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FEM-BEM Analysis of Arbitrarily-Shaped Structures Supported by Pile Groups

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Abstract: This paper presents a numerical model of the dynamic response of arbitrarily-shaped piled structures. The structure was modeled using classical finite element formulation, which enables structures of arbitrary shapes to be studied, as well as arbitrary time-harmonic loadings. The piles were modeled using the thin-layer formulation of embedded pile groups, which enables layered bounded or unbounded soils to be studied, under both external and internal (seismic) excitations. The coupling between the two systems was obtained using direct kinematic compatibility and equilibrium at the interface between the two systems. For the present case, the interfaces consist of the discrete contact points in which nodes from the finite element mesh of the structure exchange energy with the pile heads. The method is used to analyze a representative example of rigid structure supported by a group of piles, and the effect of the flexibility of the structure in the response of the system.

Keywords: Soil-foundation interaction, Pile groups, Pile-structure interaction

INTRODUCTION

Piled structures comprise an embedded group of elastic piles supporting an elastic surface structure, where the piles are used to transfer load from the structures they support to the underlying soil. Understanding the response of piled structures under dynamic vertical, horizontal and rocking loads is fundamental in the design of large-scale, vibration-sensitive structures such as concert halls, wind turbines, and particle accelerators.

Early attempts to model the response of pile groups supporting a surface plate relied on Winkler's (1867) idea of modeling the soil as discrete mass-spring-dashpot systems (Novak, 1974; Mylonakis and Gazetas, 1999, Pacheco et al., 2008). These fail to represent the soil as a loaded, deformable continuum, and require the soil's mass-spring-dashpot properties to be obtained for each case (Nguyen et al., 2016). One of the solutions to these issues was proposed by Rajapakse and Shah (1987). Their model of embedded elastic pile was based on a variational approach involving generalized coordinates, and accounted for wave propagation from the pile to its surrounding, continuous half-space. Despite the axisymmetry of the Green's function used to model the soil in Rajapakse and Shah's work (1987), their formulation has recently been successfully extended to model three dimensional pile groups (Labaki and Mesquita, 2016). Other line of thought resorts to full BEM and/or FEM discretization of the foundation and soil to model piled-raft foundations (Kaynia and Kausel, 1991; Maeso et al., 2005; Taherzadeh et al., 2009; Ai, Li and Wang, 2016; Barros, Labaki and Mesquita, 2018). Notable within this category, Kaynia and Kausel (1991) derived a considerably general model of pile group–soil interaction, in which piles within the group may have different geometries, material parameters, and distribution within the soil, while in turn the soil may be a finite layer over rigid bedrock, or an unbounded half-space, both constituted of an arbitrary number of transversely isotropic, elastic layers of arbitrary thickness. The pile group in their model may be under vertical, horizontal, rocking or torsional time-harmonic loads. Their model includes the case in which the piles are connected by a rigid surface plate, which is not in contact with the soil. On the other hand, models of elastic plates with soil contact were initially based on the coupling of classical FEM discretization for the plate with some Green's function for the soil part (Savidis and Richter, 1979; Igushi and Luco, 1981; Auersch, 1996; Guerra et al., 2016). The coupling between the plate and the soil in these works is described through kinematic compatibility and equilibrium at discrete points of the plate–soil interface. Models of elastic plates in continuous bonding contact with the soil were presented by Rajapakse (1988), and more recently extended by Labaki et al. (2014). These took on the variational approach used by Rajapakse and Shah (1987) to model embedded piles, mentioned previously. Due to the axisymmetry of the Green's function used for the soil, these plate models are limited to circular plates under vertical excitations.

This article presents a model of the dynamic response of elastic piled structures under arbitrarily distributed vertical, horizontal, and rocking time-harmonic loads (Fig. 1). The surface structure is modeled by classical linear-viscoelastic, 8-noded hexahedral finite elements. The viscoelasticity of the structure may be incorporated according to Christensen's elastic-viscoelastic principle (Christensen, 2010). The embedded pile group part of the problem is obtained through an implementation of the model by Kaynia and Kausel (1991), the formulation of which is described in the following section. The coupling between the two systems is obtained through direct superposition of the stiffness matrix of the surface plate and pile group. The coupling scheme between the stiffness matrices arises upon establishing kinematic compatibility and equilibrium between nodes of the plate and the pile heads with which they interact.

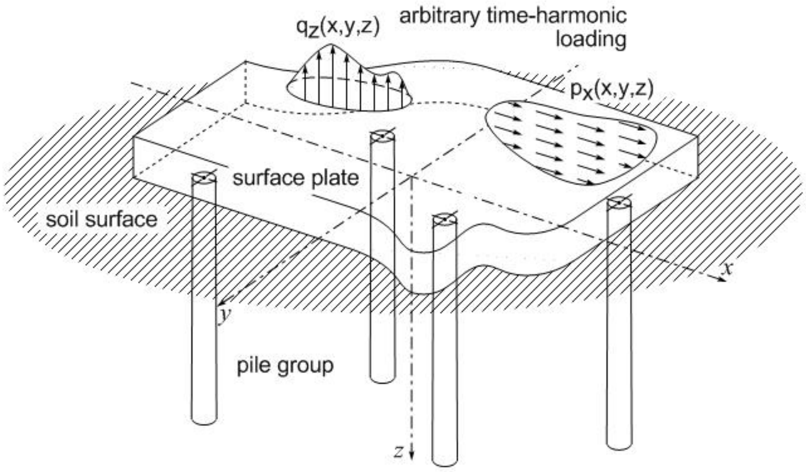


Figure 1 – Example of arbitrary structure supported by embedded pile group.

FORMULATION

Formulation of the Piles

The formulation used for the piles group was derived by Kaynia and Kausel (1991). This formulation presents a solution for a pile group, connected or not to a rigid plate, embedded in a viscoelastic, layered soil media, supported by a half-space or a rigid bed rock. The pile-soil interface is discretized in *l* arbitrary segments plus 1 (one) circular element at the pile end. The external forces acting on the pile group are harmonic and can be vertical, horizontal, rocking or torsional. It is possible to simulate also a vertically propagating seismic shear wave.

The loads and the displacements of the pile heads of the N piles in the group are related through Eq. (1):

$$P_e = K_e u_e \tag{1}$$

where P_e is a vector of forces and moments and u_e is a vector of displacements and rotations at the piles heads. The stiffness matrix of the pile group can be written as

$$K_e = K_p + \psi^T (F_s + F_p)\psi \tag{2}$$

where Ψ is the shape matrix of the displacements at each node in a pile with fixed ends, F_p is the flexibility matrix of the (l+1) nodes of a fixed-end pile, K_p is the pile stiffness matrix, simply the stiffness matrix of a one-dimensional finite beam element, and F_s is the flexibility matrix of the soil media. F_s is defined by Kaynia and Kausel (1991) as the matrix relating piecewise-constant segmental loads to the average displacements along the segments and is assembled using the Green's Functions developed using the method proposed by Apsel (1979) using the layer stiffness approach (Kausel and Rousset, 1981). Kaynia and Kausel (1991) state that the method with the best approximation for finding F_s is by calculating the soil flexibility without the pile cavities, as long as the spaces where the piles were embedded are modeled with an elasticity modulus and mass density equal to the ones of the piles, subtracted from the ones of the soil. A full description of the quantities involved in Eq. 2 and their computation is beyond the scope of this article. For further details, please refer to Kaynia and Kausel (1991).

In this work, the main interest from the implementation of this pile model is K_e, the stiffness matrix of the pile group. Matrix K_e will be incorporated in the stiffness matrix of a piled structure, according to the coupling scheme described below. The structure then undergoes external loads, the effect of which is transferred to the pile heads, rather than loads prescribed at the pile heads through load vector P_e (Eq. 1).

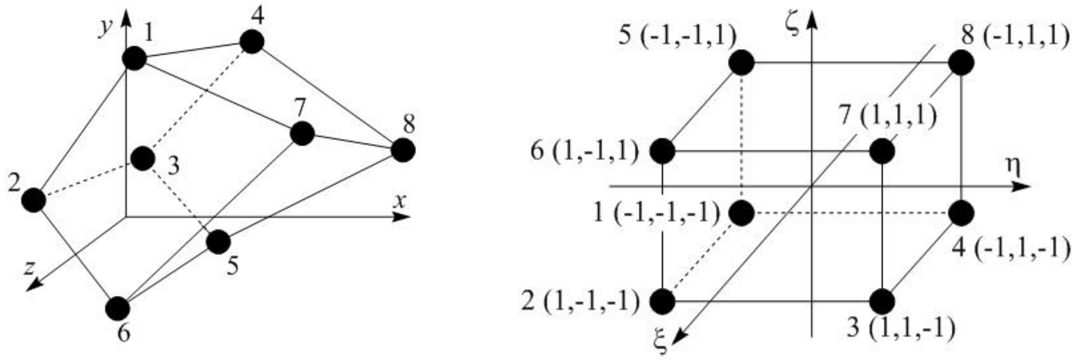
Formulation of the Structure

In this paper, the structure that is coupled with the pile group is modeled using the classical finite element method. The structure is discretized using hexahedral 8-nodes elements, with mass density ρ, and defined by coordinates (x_i,y_i,z_i) in the physical domain and $(\xi_i,\eta i,\zeta_i)$ in the natural domain. In this formulation, each node has 3 degrees of freedom and stiffness and mass matrices defined by

$$k_e = \int_{V_e} B^T DBdV = \int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} B^T DB \det(J) d\xi d\eta d\zeta \tag{3}$$

$$m_e = \int_{V_e} \rho N^T N dV = \int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} \rho N^T N \det(J) d\xi d\eta d\zeta \tag{4}$$

where V_e is the volume of each element, D is the constitutive matrix, N and B are the shape function vector and a matrix of its derivatives, and J is the Jacobian matrix that relates the physical and natural domains.



(a) (b)

Figure 2 8-noded hexahedral finite element in the (a) physical and (b) natural domains.

The inertial stiffness matrix for a specific frequency ω is given by:

$$K_{st} = K_G - \omega^2 M_G \tag{5}$$

where K_g and M_g are the stiffness and mass global matrices, assembled using classical finite element method. The nodal solution for the structure is given by:

$$P_{st} = K_{st} u_{st} \tag{6}$$

where u_{st} is the vector of nodal displacement and P_{st} is the vector of nodal loads, with the subscript st referring to the structure. A detailed deduction for each of these terms can be found in Cook et al (2001) and most finite element method textbooks.

Pile-Structure Coupling Scheme

The coupling between the structure and pile formulations described above is obtained by establishing kinematic compatibility and equilibrium in the nodes where the mesh of the structure connects with the pile heads of the pile group. In order to ensure this coupling, one must generate a mesh where there is a node that corresponds to the location of each pile head in the pile group. An example is given by Figure 2, where nodes i,j,k of the structure correspond to the location of the heads of piles 1,2,3.

The relation between nodal displacements and forces in the piled structure may be established by combining Eqs. (1) and (6) under the conditions that the displacement of the pile head is equal to the displacement of the corresponding node of the structure ($u^n_{st}=u^j_e$) and the loads at the interface are in equilibrium ($P^n_{st}=P^j_e$). These conditions yield, after some manipulation,

$$F = Ku \tag{7}$$

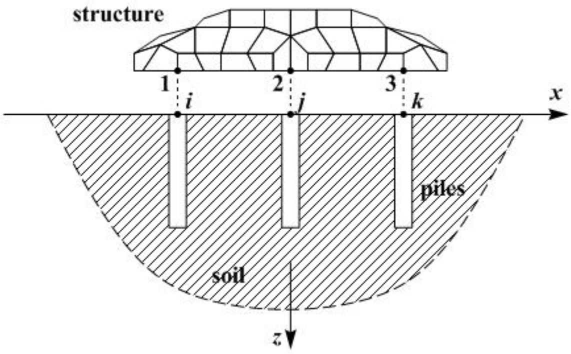


Figure 3 – Example of coupling between a pile group and structure

in which

$$F = \{f_x^1 \quad f_y^1 \quad f_z^1 \quad f_x^2 \quad f_y^2 \quad f_z^2 \quad \cdots \quad f_x^N \quad f_y^N \quad f_z^N\}^T \tag{8}$$

$$u = \{u_x^1 \quad u_y^1 \quad u_z^1 \quad u_x^2 \quad u_y^2 \quad u_z^2 \quad \cdots \quad u_x^N \quad u_y^N \quad u_z^N\}^T \tag{9}$$

where f^n_i and u^n_i are the forces and displacements of node n of the piled structure (n=1,N) in the i-direction (i=x,y,z) and

$$K = \begin{bmatrix} k_f^{1,1} & \cdots & k_f^{1,n} & \cdots & k_f^{1,m} & \cdots & k_f^{1,N} \\ \vdots & \ddots & \vdots & & \vdots & & \vdots \\ k_f^{n,1} & & k_f^{n,n} + k_p^{i,i} & & k_f^{n,m} + k_p^{i,j} & & k_f^{n,N} \\ \vdots & & \vdots & \ddots & \vdots & & \vdots \\ k_f^{m,1} & & k_f^{m,n} + k_p^{j,i} & & k_f^{m,m} + k_p^{j,j} & & k_f^{m,N} \\ \vdots & & \vdots & & \vdots & \ddots & \vdots \\ k_f^{N,1} & \cdots & k_f^{N,n} & \cdots & k_f^{N,m} & \cdots & k_f^{N,N} \end{bmatrix} \tag{10}$$

in which k are 3×3 matrices containing stiffness terms in the x-, y-, and z-directions, the sub-indices f and p refer to nodes of the structure or the pile head, and super-indices n and m indicate nodes of the structure that are connected to pile heads i and j, respectively. For example, $k^{i,j}_p$ is a 3×3 stiffness matrix relateing the displacement of the head of pile j (u^j_{px}, u^j_{py}, and u^j_{pz}) due to forces applied on the head of pile i (P^i_{px}, P^i_{py}, and P^i_{pz}). This relation is given in Eq. (1). Conversely, $k^{n,m}_f$, is a 3×3 stiffness matrix that relates the displacement of node m of the structure (u^m_{fx}, u^m_{fy}, and u^m_{fz}) due to forces applied on node n of the structure (P^n_{fx}, P^n_{fy}, and P^n_{fz}). This relation is given in Eq. (6). In this case, node n is connected to the head of pile i and node m is connected to the head of pile j.

NUMERICAL RESULTS

The present method was used to calculate the dynamic response of a representative piled structure. The results were compared with the ones obtained by Kaynia and Kausel (1991) for the vertical response of a pile group connected through a rigid surface plate. In order for this comparison to be valid, the surface structure was modeled as a square plate with high elasticity modulus, simulating a rigid plate. The pile group had the following characteristics: E_i, ρ_i, ν_i and β_i (i=s,p) that represent the elasticity modulus, mass density, Poisson ratio and the damping ratio of the soil (i=s) and the piles (i=p). The piles have length L and diameter d and distance s between adjacent piles. The pile group was defined as a 2x2 group with the following characteristics: L/d=37.5, $\pi GL^2/E_pA=1$, and s/d=5 (Fig. 4). The soil medium is a homogeneous layer of depth H=75d on top of a rigid base. The results are presented in terms of the nondimensional frequency $a_0=\omega d/c_s$, where $c^2_s=\mu_s/\rho_s$ is the largest shear wave velocity in the soil media, and the complex-valued vertical dynamic impedance of the pile group, $K_{ZZ}=k_{ZZ}+ia_0c_{ZZ}$, where k_{ZZ} and c_{ZZ} are usually referred to as stiffness and damping of the pile foundation.

Figure (5) shows a comparison of the present results with those of Kaynia and Kausel (1991) for a range of frequencies. The results show a good agreement between the two solutions.

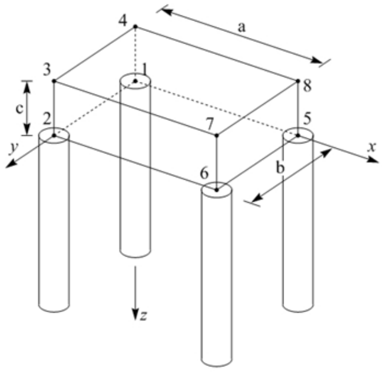


Figure 4 – Elastic plate supported by a 2×2 grid of piles

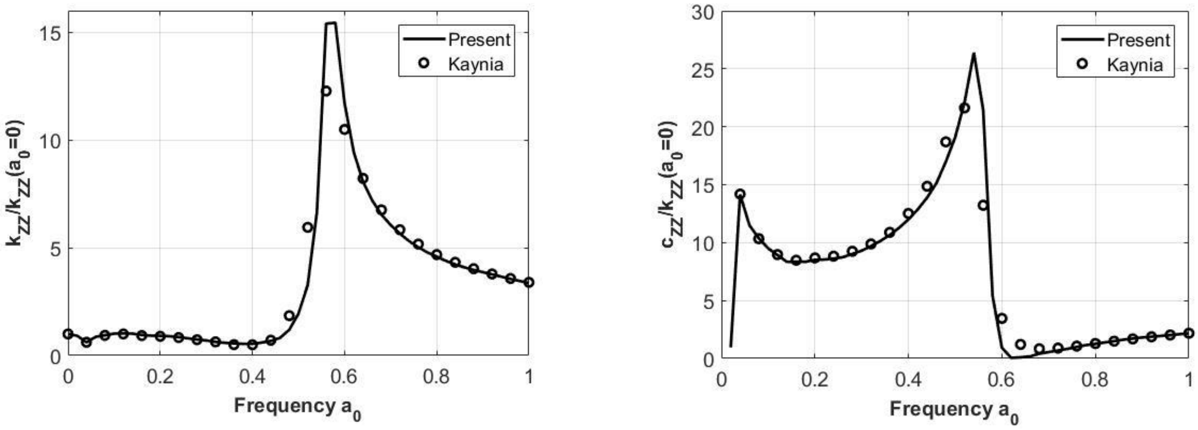


Figure 5 – Vertical stiffness and damping of a 2x2 pile group.

The present scheme allows the case of an elastic surface plate to be modeled, other than the rigid plate considered in Fig. 5. The following results, Fig. 6, were obtained from the variation of the elasticity modulus of the structure, E_f. The structure used in this analysis is modeled by only one hexahedral 8-nodes element, coupled with a group of four piles, with dimensions a=b=c=5d (Fig. 4).

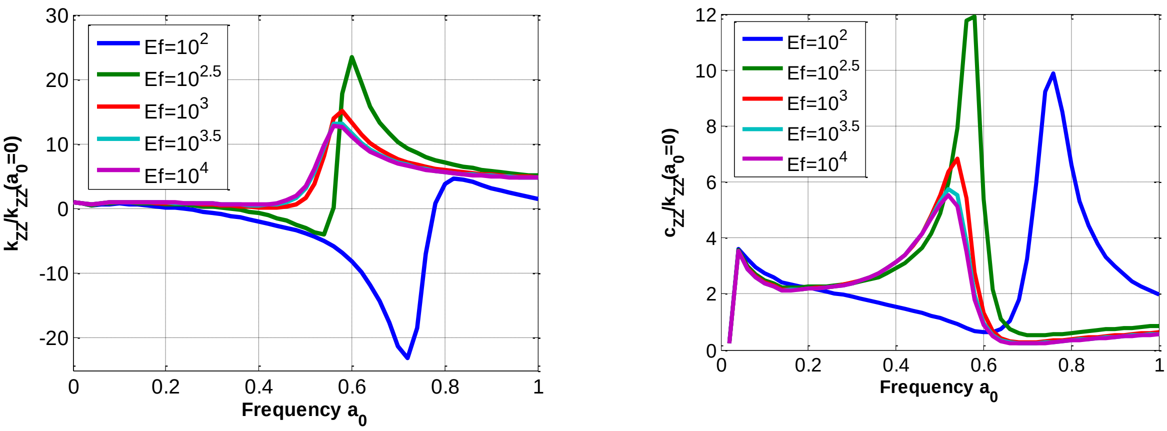


Figure 6 – Vertical stiffness and damping of a 2x2 pile group, with different elasticity modulus

As expected, the results show that the vibration of the elastic piled structure differs significantly from that of its rigid counterpart. The flexibility of the surface structure has a significant influence in the vibration of the system. The results tend non-monotonically to that of the perfectly rigid plate as the stiffness of the plate increases. A significant variation in the amplitude of the response is observed for all values of E_f. A variation in the frequency in which the largest vibration amplitudes occur is also observed.

CONCLUDING REMARKS

In this paper, we presented a numerical model of arbitrarily-shaped piled structures. The surface structure is modeled with classical finite elements, and the embedded pile group is modeled according to the thin-layer model for pile groups. The coupling was obtained by enforcing kinematic compatibility and equilibrium at discrete node–pile head interfaces, which results in the direct superposition of stiffness matrices of the structure and pile group. The results showed good agreement with the literature for the rigid plate. The influence of the flexibility of the structure in the vibration of the piled system was studied for a representative case.

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