrelation function, with correlation length of 1 year.

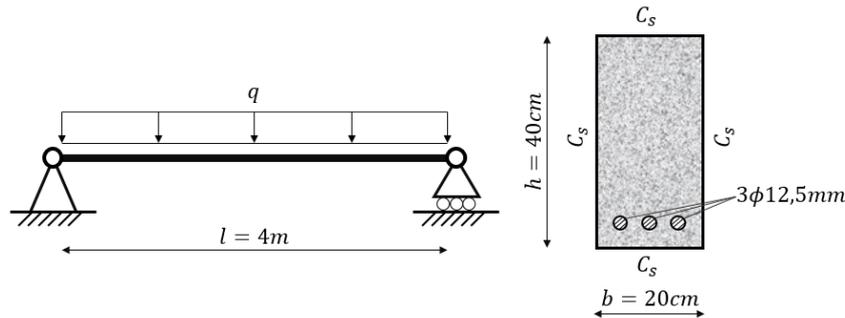


Figure 3 – Reinforced concrete beam modeled

Table 1 shows the random quantities involved in the chloride diffusion problem, and Table 2, the random quantities involved in the reliability analysis. The boundary was discretized in 8 quadratic discontinuous and isoparametric boundary elements. The number of Gauss points per element is 8. The simulation of chloride ion penetration was performed until 200 years, discretized in 50 time steps. Depassivation occurrence time of each rebar was determined by means of interpolation between two successive time steps in which in the first step, the depassivation did not occur and in which in the second step it occurred. A mesh convergence analysis was previously done in order to determine a suitable boundary discretization for the problem.

Table 1 – Statistical parameters for the diffusion analysis with the BEM

Parameter	Distribution type	Mean	COV
Chloride threshold value $C_{limit}$	Uniform	0.90 kg/m <sup>3</sup>	0.19
Surface chloride concentration $C_s$	Lognormal	1.15 kg/m <sup>3</sup>	0.50
Diffusion coefficient in concrete $\kappa$	Lognormal	0.6742277 cm <sup>2</sup> /year	0.75
Cover Thickness $CT$	Normal	3.00 cm	0.50

**Table 2 – Statistical parameters for the Time-Variant Reliability Analysis**

Parameter	Distribution type	Mean	COV
Concrete Strength $f_{ck}$	Lognormal	30 MPa	0.20
Steel Strength $f_{yk}$	Lognormal	500 MPa	0.20
Load $q$	Gaussian Random Process	10 kN/m	0.20

Reliability analysis was carried out over a 50 years' time-span, considering  $10^6$  samples in each of the 500 discrete time-points, so that each correlation length is discretized in 10 time steps. The limit state considered refers to a failure by excessive bending moment, following normative recommendations of the Brazilian standard for reinforced concrete design (NBR6118, 2014):

$$g(\mathbf{x}, t) = \text{Barrier}(\mathbf{x}, t) - \text{Solicitation}(\mathbf{x}, t)$$

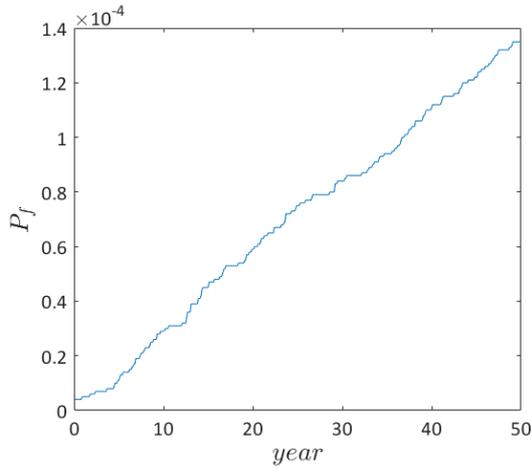
$$\text{Barrier}(\mathbf{x}, t) = A_s(t) f_{yk} \left( d - \frac{\lambda x}{2} \right) \quad (24)$$

$$\text{Solicitation}(\mathbf{x}, t) = \frac{q(t) l^2}{8}$$

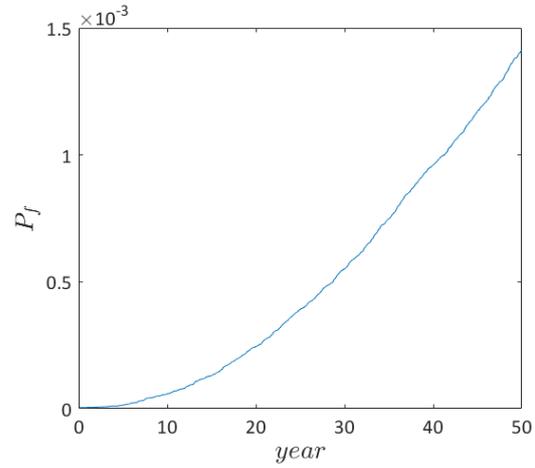
Where  $\lambda = 0.8$  when  $f_{ck} < 50 \text{ MPa}$  and  $\lambda = 0.8 - \frac{f_{ck} - 50}{400}$  when  $50 \text{ MPa} \leq f_{ck} \leq 90 \text{ MPa}$ ,  $x$  is the height of the neutral axis in the cross section, and  $A_s$  is the remaining area of steel. The reliability analysis is carried out disregarding barrier degradation until the first depassivation time occurs, and from that point on, corrosion is considered on each of the reinforcement bars already affected by corrosion. Time-independent corrosion is considered in the first year (Eq. 17), and the time-dependent equation is adopted from this point on (Eq. 18). The structure is also considered to fail when the normative ductility criterion is violated, once again considering NBR6118 specification, which is indicated by:

$$\begin{cases} \frac{x}{d} \leq 0.45 \text{ if } f_{ck} \leq 50 \text{ MPa} \\ \frac{x}{d} \leq 0.35 \text{ if } f_{ck} > 50 \text{ MPa} \end{cases} \quad (25)$$

Results for cumulative probability of failure both considering and ignoring corrosion are shown in Figure 4, and one example of Barrier and Solicitation evolution in time is shown to both cases in Figure 5.



(a) Cumulative probability of failure disregarding corrosion



(b) Cumulative probability of failure considering corrosion

**Figure 4 – Results for cumulative probability of failure**

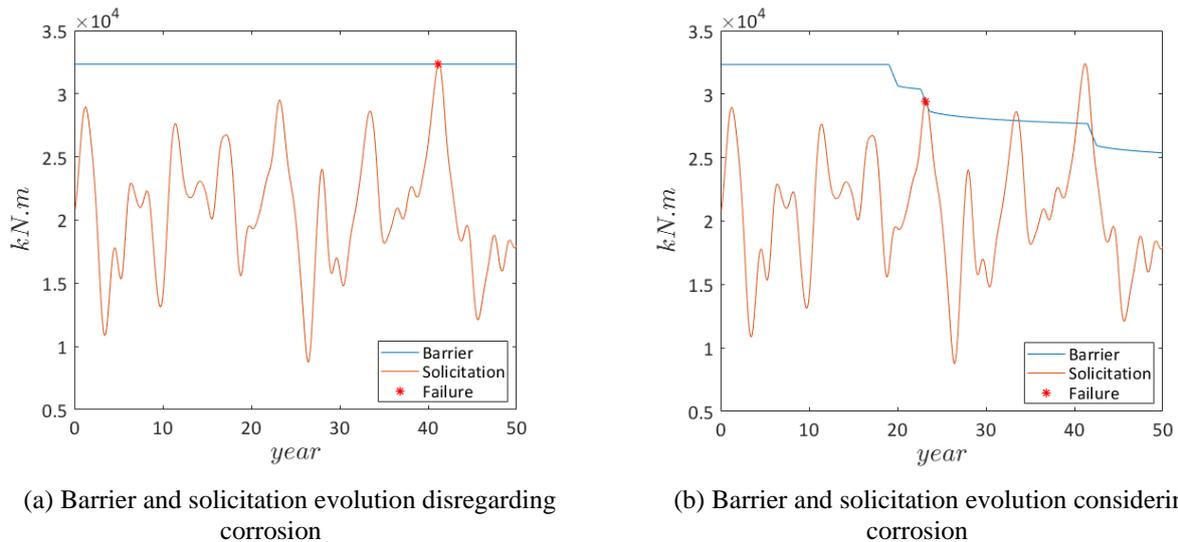


Figure 5 – Example of time-evolution of the problem

## 7. CONCLUSION

An innovative methodology for studying the safety of reinforced concrete structures over time was proposed, considering a time-dependent reliability analysis where both the solicitation changes in time due to the presence of time varying loads, and the barrier changes in time due to chloride penetration induced reinforcement bars corrosion. The time until corrosion starts in each reinforcement bar was treated as a separated reliability problem, considering a robust BEM model to study the evolution of chloride penetration in the structure. One example considering a reinforced concrete beam was studied, considering normative recommendations. The consideration of scenarios with and without corrosion lead to very different results, showing the importance of a careful consideration of the evolution and contribution of such degradation processes. The potential of such analysis is demonstrated, which shall lead to a next study considering more complex reinforced concrete structures, where surrogate models (Kroetz et al. 2017) can be used to ensure that the proposed approach does not become excessively expensive in terms of computational costs.

## ACKNOWLEDGMENTS

This study was financed in part by the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior - Brasil (CAPES) - Finance Code 001.

## REFERENCES

- Andrieu-Renaud, C.; Sudret, B.; Lemaire, M., 2004. The PHI2 method: a way to compute timevariant reliability. *Reliab. Eng. Sys. Safety* 84, 75–86.
- Apostolopoulos, C.A.; Papadakis, V.G., 2008. Consequences of steel corrosion on the ductility properties of reinforcement bar. *Construction and Building Materials*, v. 22, n. 12, p. 2316-2324.
- Associação Brasileira de Normas Técnicas (ABNT), 2014. Projeto de estruturas de concreto: procedimento (NBR 6118), Rio de Janeiro, 238 p.
- Bastidas-Arteaga, E.; Chateauneuf, A.; Sánchez-Silva, M.; Bressolette, P.; Schoefs, F., 2011. A comprehensive probabilistic model for chloride ingress in unsaturated concrete. *Engineering Structures*, v. 51, p. 259-266.
- Cascudo, O., 1997. O controle da corrosão de armaduras em concreto: inspeção e técnicas eletroquímicas. São Paulo: Pini; Goiania: Editora UFMG.
- Comité Euro-Internacional du Béton - Durable concrete structures – design guide (CEB), 1992. Bulletin d'information n. 183, London, Thomas Telford.
- Dietlevsen, O.; Madsen, H.O., 2007. Structural reliability methods. Technical University of Denmark.
- Gonzalez, J.A.; Andrade, C.; Alonso, C.; Feliu, S., 1995. Comparison of rates of general corrosion and maximum pitting penetration on concrete embedded steel reinforcement. *Cement and Concrete Research*, v. 25, p. 257-264.
- Guzmán, S., Gálvez, J.C., Sancho, J.M., 2011. Cover cracking of reinforced concrete due to rebar corrosion induced by chloride penetration. *Cement and Concrete Research*, v.41, p. 893-902.
- Hasofer, A.M.; Lind, N.C., 1974. Exact and invariant second moment code format. *ASCE JEng Mech*, 100:111–21 .
- Kroetz, H.; Tessari, R.; Beck, A., 2017. Performance of global metamodeling techniques in solution of structural reliability problems. *Adv. in Eng. Soft.* 114, 394-404.

- Li, C.; Der Kiureghian, A., 1993. Optimal discretization of random fields. *J. Eng. Mech.* 119(6), 1136–1154.
- Mangat, P.S.; Molloy, B.T., 1994. Prediction of long term chloride concentration in concrete. *Materials and Structures*, v.27, p. 338-346.
- Marelli, S.; Sudret, B., 2014. UQLab: a framework for uncertainty quantification in Matlab. In: *Vulnerability, uncertainty and risk, proceedings of the 2nd international conference on vulnerability, risk and analysis management (ICVRAM2014)*, Liverpool, PP 2554–2563
- Moustapha, M.; Sudret, B.; Bourinet, J.-M.; Guillaume, B., 2016. Quantile-based optimization under uncertainties using adaptive Kriging surrogate models. *Struct. Multidisc. Optim.* 54(6), 1403–1421.
- Pellizzer, G.P.; Leonel, E.D. 2018. Modelagem do período de iniciação da corrosão em estruturas de concreto armado via método dos elementos de contorno. In: *DEGRADA 2018 - 3º Encontro Luso-Brasileiro De Degradação Em Estruturas De Concreto*, 2018, São Carlos. *Anais do 3º Encontro Luso-Brasileiro De Degradação Em Estruturas De Concreto - DEGRADA 2018*.
- Samson, E.; Marchand, J.; Snyder, K.A., 2003. Calculation of ionic diffusion coefficients on the basis of migration test results. *Mater Struct*, v.36, p. 156–165.
- Sudret, B., 2008. Analytical derivation of the outcrossing rate in time-variant reliability problems. *Struct. Infrastruct. Eng* 4(5), 353–362.
- Sudret, B.; Der Kiureghian, A., 2000. Stochastic finite elements and reliability: a state-of-the-art report. Technical Report UCB/SEMM-2000/08, University of California, Berkeley (173 pages).
- Suo, Q.; Stewart, M.G., 2009 Corrosion cracking prediction updating of deteriorating RC structures using inspection information. *Reliability Engineering and System Safety*, v. 94, p. 1340-1348.
- Thoft-Christensen, P.; Hansen, H.I., 1994. Optimal strategy for maintenance of concrete bridges using expert systems. *Proc., ICOSSAR'93*, A.A.Balkema, Rotterdam, The Netherlands, p. 939-946.
- Tuutti, K., 1982. Corrosion of steel in concrete Swedish. Cement and concrete research institute Stockholm, 460 p.
- Val, D.V.; Chernin, L.; Stewart, M.G., 2009. Experimental and numerical investigation of corrosion-induced cover cracking in reinforced concrete structures. *Journal of Structural Engineering, ASCE*, v.135, p. 376–385.
- Val, D.V.; Melchers, R.E., 1997. Reliability of deteriorating RC slab bridges. *Journal of Structural Engineering (ASCE)*, v. 123, n. 12, p. 1638-1644.
- Val, D.V.; Stewart, M.G. 2003, Life-cycle cost analysis of reinforced concrete structures in marine environments. *StructuralSafety*, v. 25, n. 4, p. 343-362.
- Vu, K.A.T.; Stewart, M.G., 2000. Structural reliability of concrete bridges including improved chloride-induced corrosion models. *Structural Safety*, v.22, n.4, p. 313-333.
- Wrobel, L.C., 2002. *The Boundary Element Method Volume 1: Applications in Thermo-Fluids and Acoustics*. 1 ed., Wiley, 468 p.
- Zhang, R.; Castel, A.; François, R., 2010. Concrete cover cracking with reinforcement corrosion of RC beam during chloride-induced corrosion process. *Cement and Concrete Research*, v. 40, p. 415-425.