MECSOL 2019 - Proceedings of the 7th International Symposium on Solid Mechanics,
V. Tita, A. Ferreira, M.L. Ribeiro, A. Vieira and A.T. Beck (Editors), ABCM, Sao Carlos, SP, Brazil, April 15th to 17th, 2019

Instability caused by a follower force on an elastic rocket structure mathematical model

Victor Casulli de Oliveira [1], Reyolando Manoel L. R. da Fonseca Brasil [1]

[1] Federal University of ABC, UFABC, Santo Andre, Brazil

Abstract: We present a two degree of freedom model of an elastic rocket structure. To represent the motor thrust, we use a follower (circulatory) nonconservative force. It is supposed that this force is always in the direction of the tangent to the deformed shape of the device at its lower tip. The model is described by two massless rigid pinned bars connected by rotational springs, and lumped masses and dampers are considered at the connections. The starting position is the vertical position and the generalized coordinates are the angular displacements of the bars. We derive the equations of motion via Lagrange's equations and simulate its time evolution using Runge-Kutta 4th order time step-by-step numerical integration algorithm. Results indicate possible occurrence of stable and unstable vibrations, such as limit cycles and flutter.

Keywords: elastic structure, rocket, follower force.

INTRODUCTION

Rockets are used to transport people and equipment into space. This provides research conditions that would not be possible on Earth due to such effects as atmospheric interference, influence of gravitational fields, electromagnetic radiation, cosmic rays and magnetic fields of unknown distribution (Hennemann, 2016). As any other structure, rockets are not absolutely rigid, so, when they are excited by external loads, the structural deformations tend to affect the flight dynamics. In this work, a mathematical model was developed of an elastic space rocket structure as a Beck column excited by a follower (or circulatory) force. This force represents the rocket motor thrust that should be always in the direction of the tangent to the structure deformed axis at the base of the vehicle. We considered for this problem a simplified two degree of freedom rigid bars discrete model. The two second order nonlinear ordinary differential equations of motion are derived via Lagrange's energy method, for a general understanding of the main characteristics of the problem. The obtained equations consider up to third order (cubic) inertia, stiffness and forcing terms. Among other rich nonlinear dynamic behavior of this model, depending on parameters and initial conditions choices, either stable or unstable limit cycle solutions are possible. The unstable solution is, of course, an example of flutter instability.

Beck's column is a widely used method for problems of dynamical instability of a viscoelastic column subjected to the action of a circulatory charge. In (Mazzilli, 1988) it is possible to find an example for this type of problem. Many studies by (Herrmann and Jong, 1965) show that the critical load for a damped system is about 70% of that for the non-damping system. Other studies were carried out using this method such as (Thompson, 1982) and (Hanaoka and Washizu, 1980), proving to be a reasonable model for our problem.

There are many studies in the technical literature on elastic systems loaded by follower forces associated with beams and shells. However, literature of follower forces associated with aerospace structures is still rare, as presented in (Chae, 2004). The mechanics of linear flight of remote rotating projectiles began their studies in the 20th century, after World War II (Foweler, Gallop, Lock and Richmond, 1920). Subsequently, nonlinear flight mechanics was extensively addressed in (Murphy, 1963).

Platus derived a nonlinear motion equation for spinning thin missiles using a Lagrangian approach (Platus, 1992). He showed that the viscous structural damping has a destabilizing effect on the stability of such systems above the critical frequency. Using an inverted pendulum model, it is possible to show that the critical flutter load of the undamped system was considerably higher than that of the corresponding damped system (Ziegler, 1952). Structural instability of a damped system depends on the initial conditions of the problem and the magnitude of the force.

Modeling hypothesis and structural physical model

Figure 1 is the simplified physical model of the structure of a launcher vehicle. It is constructed of two rigid massless bars $\bar{\bar{A}}B$ and $\bar{\bar{B}}C$, $L1$ and $L2$ long, respectively, pinned to nodes A and B. Displacements are restricted at point A. We consider lumped masses $M1$, $M2$ and $M3$ attached to nodes A and B where torsional springs $k1$ e $k2$ provide elastic restoring forces. Viscous dampers with damping constants $\mu 1$ and $\mu 2$ are added to the joints of the system.

The following simplifications are made.

1. It is adopted $L1 = L2 = L$.
2. The bars are rigid and massless.
3. Lumped masses $M1$, $M2$, and $M3$ represent the actual masses of half the bars connected to that point. If 2m is the mass of each bar, $M1 = M3 =$ m and $M2 =$ 2m.
4. The stiffness constants of the torsion springs of the system are considered to represent the stiffness of the constituent material of the bars. In particular, the relation $k1 = k2$ is adopted for this model. These springs are assumed to be linear, that is, the restoring moments they apply to the structure (elastic moments) are proportional to the angular displacements comprised between the longitudinal axis of each bar and the local vertical.
5. The viscous damping constants of the shock absorbers of the system are considered to represent the dissipative tendency of energy by the structure of the vehicle given its deformation. In particular, the relation $\hat{\mu}1 = \hat{\mu}2 = \hat{\mu}$ is adopted for this model. The dampers are assumed to be linear, i.e. the dissipation moments they apply to the structure are proportional to the first derivatives of the angular displacements comprised between the longitudinal axis of each bar and the local vertical.
6. Motions are restricted to the Axy plan.
7. Initially, only self-weight forces act. This is the fundamental static equilibrium configuration of the system, representing the vehicle at rest in its launch platform.

The reference coordinate system is fixed to the nose of the rocket. In this model, the reference frame origin is A, as shown in Fig. 1. This system is non-inertial for an observer on ground but is assumed to be inertial in the reference frame of the vehicle and travels with it. For this reason, an articulated support in A and elastic link between this support and the vehicle is assumed.

Physical loading model

In C is applied a follower non-conservative force, F, in the direction of bar BC. In this model, we do not consider variation in time of the thrust force due to the combustion of the expanded gases in the vehicle's engines, so the force modulus F is constant.

The action of force F applied to C excites the system to depart from its fundamental equilibrium position. Our generalized coordinates are angular displacements θ1 and θ2 of bars $\bar{\bar{AB}}$ and $\bar{\bar{BC}}$ computed from their original vertical equilibrium positions. They are, of course, implicitly time dependent, that is, $\theta 1 = \theta 1(t)$ and $\theta 2 = \theta 2(t)$. We denote $\theta 1(t) \equiv q1(t)$ and $\theta 2(t) \equiv q2(t)$. Nonzero values represent the vehicle in flight conditions as represented in Figure 1.

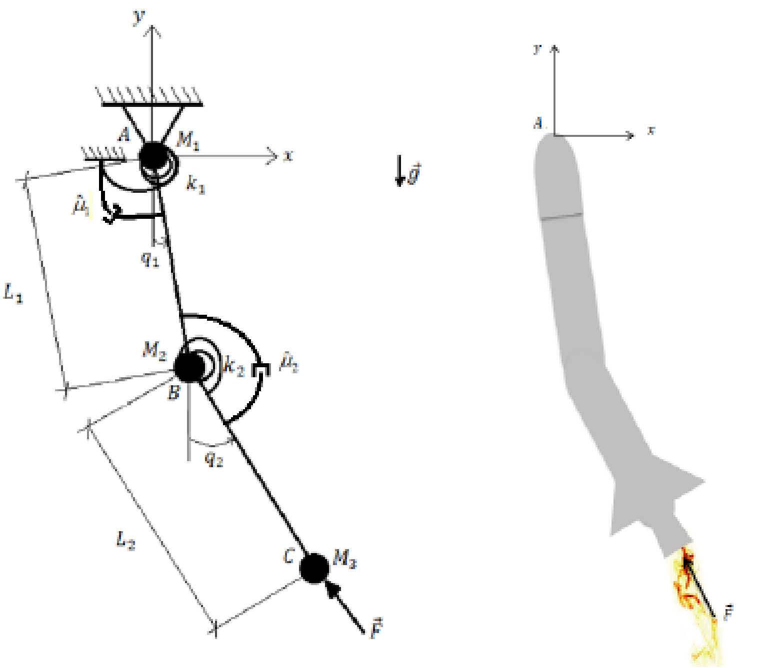

Figure 1- (left) System motion under follower force F; (right) rocket appearance under follower force F.

Mathematical model

Generalized non-conservative forces

The generalized non-conservative forces, due to the follower force F are:

$$f_1^{nc} = f_2^{nc} = 0, \qquad f_3^{nc} = F(-sin(q_2)\hat{\imath} + cos(q_2)\hat{\jmath}) \tag{1}$$

$$F_1^{nc} \cong -FL\left(-q_1 + q_2 + \frac{{q_1}^3}{6} - \frac{{q_2}^3}{6} + \frac{q_1{q_2}^2}{2} - \frac{q_2{q_1}^2}{2}\right), \qquad F_2^{nc} = 0 \tag{2}$$

where F is the scalar value of the follower force, considered time independent.

Kinematics

Position vectors of the lumped masses

$$r_1 = 0 \tag{3}$$

$$r_2 = L(sin(q_1)\hat{\imath} - cos(q_1)\hat{\jmath}) \tag{4}$$

$$r_3 = L[(sin(q_1) + sin(q_2))\hat{\imath} - (cos(q_1) + cos(q_2))\hat{\jmath}] \tag{5}$$

Velocity vectors of the lumped masses

$$\dot{r}_1 = 0 \tag{6}$$

$$\dot{r}_2 = L(\dot{q}_1 cos(q_1)\hat{\imath} + \dot{q}_1 sin(q_1)\hat{\jmath}) \tag{7}$$

$$\dot{r}_3 = L[(\dot{q}_1 cos(q_1) + \dot{q}_2 cos(q_2))\hat{\imath} + (\dot{q}_1 sin(q_1) + \dot{q}_2 sin(q_2))\hat{\jmath}] \tag{8}$$

Approximations

We adopt a third order truncated polynomial McLaurin approximations to the sinusoidal functions

$$cos(q_i) \cong 1 - \frac{{q_i}^2}{2}, \qquad sin(q_i) \cong q_i - \frac{{q_i}^3}{6} \tag{9}$$

Energy computation

In the following equations, we neglect terms of higher order than third.

Kinetic energy

$$T \cong \frac{1}{2}mL^2\left[3{\dot{q}_1}^2 + {\dot{q}_2}^2 + 2\left(\dot{q}_1\dot{q}_2 - \dot{q}_1\dot{q}_2\frac{{q_1}^2}{2} - \dot{q}_1\dot{q}_2\frac{{q_2}^2}{2} + \dot{q}_1\dot{q}_2 q_1 q_2\right)\right] \tag{10}$$

Total potential energy

$$V = U - W = \frac{1}{2}k(2{q_1}^2 + {q_2}^2 - 2q_1q_2) - \frac{1}{2}mgL(3{q_1}^2 + {q_2}^2) \tag{11}$$

where U is the strain energy of the springs and W is the work of the conservative forces (masses weight).

Derivation of the equations of motion

Next, we apply Euler-Lagrange equations to the Lagrangian functional $\mathcal{L} = T - V$:

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_i}\right) - \frac{\partial \mathcal{L}}{\partial q_i} = F_i^{NC} = \sum_{j=1}^{N}\left(f_j^{NC} \times \frac{\partial}{\partial q_i} r_j\right) \tag{12}$$

where the non-conservative generalized forces are in the left-hand term. The detailed mathematical deductions of this study can be found in (Mazzilli, 1988), (Brasil, 1996) and (Brejão and Brasil, 2016).

Equations of motion

Although gravity has been considered in the process of mathematical modeling of the problem, present in the working terms of the conservative forces acting on the vehicle (weight forces), from now on, the gravitational effect on the system dynamics is ignored, as we are only interested on the effects of the propulsion force. In fact, as we know, the acceleration of Earth's gravity decays with the square of the distance between the center of mass of the body under study (in this case the vehicle) and the center of mass of the Earth. With this, the equation of motion is given by:

$$\begin{aligned} &\left[\begin{bmatrix} 3 & 1 \\ 1 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -\frac{1}{2}(q_1 - q_2)^2 \\ -\frac{1}{2}(q_1 - q_2)^2 & 0 \end{bmatrix}\right] \begin{Bmatrix} \ddot{q}_1(t) \\ \ddot{q}_2(t) \end{Bmatrix} \\ &\quad + \left[\begin{bmatrix} 0 & \dot{q}_2(q_1 - q_2) \\ -\dot{q}_1(q_1 - q_2) & 0 \end{bmatrix} + \mu \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}\right] \begin{Bmatrix} \dot{q}_1(t) \\ \dot{q}_2(t) \end{Bmatrix} \\ &\quad + \left[c \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix} - \frac{g}{L}\begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}\right] \begin{Bmatrix} q_1(t) \\ q_2(t) \end{Bmatrix} \\ &\quad = -\lambda c \begin{Bmatrix} -q_1 + q_2 + \frac{q_1^{\,3}}{6} - \frac{q_2^{\,3}}{6} + \frac{q_1 q_2^{\,2}}{2} - \frac{q_2 q_1^{\,2}}{2} \\ 0 \end{Bmatrix} \end{aligned} \tag{13}$$

where

$$c = \frac{k}{mL^2}, \quad \lambda = \frac{FL}{k}, \quad \mu = \frac{\hat{\mu}}{mL^2} \tag{14}$$

Results

We performed computational simulations of the parametric model of Eq. (13), for a case in which the launch vehicle model is stable and a case of limit cycle. The initial conditions of the system are always given by $\theta 1(t)$ = q1(t) = 0.01 rad, $\theta 2(t)$ = q2(t) = 0.01, $\theta 1(t)$ = q1(t) = 0 rad and $\theta 2(t)$ = q2(t) = 0 rad. These conditions represent the axis of symmetry of the rocket slightly misaligned with the vertical reference direction.

A case of asymptotic stability

As an example of a case of stability of the system modeled under load action at its base, the arrangement m = 1 kg, L = 1 m, k = 1 Nm/rad, $\hat{\mu}$ = 0.1 kg/s kg/s and P = 1 N is considered. The results in this configuration are shown in Figure 2.

A case of post-critical stationary regime type limit cycle

As an example of a case of limit cycle of the system modeled under load action at its base, the arrangement m =1 kg, L=1 m, k=1 Nm/rad, $\hat{\mu}$=2 kg/s and P = 4 N is considered. The results in this configuration are shown in Figure 3.

CONCLUSIONS AND FUTURE RESEARCH

We presented a two degree of freedom lumped parameters model of an elastic rocket structure excited by the follower force given by the motor thrust that is supposed to be always in the direction of the tangent to the deformed shape of the device at its lower tip. We derived the equations of motion via Lagrange's equations and simulate its time evolution using Runge-Kutta 4th order time step-by-step numerical integration algorithm. Results indicate possible occurrence of stable and unstable vibrations, such as limit cycles. In the future, we will implement non-linear control methods for that structure. One of the methods that will be explored is SDRE and feedback linearization.

ACKNOWLEDGMENTS

We acknowledge support by FAPESP, CAPES and CNPq, all Brazilian research funding agencies.

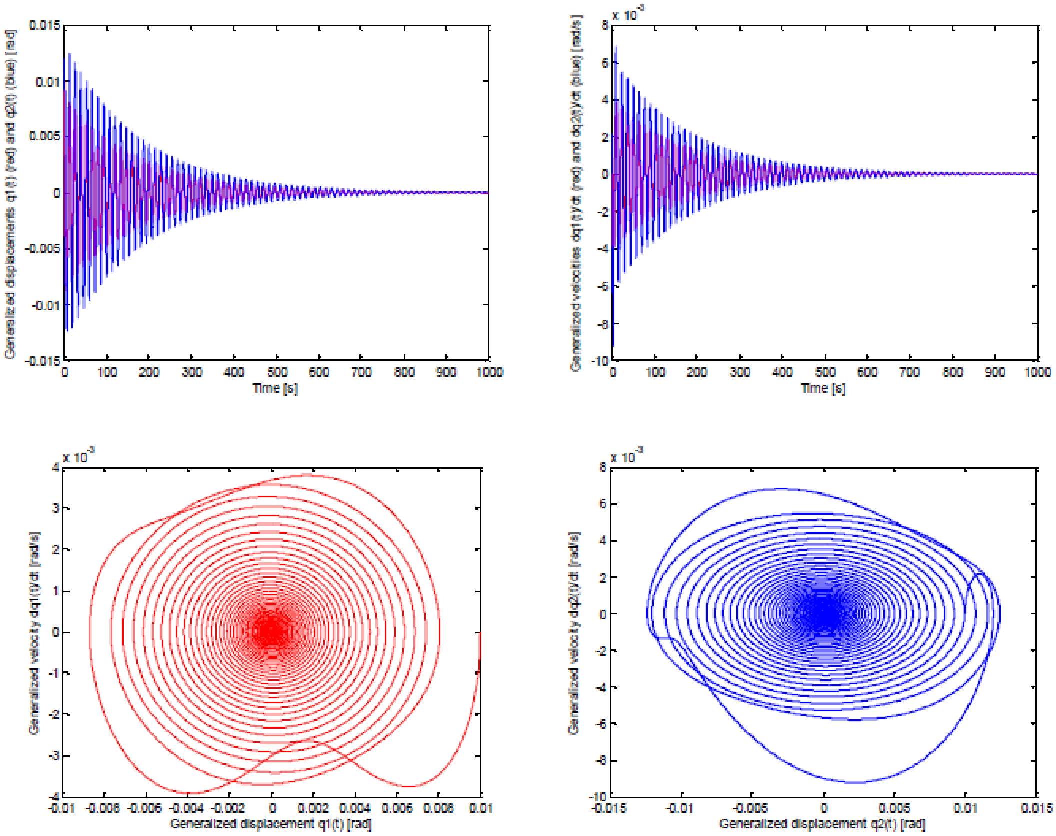

Figure 2: (above and left) Generalized displacements in stable condition; (above and right) rates of generalized displacements in stable condition; (below and left) phase plane of generalized displacement $q1$ in stable condition; (below and right) phase plane of generalized displacement $q2$ in stable condition.

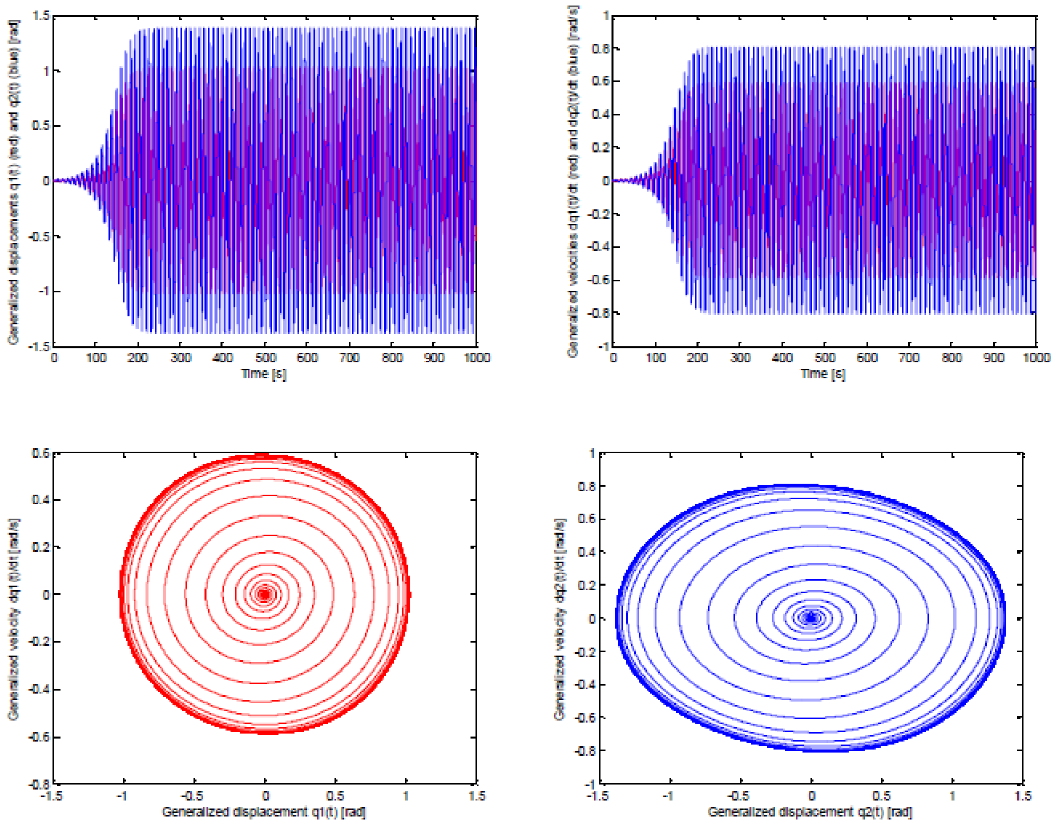

Figure 3: (above and left) generalized displacements in limit cycle case; (above and right) rates of generalized displacements in limit cycle case; (below and left) phase plane of generalized displacement $q1$ in limit cycle case; (below and right) phase plane of generalized displacement $q2$ in limit cycle case.

REFERENCES

Hennemann, L. Importância da pesquisa e desenvolvimento de foguetes e propulsão. Jornal of Aeronautical Sciences, v. 7 number 1, 2016.

Chae, S. Effect of Follower Forces on Aeroelastic Stability of Flexible, School of Aerospace Engineering, Georgia Institute of Technology, July 2004.

Foweler, R. H.; Gallop, J. L.; Lock, C. N.; and Richmond, H. W. Aerodynamics of a Spinning shell. Philosophical Transactions of Royal Society of London, Vol. 221, 1920, pp. 295 – 387.

Murphy, C. H. Free Flight Motion of Missiles of Symmetric Missiles. Tech. Rep. BRL-TR-1216, U.S. Army Ballistic Research Laboratories, Aberdeen Proving Ground, MD, 1963.

Platus, D. H. Aeroelastic Stability of Slender, Spinning Missiles. Journal of Guidance, Control, and Dynamics, Vol. 15, No. 1, Jan.-Fev. 1992, pp. 144 – 151.

Ziegler, H. Die Stabilititskriterien der Elastomechanik. Ingenieur-Archiu 20,49-56 (1952).

C. E. N. Mazzilli, Dinâmica não linear e estabilidade: uma formulação para sistemas submetidos a excitação dedsuporte ou carregamentos não conservativos. Tese de Livre Docência, Escola Politécnica da USP, São Paulo (1988).

R. M. L. R. F. Brasil, O Fenômeno de Localização de Modos em Dinâmica e Estabilidade de Estruturas Modulares de Comportamento Linear ou Não-Linear, Tese de Livre Docência, Escola Politécnica da USP, São Paulo (1996).

L.F. Brejão; R.M.L.R.F. Brasil, Modeling a Rocket Elastic Structure as a Beck's Column Under Follower Force. In: Proceedings of CILAMCE 2016. Brasília (2016).

G. Herrmann; I. Jong, On the Destabilizing Effect of Damping in Nonconservative Elastic Systems. Journal of Applied Mechanics, 1965.

J. M. T. Thompson, Instabilities and Catastrophes in Science Engineering. John Wiley. Chichester. 1982.

M. Hanaoka; K. Washizu. Optimum Design of Beck's column. Computers and Structurs. Volume 11. Issue 6. Pages 473-480. June 1980.

RESPONSIBILITY NOTICE