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Optimal Probabilistic Design of Redundant Structural Systems for Progressive Collapse

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Abstract: Structural design is nowadays still made on a member-by-member basis. But the reliability of redundant or hyperstatic systems is known to be larger than that of isostatic or non-redundant systems. This paper makes an objective investigation on the optimal design of redundant systems. The study is based on different proportioning for the active and passive parts of structural redundancy, and brittle and ductile materials, strength correlation, progressive collapse and impact factors in load redistribution. The differentiation of consequences of failures, which occur in a direct or progressive manner, is a significant aspect of the study.

Keywords: progressive collapse, redundant systems, RBDO, risk optimization

INTRODUCTION

Until today, basic structural design is made on a member-by-member basis. This includes design of redundant hyperstatic structures, which are known to be more reliable than non-redundant or isostatic structures. Reliability-based calibration of partial factors in use in modern design codes is also made at member level. Target reliabilities used in calibration work are member-based. Somehow, this is a consequence of the fact that the profession never accomplished development of partial factors that could be related to system reliability.

This paper presents a fundamental investigation of the progressive collapse of redundant structural systems (Figure 1). The study provides a framework for the semi-probabilistic design of such systems, which is based on differentiating partial factors for active and passive redundancy. These factors are calibrated accounting for progressive collapse and for the consequences of failure. Another fundamental point is to differentiate between consequences of collapses of redundant hyperstatic systems, which occur in a progressive manner (with warning), and collapses of isostatic or non-redundant systems, which occur directly.

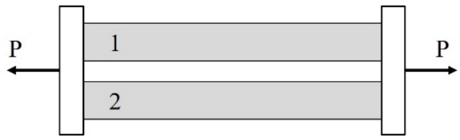

Figure 1: Two bar redundant structural system.

Material model

Progressive failure, and reserve strength of "failed" members depends on material behavior. In this development, a simple fragile-ductile material model is considered, as shown in Figure 2. The rupture or yielding stress is S, and the post-failure strength is given by ηS. For $\eta = 0$ we have elastic-fragile material; for $\eta = 1$ the material is elastic-perfectly plastic (ductile). Intermediate behavior is obtained for $0 < \eta < 1$.

In order to make development as analytical and linear as possible, material strength (and loading, for this matter) is modelled as Gaussian random variables. For two-bar parallel systems, rupture or yielding stress is given by:

$$S_1 \sim N(\mu_1, \sigma_1),$$
$$S_2 \sim N(\mu_2, \sigma_2), \quad (1)$$

where the subindex $()_i$ indicates bar number. Moreover, correlation between material strengths is given by $\rho_{12} \in [0,1)$. Uncertainty in material strength is specified by means of the coefficient of variation $\delta = \sigma/\mu$.

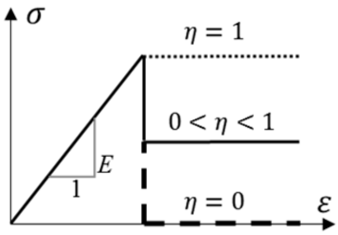

Figure 2: Fragile-ductile material model.

Load model

In order to investigate progressive failure of a two-bar parallel system, a minimum of two loads applications need to be considered. In this paper, load is modelled as an independent sequence of two load pulses, with identical intensity modelled as a Gaussian random variable:

$$P_1 \sim N(\mu_P, \sigma_P);\ P_2 \sim N(\mu_P, \sigma_P). \tag{2}$$

Each load pulse represents the maximum load that would be sustained during two periods of operation; the divider between these periods would be an eventual primary failure. After primary member failure, an impact factor $f_i \geq 0$ is considered in load re-distribution. This impact factor is particularly relevant in primary failure of brittle members.

Optimal design considering uncertainties

Let **X** and **d** be vectors of structural system parameters. Vector **X** contains random variables, such as member dimensions and geometry, strength of materials, loads and model error variables. Vector **d** contains design variables whose values are to be determined, in order to maximize performance of the system, or in order to minimize weight, cost, etc. Typical variables in vector **d** are nominal member dimensions, partial safety factors, reinforcement ratio, design life, parameters of inspection and maintenance programs, etc.

To keep the development as simple as possible, in this paper vector **X** contains only the rupture or yielding stress and the loads, such that: $\mathbf{X} = \{S_1, S_2, P_1, P_2\}$. Vector **d** contains the partial safety factors to be used in design of the redundant systems: $\mathbf{d} = \{\lambda_A, \lambda_P\}$, which are introduced shortly.

The existence of uncertainty implies in the possibility of structural failure. The boundary between safe and failure domains is given by limit state functions $g_i(\mathbf{d}, \mathbf{X}) = 0$, written for each failure mode, and/or for each member of the structure, such that:

$$\Omega_{fi}(\mathbf{d}) = \{\mathbf{x} | g_i(\mathbf{d}, \mathbf{X}) \leq 0\},$$

$$\Omega_{si}(\mathbf{d}) = \{\mathbf{x} | g_i(\mathbf{d}, \mathbf{X}) > 0\}, \qquad i = 1, \dots, n_{LS}, \tag{3}$$

where $\Omega_{fi}(\mathbf{d})$ is the failure domain, $\Omega_{si}(\mathbf{d})$ is the survival domain, and n_{LS} is the number of limit state functions. Limit states describe primary and conditional member failures, as well as simultaneous system failure. System failure resulting from progressive collapse is characterized by specific combinations of primary and conditional failures, as shown in the sequence. Probabilities of primary and conditional member failures are given generically by:

$$p_{fi} = P\left[\mathbf{X} \in \Omega_{fi}\right] = \int_{\Omega_{fi}} f_{\mathbf{X}}(\mathbf{x})\, d\mathbf{x}], \tag{4}$$

where $P[\ ]$ is the probability operator and $f_{\mathbf{X}}(\mathbf{x})$ is the joint density function of the random variable vector. In this manuscript, limit state functions are linear, and random variables have Gaussian distribution, hence Eq. (4) has analytical closed form solution (Ang & Tang, 2003; Melchers & Beck, 2018):

$$p_{fi} = \Phi[-\beta_i], \text{ with } \beta_i = \frac{E[g_i(\mathbf{d}, \mathbf{X})]}{\sqrt{Var[g_i(\mathbf{d}, \mathbf{X})]}}, \tag{5}$$

where β_i is the reliability index, $E[\ ]$ is the expected value operator, and $Var[\ ]$ is the variance operator.

DESIGN AND RELIABILITY OF SYSTEMS WITH ACTIVE-PASSIVE REDUNDANCY

Load sharing

Consider a two-bar hyperstatic parallel system. In this type of (conventional) redundant system, the two bars share the workload, until one of them fails. If there is passive or standby redundancy, the remaining bar will absorb the total load. If the displacement u of both bars is the same, then the fraction of load P carried by the i^{th} bar is:

$$P_i = k_i u = P \frac{k_i}{(k_1+k_2)}, \quad (6)$$

where k_i is the stiffness of the i^{th} bar, given by: $k_i = E_i a_i / L_i$, with E_i the elasticity modulus and L_i the length of each bar. If elasticity modulus and length are the same, the load fraction reduces to:

$$P_i = P \frac{a_i}{(a_1+a_2)}. \quad (7)$$

Usual design

Usual design of redundant hyperstatic systems is made on a member-by-member basis, without differentiating for number of parallel elements, or between the type of redundancy (active or passive). Moreover, load sharing is decided beforehand, i.e., on a basis of analyzing influence areas for loads. For the simple two=bar redundant system, for instance, one has:

$$a_1\mu_1 + a_2\mu_2 = \lambda_E \mu_P, \quad (8)$$

where λ_E is a conventional safety factor. For proportional load sharing, one has $a_1\mu_1 = a_2\mu_2$, which inserted in Eq. (8) leads to:

$$a_i(\lambda_E) = \frac{\lambda_E \mu_P}{2\mu_i}. \quad (9)$$

This design approach is not adopted herein; but results of a proposed design approach are compared to the traditional design in terms of λ_E in Eqs. (8) and (9).

Proposed alternative design procedure

One of the key points of the proposed framework is to design active-passive systems with an independent proportioning of each element. Hence, the design procedure follows that of passive standby systems, with element 1 designed for active redundancy, and element 2 designed for passive redundancy. In practice, both elements will share the load, and one does not need to guess which element will fail first. Hence, following Eqs. (8) and (9), the primary element is designed as:

$$a_1 = \frac{\lambda_A \mu_P}{\mu_1}, \quad (10)$$

where $\lambda_A > 1$ is the partial safety factor for active redundancy. The redundant element is designed as:

$$a_2 = \frac{\lambda_P \mu_P}{\mu_2}, \quad (11)$$

where $\lambda_P \geq 1$ is the partial safety factor for passive, or standby redundancy. The passive safety factor is allowed to be equal to one ($\lambda_P = 1$), as the result of system optimization may be that the standby part of redundancy is not necessary.

Reliability

Figure 3 illustrates the event three for active-passive system submitted to two load applications, with F_i denoting the event "failure of bar *i*".

Regardless of independent dimensioning (Eqs. 6 and 7), any bar can fail first. The limit state function for primary failure of the i^{th} bar (F_i) is given by:

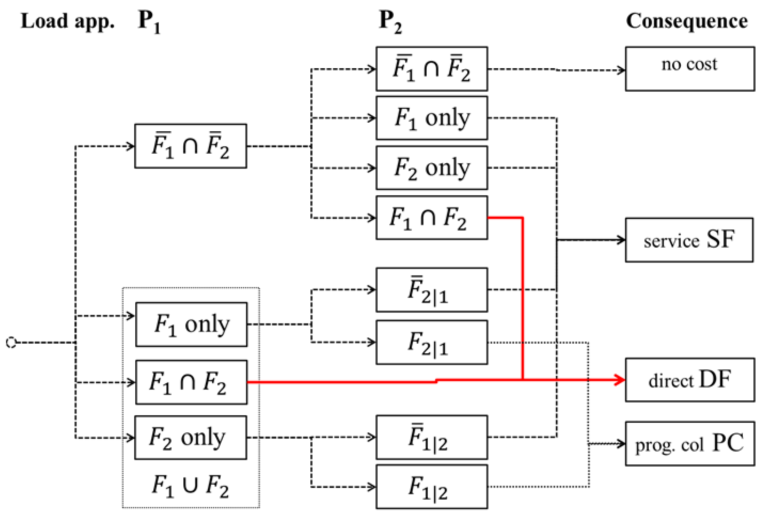

Figure 3: Event tree for active-passive two-bar system submitted to two load applications.

$$g_i(\mathbf{d}, \mathbf{X}) = a_i S_i - P_i = a_i S_i - P \frac{a_i}{(a_1+a_2)} = (a_1 + a_2) S_i - P = 0. \quad (12)$$

The reliability index for primary member failure is:

$$\beta_i(\mathbf{d}) = \frac{(a_1+a_2)\mu_1 - \mu_P}{\sqrt{(a_1+a_2)^2\sigma_1^2 - \sigma_P^2}}\,. \quad (13)$$

The limit state function for failure of the second element, given failure of the primary element ($F_{2|1}$), is given by:

$$g_{2|1}(\lambda_P, \mathbf{X}) = a_2 S_2 + \eta a_1 S_1 - f_i P. \quad (14)$$

The reliability index for conditional failure is given by:

$$\beta_{2|1}(\lambda_P, f_i) = \frac{a_2\mu_2 + \eta a_1 \mu_1 - f_i \mu_P}{\sqrt{a_2^2\sigma_2^2 + \eta^2 a_1^2 \sigma_1^2 + \eta a_1 a_2 \sigma_1 \sigma_2 \rho_{12} - f_i^2 \sigma_P^2}}\,. \quad (15)$$

The limit state function for simultaneous failure of both elements, in the first load application ($F_{1\cap 2}$), is given by:

$$g_{1\cap 2}(\lambda_A, \lambda_P, \mathbf{X}) = a_1 S_1 + a_2 S_2 - P. \quad (16)$$

The reliability index for the joint failure event is:

$$\beta_{1\cap 2}(\lambda_A, \lambda_P) = \frac{a_2\mu_2 + a_1\mu_1 - f_i\mu_P}{\sqrt{a_1^2\sigma_1^2 + a_2^2\sigma_2^2 + a_1 a_2 \sigma_1 \sigma_2 \rho_{12} - f_i^2 \sigma_P^2}}\,. \quad (17)$$

The event in dashed lines, at the bottom left in Figure 3, corresponds to failure of bars 1 and/or 2: $F_1 \cup F_2$. The complementary event to $F_1 \cup F_2$ is $\bar{F}_1 \cap \bar{F}_2$, as stated in Figure 3. The probability of the union is written as:

$$P[F_1 \cup F_2] = P[F_1] + P[F_2] - P[F_1 \cap F_2]\,. \quad (18)$$

By adding and substracting the term $P[F_1 \cap F_2]$:

$$\begin{aligned} P[F_1 \cup F_2] = P[F_1] - P[F_1 \cap F_2] \\ +P[F_2] - P[F_1 \cap F_2] \\ +P[F_1 \cap F_2], \end{aligned} \quad (19)$$

we obtain, in the first line of Eq. (26), the probability that only bar 1 fails :

$$P[F_1 \text{ only}] = P[F_1] - P[F_1 \cap F_2] = \Phi[-\beta_1] - \Phi[-\beta_{1\cap 2}]\,. \quad (20)$$

In the same way, the second line in Eq. (19) is the probability that only bar 2 fails.

There are tree mutually exclusive ways in which system failure can occur: progressive failure of bars in order 1-2, progressive failure in order 2-1, or simultaneous failure of both bars. Hence, the system failure probability is evaluated as:

$$\begin{aligned} p_{fsys}(\lambda_A, \lambda_P) = (\Phi[-\beta_1] - \Phi[-\beta_{1\cap 2}])\Phi[-\beta_{2|1}] \\ +(\Phi[-\beta_2] - \Phi[-\beta_{1\cap 2}])\Phi[-\beta_{1|2}] + \Phi[-\beta_{1\cap 2}], \end{aligned} \quad (21)$$

where the term $\beta_{1|2}$ is evaluated from Eq. (15), with a proper change of indexes.

DESIGN OPTIMIZATION

From Eqs. (16) and (21), it is clear that the reliability of the redundant system, $\mathcal{R} = 1 - p_{fsys}(\lambda_A, \lambda_P)$, is a function of the partial safety factors to be used in design: $\mathbf{d} = \{\lambda_A, \lambda_P\}$. Optimum design considering progressive failure is sought, with the objective of minimizing cost of materials:

$$f(\mathbf{d}) = a_1(\lambda_A)\mu_1 + a_2(\lambda_P, f_i)\mu_2\,, \quad (22)$$

where unit material cost per volume is simply assumed proportional to mean strength ($\mu_i, i = 1,2$). Two different approaches can be used to formulate the design problem: reliability-constrained design optimization, or optimization considering failure consequences, as follows.

Reliability-constrained design optimization problem

In reliability-constrained design optimization, also known as Reliability-Based Design Optimization (RBDO) in the literature, minimization of an objective function $f(\mathbf{d})$ is sought, with constraints expressed in terms of a target (or minimum) system reliability (Enevoldsen and Sørensen, 1993; Royset et al., 2001; Liang et al., 2007; Aoues and Chateauneuf, 2008; McDonald and Mahadevan, 2008;Nguyen et al., 2010; Song and Kang, 2009):

$$\text{Find } \mathbf{d}^* = \{\lambda_A^*, \lambda_P^*\} \text{ which minimizes } f(\mathbf{d})$$
$$\text{subject to: } \mathcal{R}(\mathbf{d}) = 1 - p_{fsys}(\mathbf{d}) \geq \mathcal{R}_T;\ \mathbf{d} \in \mathcal{D}, \quad (23)$$

where $\mathcal{R}_T$ is the target system reliability and $\mathcal{D} = \{\mathbf{d}_{\min}, \mathbf{d}_{\max}\}$ are side constraints.

Results of RBDO are clearly dependent on the target system reliability $\mathcal{R}_T$ used as design constraint. The target reliability is specified a priori by the designer, and should be based on a cost-benefit analysis. The RBDO formulation is justifiable when the failure consequences cannot be quantified, as follows.

Failure consequence-driven design optimization

Failure consequence-driven design optimization, also known in the literature as Life-Cycle Risk Optimization or simply Risk Optimization (Beck and Verzenhassi, 2008; Beck and Gomes, 2012; Beck et al., 2012, 2014, 2015) increases the scope of RBDO by including the expected consequences of failure.

Each failure event described in Figure 3 has different consequences. The failure of a primary element can be considered as a service failure; which demands corrective maintenance. Failure of the redundant element leads to structural collapse. This may result in damage to equipment, damage to third parties, injury, death, and environmental damage. The cost of failure, or a measure of the consequences of failure, is given by C_f. The expected cost of failure (C_{EF}) is obtained by multiplying failure costs (C_f) by failure probabilities (p_f):

$$C_{EF}(\mathbf{d}) = C_f p_f(\mathbf{d}), \quad (24)$$

The cost of a primary element failure, or service failure (*SF*), is assumed proportional to the cost of materials for replacing the failed element:

$$C_{fi} = k_{SF}\mu_i a_i(\mathbf{d}), \quad (25)$$

where $k_{SF} > 1$ is a multiplication factor to account for workmanship and repair downtime.

The cost of ultimate failures accounts for damage to equipment, damage to third parties, payment of compensation for injury, death, and environmental damage, and for bad reputation. These costs are assumed constant, or independent of the design vector.

One very relevant feature of failure consequence-driven design optimization is the possibility to differentiate for the consequences of collapse that occurs directly, or without warning, and collapse that occurs in a progressive manner. Failure of a primary element, which is not immediately followed by failure of the redundant element, serves as warning, allowing evacuation and reducing costs of compensation for injury and death. Such warning also allows preventive and consequence mitigation actions to be taken. Hence, consequences of stepped progressive collapse (*PC*) are measured by k_{PC}, and consequences of direct collapse (*DC*) are given by $k_{DC} > k_{PC}$.

The objective function for consequence-based optimization is obtained by adding all cost over the life-cycle of the structure. This can include costs of operation, inspection, maintenance, and cost of disposal; all of which are very application-dependent. In this paper, only cost of materials, and expected costs of failure are considered.

For failure paths and failure consequences of the active-passive redundant system, one needs to refer to the event three in Figure 3. There are two paths leading to service failure for bar 1: one with probability $(1 - \Phi(-\beta_1))(\Phi(-\beta_1) - \Phi[\beta_{1\cap 2}])$, and one with probability $(\Phi(-\beta_1) - \Phi[\beta_{1\cap 2}])(1 - \Phi[\beta_{2|1}])$; similarly for service failure of bar 2. There are two failure paths leading to progressive collapse, corresponding to failure sequences 1-2 and 2-1. For failure sequence 1-2, the probability is $(\Phi(-\beta_1) - \Phi[\beta_{1\cap 2}])\Phi[\beta_{2|1}]$. There are two failure paths leading to direct collapse, corresponding to the first and second load applications, respectivelly. The probability of direct collapse is given by $\mathrm{P}[\bar{F}_1 \cap \bar{F}_2]\Phi[\beta_{1\cap 2}] + \Phi[\beta_{1\cap 2}]$. With these preliminaries, the objective function for active-passive redundant systems is given by:

$$\begin{aligned} f(\mathbf{d}) = {} & \tfrac{(a_1(\lambda_A)\mu_1 + a_2(\lambda_P)\mu_2)}{(a_1(1)\mu_1 + a_2(1,1)\mu_2)} \\ & + k_{SF}\frac{a_1(\lambda_A)}{a_1(1)}(\Phi(-\beta_1) - \Phi[\beta_{1\cap 2}])(1 - P[F_1 \cup F_2])\left(1 - \Phi\left[\beta_{2|1}\right]\right) \\ & + k_{SF}\frac{a_2(\lambda_P)}{a_2(1)}(\Phi(-\beta_2) - \Phi[\beta_{1\cap 2}])(1 - P[F_1 \cup F_2])\left(1 - \Phi\left[\beta_{1|2}\right]\right) \\ & + k_{PC}(\Phi[-\beta_1] - \Phi[\beta_{1\cap 2}])\Phi\left[-\beta_{2|1}\right] + k_{PC}(\Phi[-\beta_2] - \Phi[\beta_{1\cap 2}])\Phi\left[-\beta_{1|2}\right] \\ & + k_{DC}\Phi[\beta_{1\cap 2}][1 + (1 - P[F_1 \cup F_2])]. \end{aligned} \quad (26)$$

The last term in this equation is evaluated as $P[F_1 \cup F_2] = \Phi[-\beta_1] + \Phi[-\beta_2] - \Phi[\beta_{1\cap 2}]$.

The failure consequence-driven design optimization problem is stated as:

$$\text{Find } \mathbf{d}^* = \{\lambda_A^*, \lambda_P^*\} \quad \text{which minimizes } f(\mathbf{d})$$
$$\text{subject to: } \mathbf{d} \in \mathcal{D}, \tag{27}$$

with $\mathcal{D} = \{\mathbf{d}_{\min}, \mathbf{d}_{\max}\}$ being side constraints.

In the above equations, failure cost multipliers k_{SF}, k_{PC} and k_{DC} can be interpreted as the actual cost of failures, or simply as parameters to balance the relative consequences of failure, w.r.t. the cost of materials for the reference structure. Typical values of these cost multipliers for conventional civil engineering structures are given in JCSS (2001). In this paper, the following values are used: $k_{SF} = 2$, $k_{PC} = 10$ and $k_{DC} = 40$. Results to be presented depend directly on these cost multipliers. However, the overall conclusions should be valid for similar relative magnitudes of cost multipliers.

NUMERICAL RESULTS

Parametric analysis

The key parameters that characterize design of redundant systems for progressive failure are the impact factor in load re-distribution (f_i), the correlation between material strengths (ρ_{12}), material behavior (fragile-ductile) and the type of redundancy (active-passive). In the following, two levels of each parameter are considered: $f_i = (1; 1.3)$, $\rho_{12} = (0; 0.9)$, $\eta = (0;\ 1)$, for active-passive redundant assemblies. Results for active redundancy are presented elsewhere. This leads to $2^3 = 8$ combinations. Further, we study different ratios between mean material strengths (μ_2/μ_1) and between strength COVs (δ_1/δ_2). Hence, in order to limit results, only the most usual combinations are discussed. For instance, for materials with different strengths, only $\rho_{12} = 0$ is considered.

In the following, we discuss redundant assemblies made with two bars of brittle or ductile materials. Mixed material assemblies are not considered, but the ductile-ductile combination also describes what would be obtained for a ductile-brittle combination, if the primary failure is in the ductile material (post-failure behavior of the second bar does not matter). In this regard, the brittle-ductile combination does not make much sense.

Results in this section are evaluated for ($\mu_P = 10, \delta_P = 0.3$) and $\delta_1 = \delta_2 = 0.1$. These are typical values for civil engineering structures. In the following, we consider two bars with different mean strengths: ($\mu_1 = 1, \mu_2 = 9$).

Results for reliability-constrained optimization (RBDO)

RBDO results for an active-passive assembly with different materials are illustrated in Figure 4. Results for fragile material show a non-linear dependency with partial factors λ_A and λ_P. For the ductile material, results are nearly linear with partial safety factors, but show greater dependency on strength correlation (ρ_{12}) and impact factor (f_i). In particular, results show large dependency on impact factor f_i, for $\lambda_A \lesssim 2$ and larger values of λ_P.

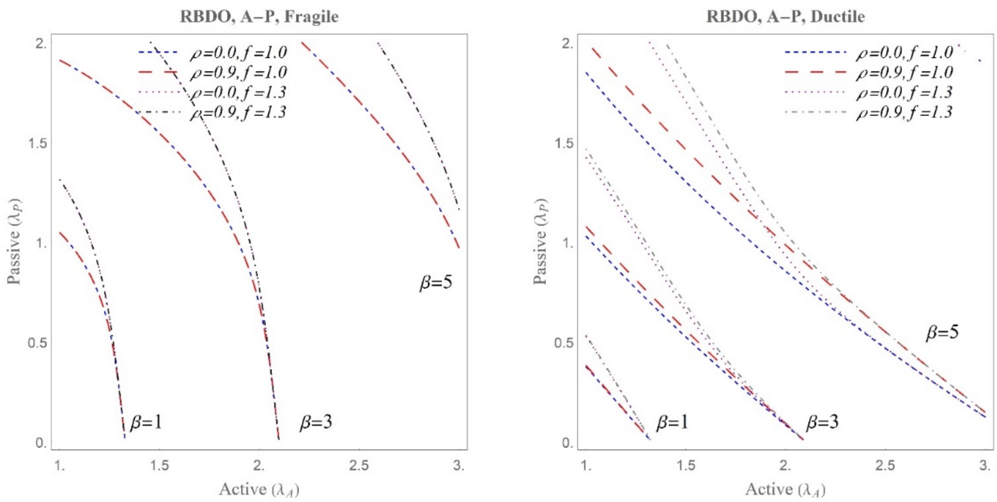

Figure 4: RBDO solutions of active-passive system with two bars of different mean material strength.

Results for consequence-driven optimization (RO)

Results for risk optimization of the active-passive system, with different material strengths, are presented in Table 1 and Figure 5. Contour plots of cost functions are shown in Figure 5. The cost functions are linear with partial factors λ_A

Table 1: Risk optimization results for active-passive system and materials of different strength ($\mu_2 = 9\mu_1$).

		Active-Passive			
Param.	**Correl.** (ρ_{12})	0.0			
	Material (η)	0		1	
	Impact (f_i)	1.0	1.3	1.0	1.3
Optimum	λ_A	2,071	2,071	2,071	2,071
	λ_P	0,000	0,000	0,000	0,000
	a_1	20,708	20,708	20,708	20,708
	a_2	0,000	0,000	0,000	0,000
Reliability	β_1	2,937	2,937	2,937	2,937
	$\beta_{2\|1}$	-3,333	-3,333	2,937	1,746
	$\beta_{1\cap 2}$	2,937	2,937	2,937	2,937
	β_{SYS}	2,937	2,937	2,937	2,937
Costs	Material	1,0354	1,0354	1,0354	1,0354
	SF	0,0000	0,0000	0,0000	0,0000
	PC	0,0000	0,0000	0,0000	0,0000
	DC	0,0678	0,0678	0,0678	0,0678
	Total	1,1032	1,1032	1,1032	1,1032

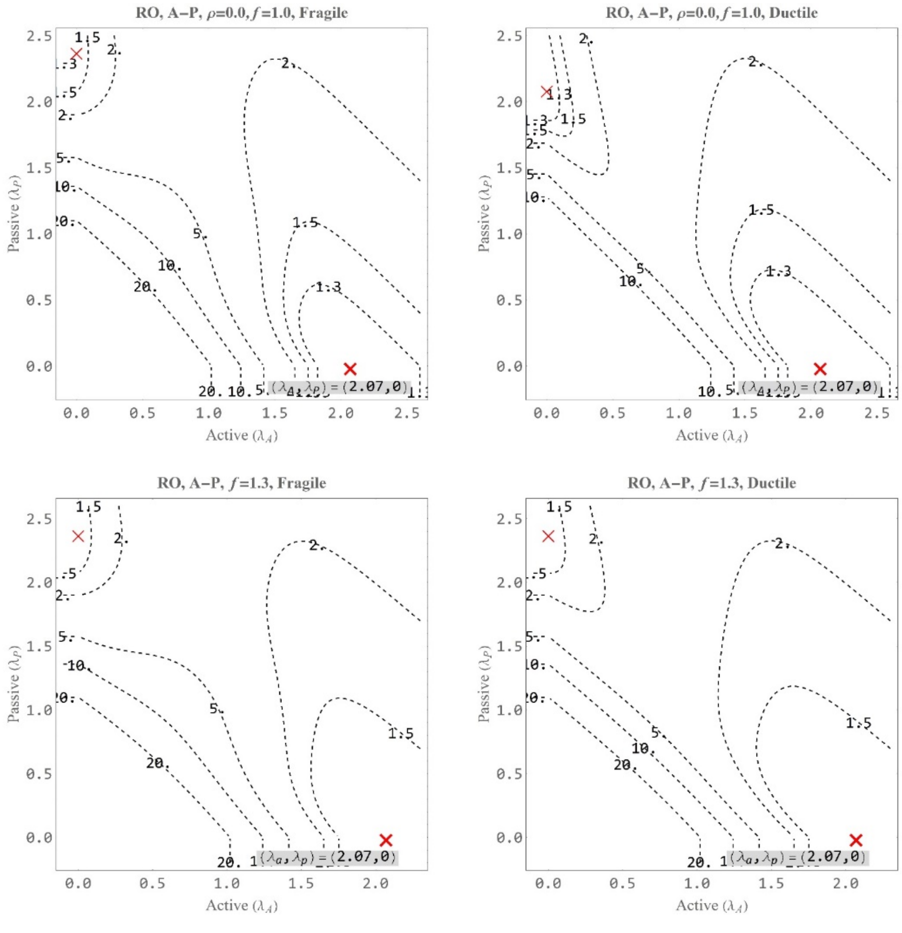

Figure 5: Level curves of cost functions and RO solutions of active-passive system with materials of different strength ($\mu_2 = 9\mu_1$).

and λ_P only for very large failure probabilities, where expected costs of failure are also large (bottom-left in Figure 5 plots). The objective function is non-convex, and there are two areas with local minima: one for large λ_A and small λ_P, one for large λ_P and small λ_A. We hence notice that the economical trade-off between alternative designs happens along the diagonal line $\lambda_P \approx 2 - \lambda_A$, where increases in λ_A are compensated by reductions in λ_P. Along this line, however, the cost functions vary in a non-linear way, leading to near-best solutions of single-bar systems, made of either material. Yet the best solution, regardless of material or impact factor, is a single bar of the cheapest material, as shown in Table 1. For this single-bar solution, joint ($\beta_{1\cap 2}$) and system reliability index (β_{SYS}) converge to the single bar failure reliability index ($\beta_1 = 2.937$). The conditional reliability index, however, is different for fragile and ductile materials. For these single bar solutions, all service and progressive failure cost terms vanish, as shown in Table 1. Figure 5 shows that cost functions are virtually insensitive to impact factor (f_i) and strength correlation (ρ_{12}).

Figure 6 shows the objective functions plotted along the diagonal line $\lambda_P \approx 2 - \lambda_A$. It is observed that the absolute minimum corresponds to $\lambda_A = 2.07$ and $\lambda_P = 0$, regardless of impact factor (f_i) and strength correlation (ρ_{12}). Figure 7 illustrates cost functions along line $\lambda_P \approx 2 - \lambda_A$, for other values of the failure cost multipliers (k_{DC} and k_{PC}). These results confirm that the optimal solutions shown in Table 1 are also valid for similar relative magnitudes of cost multipliers.

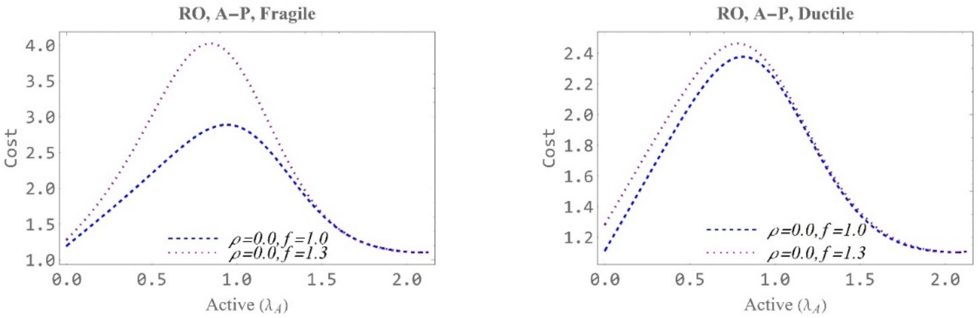

Figure 6: Influence of strength correlation and impact factor in RO solutions of active-passive systems with materials of different strength ($\mu_2 = 9\mu_1$).

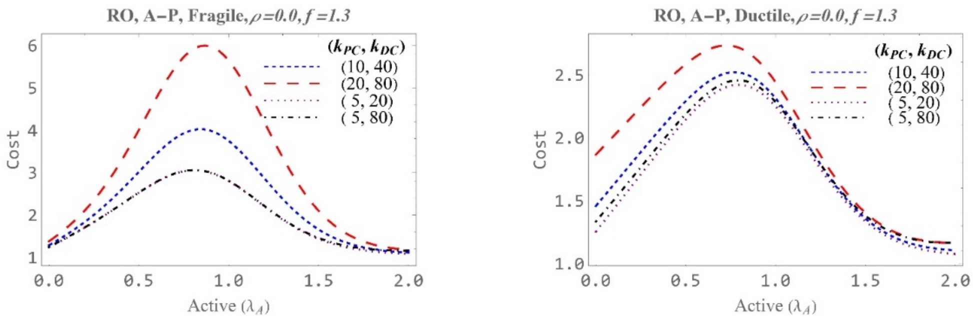

Figure 7: Influence of failure cost multipliers in RO solutions of active-passive systems with materials of different strength ($\mu_2 = 9\mu_1$).

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