

Effects of Piezoelectrically Induced Stresses Field on the Dynamic Behavior of a Plate-like Wing Aeroelastic System

Thiago de Souza Siqueira Versiani¹, Maurício Vicente Donadon¹, Flávio José Silvestre¹ and Alessandro Guimarães²

¹ Instituto Tecnológico de Aeronáutica

² Instituto de Pesquisas Tecnológicas do Estado de São Paulo

Abstract. A study about the effects of piezoelectrically induced stresses on the aeroelastic behavior of a composite plate-like wing is presented. Different positions of the piezoelectric unit were considered and, for each of them, modal and aeroelastic analyses were performed. The system was modeled considering the variational principle and approximated by finite element method employing two-node, eight-degree-of-freedom smart beam elements assuming Euler Bernoulli kinematic relations and von Kármán non-linear deformations. Results show a good effectiveness of the proposed technique on increasing natural frequencies, specially from modes that present most participation of bending movement. Moreover, a good effectiveness on increasing flutter speed was also observed, while the divergence speed was not affected.

Keywords: aeroelasticity, piezoelectric transducers, composite structures

INTRODUCTION

Flutter phenomenon is extensively studied and technological solutions for this aeroelastic issue are continually proposed for many researchers (Haghighat et al., 2012; Dixit, 2016; Silva et al., 2017). Among them, piezoelectric transducers have been shown to be a potential device in control systems for aeroelastic stability (Agneni et al., 2002; Suleman and Costa, 2004; Moulin and Karpel, 2007; Zeng et al., 2010; Wang and Inman, 2012; Bruni et al., 2014; Fonte et al., 2015).

The use of piezoelectric transducers for a general vibration suppression can be applied as passive or active control. In passive application, piezoelectric transducers can be used in shunted devices, designed to add damp in a specific frequency bandwidth (Agneni et al, 2002); and harvesting devices, in order to spend or store electrical energy provided by mechanical vibration (Bruni et al., 2014; Wang and Inman, 2012).

In active application, in turn, piezoelectric transducers can be used as actuators attached to a stability augmentation system in order to reduce vibration amplitudes (Suleman and Costa, 2004; Moulin and Karpel, 2007; Zeng et al., 2010; Fonte et al., 2015). A particular technique was studied by Donadon, Almeida and Faria (2002), which use piezoelectric transducers to control natural frequencies of a composite plate through the addition of in-plane piezoelectrically induced stresses. Numerical and experimental results showed significant effectiveness of the proposed technique.

The present work proposes to evaluate the performance of piezoelectric transducers on natural frequency augmentation through the addition of axial piezoelectrically induced stresses on a high aspect ratio composite wing. Since the natural frequencies of the elastic modes are affected, the aeroelastic response will be also affected. Flutter and divergence speed are evaluated for different conditions of activation of the piezoelectric unit and its position along the wingspan.

MODEL FORMULATION

Let's consider a composite smart flat plate clamped at one of its ends and containing a slender body (ballast) attached at its free tip, as shown in Fig. 1. At the same figure, the velocity (V_∞) of the wind acting on the smart plate-like wing is illustrated. Moreover, pairs of identical piezoelectric transducers attached in opposite surfaces can be disposed in general locations along the wingspan and their activation induces only longitudinal deformation along y axis, as illustrated at Fig 2.

The structural model is based on the variational principle and approximated by finite element method employing two-node, eight-degree-of-freedom smart beam elements assuming non-linear deformations and neglecting transverse shear effects.

The kinetic energy of the system can be given by:

$$T = \frac{1}{2} \int_{\Omega} \rho \left[(v - z\dot{w}_{,y})^2 + \dot{w}^2 + (x^2 + z^2) \dot{\alpha}^2 \right] d\Omega + \frac{1}{2} \left[m_{sb} (\dot{w}(L))^2 + I_{yy_{sb}} (\dot{\alpha}(L))^2 \right]. \quad (1)$$

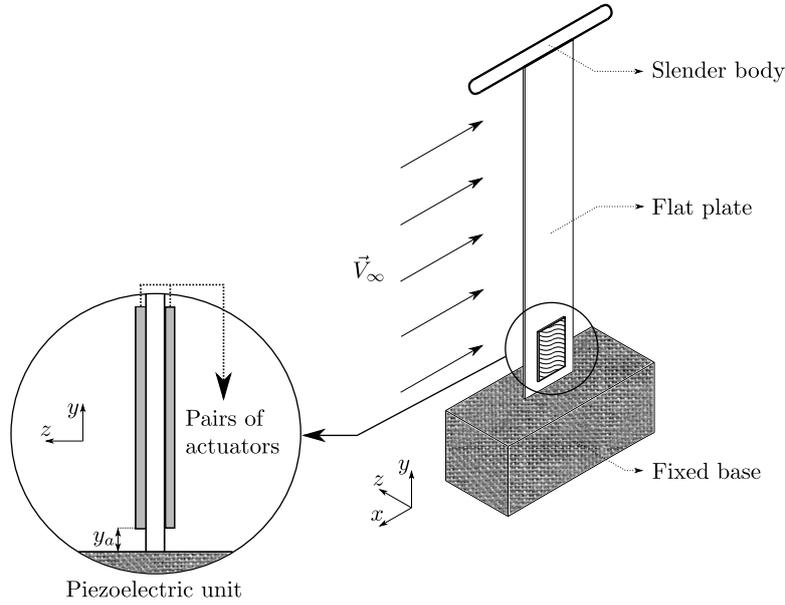


Figure 1: Plate-like wing under study.

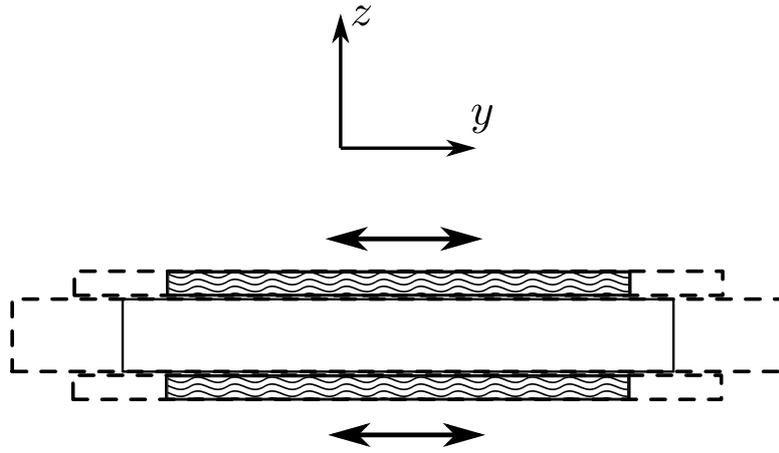


Figure 2: Local strain by activation of the piezoelectric unit.

where ρ is the respective material density, v is the displacement along y axis, w is the displacement in z axis direction, α is the rotation around y axis, $I_{yy, sb}$ is the moment of inertia around y axis, L is the length of the structure and Ω is the spatial domain integrated.

The potential energy, in turn, was calculated considering the von Kármán non-linear strain-displacement relationship. Since the piezoelectric transducers are used only as actuators and neglecting the nonlinear term of highest order, the potential energy can be given by:

$$\Pi = \frac{1}{2} \int_{\Omega} c^E (\epsilon^L)^2 d\Omega + \int_{\Omega} c^E \epsilon^L \epsilon^{NL} d\Omega - \int_{\Omega} e_{13} E \epsilon^L d\Omega - \int_{\Omega} e_{13} E \epsilon^{NL} d\Omega + \frac{1}{2} \int_0^L G J_{\alpha} \alpha_y^2 dy \quad (2)$$

where c^E is the elasticity modulus at constant electric field, e_{13} the piezoelectric constant operating in 13 mode, ξ^{σ} is the dielectric constant at constant stress, E is the electrical field, G is the shear modulus, J_{α} is the torsion constant of the cross section, ϵ^L and ϵ^{NL} are the linear and non-linear terms of the strain, respectively.

As proposed by Donadon, Almeida and Faria (2002), the portions associated by the non-linear mechanical and electrical terms were assumed as stresses per unit thickness length.

To model the aerodynamic forces and moments, the unsteady aerodynamics was assumed using the Wagner's function and Jones approximation, and then applied to the system via strip theory (Silvestre and Luckner, 2015).

Using the Modified Hamilton's Principle, the structural dynamics are given by:

$$[M] \{\ddot{q}\} + [K]_{eq} \{q\} = [A_1] \{\ddot{q}\} + [A_2] \{\dot{q}\} + [A_3] \{q\} + [A_4] \{\lambda\} \quad (3)$$

$$\{\dot{\lambda}\} = [B_1] \{\ddot{q}\} + [B_2] \{\dot{q}\} + [B_3] \{q\} + [B_4] \{\lambda\} \quad (4)$$

where $[M]$ is the mass matrix, $[K]_{eq} = [K] + N_x [K_g]$ is the equivalent stiffness matrix, $[K]$ is the elastic stiffness matrix, $[K]_g$ is the additional stiffness matrix provided by the pairs of piezoelectric actuators, N_x is the scalar term that quantify the pre-stress induced by the piezoelectric actuators when activated, $\{q\}$ is the vector of physical displacements of the structure and vector $\{\lambda\}$ refer to lag states due to the unsteady aerodynamic behavior. The matrices $[A_i]$ and $[B_i]$, with $i = 1, \dots, 4$, are obtained by the aerodynamic formulation and describe the dependence of the aerodynamic loads with respect to each degree of freedom.

The plate like wing was discretized in beam elements, the degrees of freedom of which take into account the bending and torsion movement, as illustrated in Fig. 3.

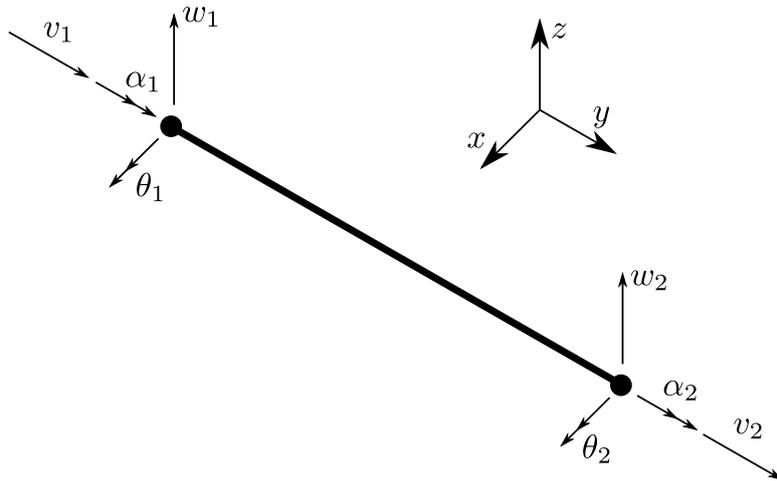


Figure 3: Two-node, eight-degree-of-freedom smart beam element.

Connecting all elements by the respective boundary conditions, grouping the terms and writing the structural dynamics in state-space realization, we get:

$$\begin{Bmatrix} \dot{q} \\ \ddot{q} \\ \dot{\lambda} \end{Bmatrix} = \begin{bmatrix} 0_{n_{GDLP} \times n_{GDLP}} & I_{n_{GDLP} \times n_{GDLP}} & 0_{n_{GDLP} \times n_{GDLA}} \\ -[M_t]^{-1} [K_t] & -[M_t]^{-1} [C_t] & [M_t]^{-1} [A_4] \\ [B_3] - [B_1] [M_t]^{-1} [K_t] & [B_2] - [B_1] [M_t]^{-1} [C_t] & [B_4] - [B_1] [M_t]^{-1} [A_4] \end{bmatrix} \begin{Bmatrix} q \\ \dot{q} \\ \lambda \end{Bmatrix} \quad (5)$$

where n_{GDLP} is the number of physical degrees of freedom and n_{GDLA} is the number of aerodynamical degrees of freedom. The matrices $[M_t]$, $[C_t]$ and $[K_t]$ are the total mass, damp and stiffness.

RESULTS

In this work, we investigated the influence of piezoelectrically induced stresses on natural frequencies and aeroelastic response of a composite plate-like wing structure. A parametric study was performed to evaluate the influence of the piezoelectric unit position along the wingspan on the first six natural frequencies. Moreover, aeroelastic analyses were performed for each position in order to investigate the increasing of the flutter and divergence speed.

The smart plate-like wing considered in this work consists in a woven-glass/epoxy composite plate as elastic structure, with stacking sequence $[(-45, 45)_3]$, which properties are described in Tab. 1; and piezoelectric transducers type QP10N as actuators, which properties are described at same table. The ballast is considered made of zinc-copper alloy with mass

Table 1: Physical properties of the materials considered. (Donadon, Almeida and Faria, 2002)

Property	Glass/epoxy	QP10N
Length [m]	340×10^{-3}	45×10^{-3}
Width [m]	30×10^{-3}	20.6×10^{-3}
Thickness [m]	0.630×10^{-3}	0.254×10^{-3}
Young's modulus, E_{11} [GPa]	30.1	34.7
Young's modulus, E_{22} [GPa]	30.1	34.7
Poisson's ratio, ν_{12}	0.14	0.3
Shear modulus, G_{12} [GPa]	5.5	13.4
Shear modulus, G_{13} [GPa]	5.5	13.4
Shear modulus, G_{23} [GPa]	5.5	13.4
Density [Kg/m^3]	1905	6895
Piezoelectric constant, e_{13} [N/Vm]	-	12.7
Acceptable voltage range [V]	-	± 200

equal to 0.03458 Kg and both moment of inertia about y and z axis equal to $1.858 \times 10^{-5} \text{ Kg}m^2$. In order to increase the bend-torsion coupling of the system, the ballast was attached in an offset of $-15 \times 10^{-3} \text{ m}$ on x direction.

Along the wingspan, the pair of actuators was positioned in a step of 0.005 m , from the clamped region to the free tip.

Modal analyzes

The modal analyzes were performed observing the evolution of the first six natural frequencies as the piezoelectric unit is displaced along the wingspan.

Figure 4 shows the first six natural frequencies as a function of the piezoelectric unit position along spanwise direction. The blue curves describe the natural frequencies when the piezoelectric unit is submitted to voltage of $0V$. The red curves, in turn, describe the natural frequencies when a voltage of $200V$ is applied.

The relative variation in frequency between the two cases of voltages applied to the piezoelectric unit is described in Fig. 5, where the variations shown in this plot are relative to the $0V$ is prescribed to the piezoelectric unit. As we can see, the effect of piezoelectrically induced stresses are greater when applied close to the modal nodes. This pattern is only not displayed by the first mode.

Since the center of mass of the ballast is displaced relative to the elastic axis, all the modes present a bend and torsion movement contribution. At Fig. 6, the first six elastic mode shapes are illustrated when a voltage of $0V$ is applied to the piezoelectric unit at $y_a = 0$. At Fig. 7, in turn, shows the six mode shapes when a voltage of $200V$ is applied.

Comparing the figures 4, 5, 6 and 7, bending dominated modes are more affected by the piezoelectrically induced stress field. This fact is expected, since the axial movement is coupled with the bending.

Comparing only the figures 6 and 7, we can see that the region of the plate that contains the piezoelectric unit (close to the clamped region) is less deformed when a voltage of $200V$ is applied, as expected. This occurs because the local bending stiffness and the voltage applied are directly proportional.

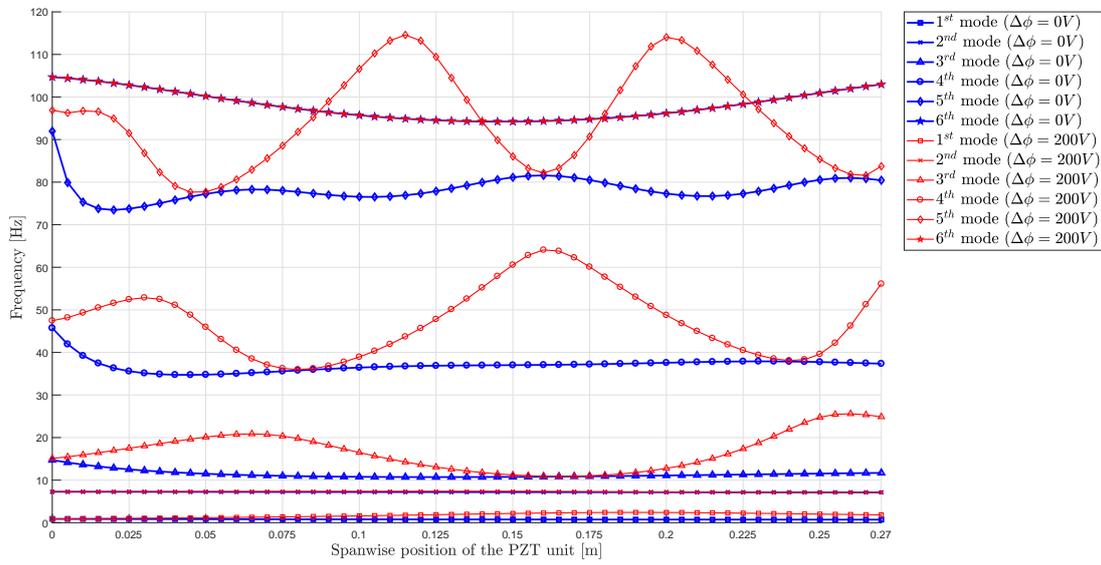


Figure 4: Modal analysis for each piezoelectric unit position.

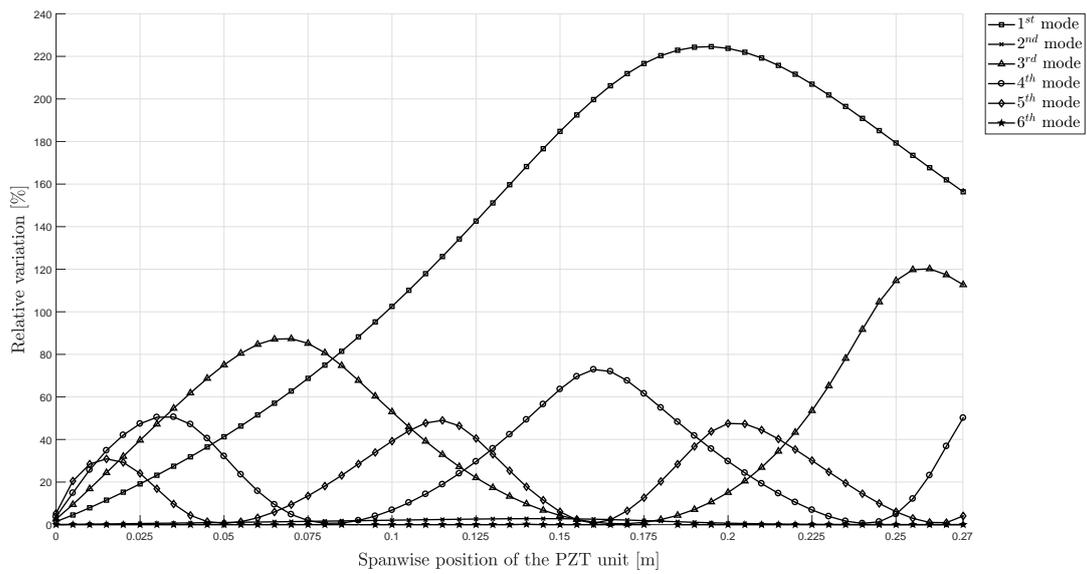
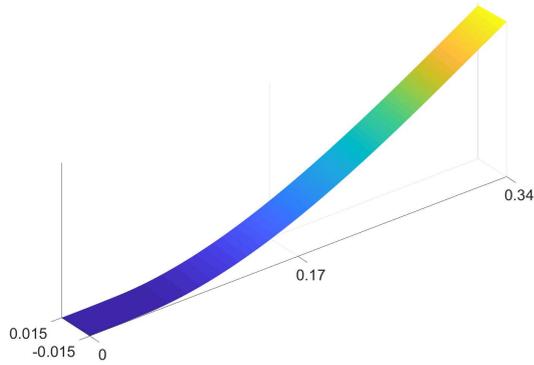
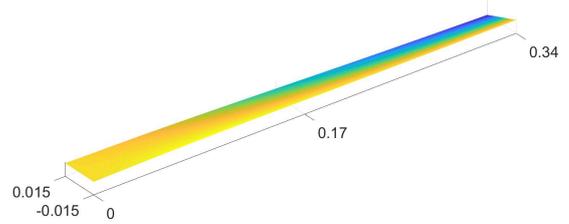


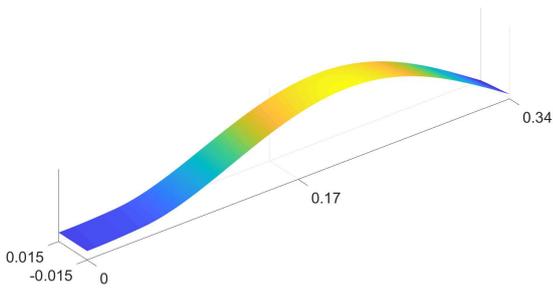
Figure 5: Relative variation in frequency [%].



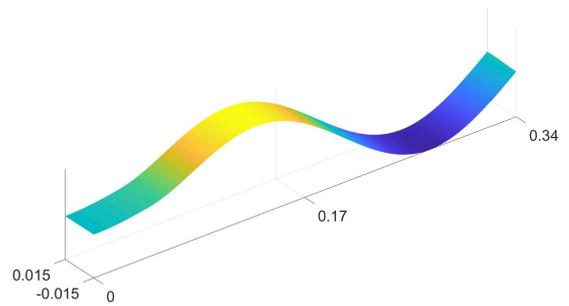
(a) 1st mode (0.88 Hz)



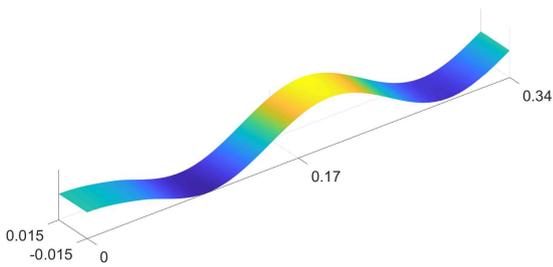
(b) 2nd mode (7.27 Hz)



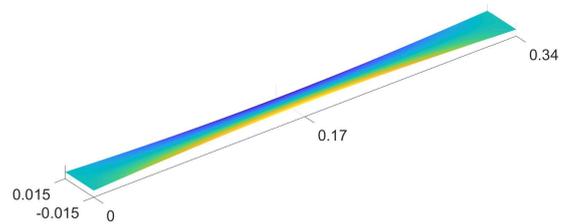
(c) 3rd mode (14.71 Hz)



(d) 4th mode (45.73 Hz)

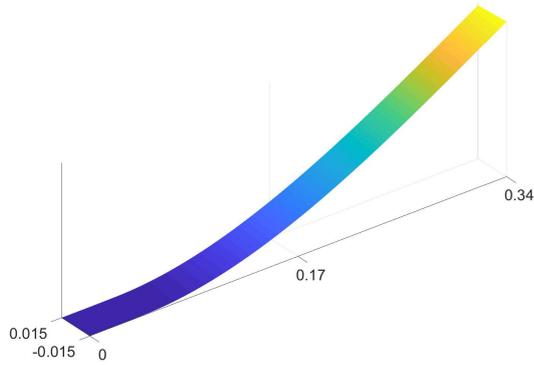


(e) 5th mode (91.96 Hz)

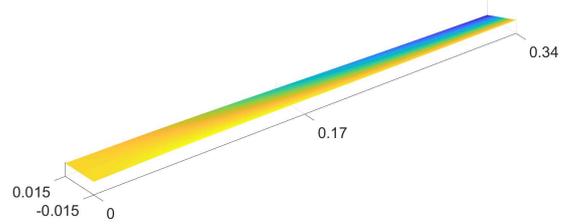


(f) 6th mode (104.69 Hz)

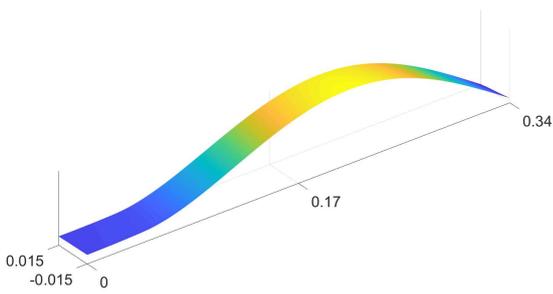
Figure 6: Flexible modes with $y_a = 0$ and $\Delta\phi = 0V$.



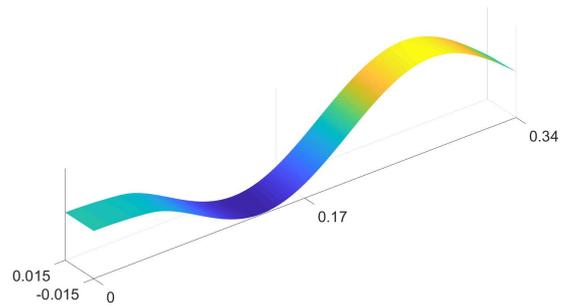
(a) 1st mode (0.89 Hz)



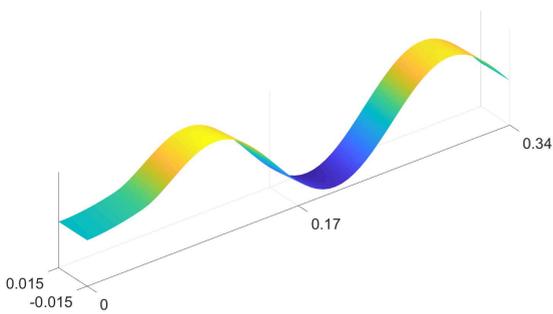
(b) 2nd mode (7.27 Hz)



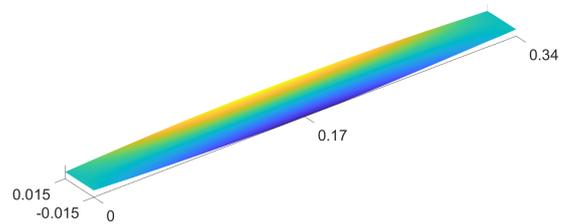
(c) 3rd mode (15.10 Hz)



(d) 4th mode (47.45 Hz)



(e) 5th mode (96.87 Hz)



(f) 6th mode (104.69 Hz)

Figure 7: Flexible modes with $y_a = 0$ and $\Delta\phi = 200V$.

Aeroelastic analyzes

Since the natural frequencies of the elastic modes are affected by the piezoelectrically induced stresses, we expect that the aeroelastic response will also change.

The aeroelastic analyzes were performed by computing the flutter and divergence speed as the piezoelectric unit is displaced along the wingspan. It is important to mention that, increasing the wind speed, if any pole with complex imaginary part present a positive real part, then the respective wind speed is registered as flutter speed; and if any negative real pole becomes positive, then the respective wind speed is registered as divergence speed.

The aeroelastic response of the structure is described in Fig. 8. Again, the blue curves describe the flutter and divergence speed applying a voltage of 0V to the piezoelectric unit; and the red curves describe when it is submitted to a voltage of 200V. The black curves, in turn, describe the relative variation with relation to the 0V voltage case. In both figures, the x axis describes the position of the piezoelectric unit along the wingspan.

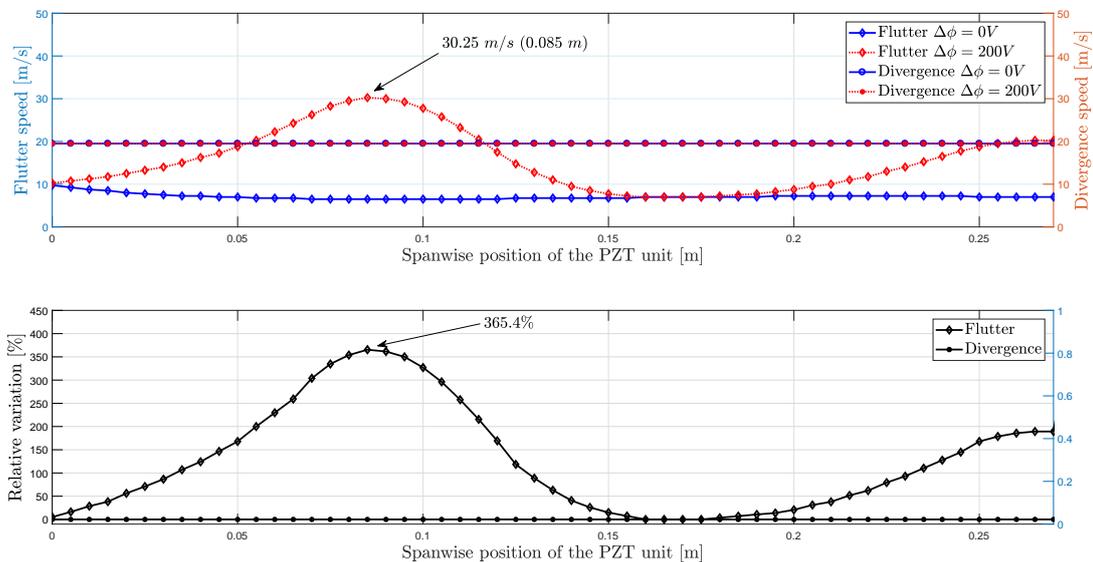


Figure 8: Flutter and divergence speed per piezoelectric unit wingspan position.

As we can see, depending on the piezoelectric unit position along the wingspan, the flutter speed can be increased by until 365.4%. Moreover, the induced stresses do not change the divergence speed, independently of the piezoelectric unit position. Because of that, depending on the unit position, the flutter speed can become higher than the divergence one.

CONCLUSIONS

In the present work, modal and aeroelastic analyzes considering different positions of the piezoelectric unit along the spanwise direction of a composite plate-like wing is presented. Moreover, two cases of voltage resulting in different conditions of piezoelectrically induced stresses field were also analyzed. Such analyzes were performed through an aeroelastic model developed by variational principle considering the von Kármán non-linear strain-displacement relationship, unsteady aerodynamics with strip theory and the constitutive relations of the linear piezoelectricity.

From the parametric study, we saw that the bending dominated modes are more affected by the piezoelectrically induced stresses, specially when it is applied close to the modal nodes. Checking the relative variation in frequency, we saw that the first bending dominated mode is the most affected by the piezoelectrically induced stresses field, followed by the second bending dominated mode and so on.

From the aeroelastic analyzes, we conclude that the piezoelectrically induced stresses effectively increased the flutter speed, reaching to 365.4% relative variation. In the cases that the flutter speed became higher than the divergence one, experimental applications would be limited by the latter speed.

Experimental tests need to be performed to check the results obtained in this work and the accuracy of the proposed

model need to be measured.

ACKNOWLEDGMENTS

The authors would like to thank Instituto de Pesquisas Tecnológicas do Estado de São Paulo (IPT) and Fundação de Apoio ao Instituto de Pesquisas Tecnológicas do Estado de São Paulo (FIPT) for the technical contribution and support; to Comissão de Aperfeiçoamento de Pessoal de Nível Superior (CAPES) for the financial support; to Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq) for the financial support by process 301053/2016-2; and to Douglas Quintanilha Tsunematsu for the helpful discussions that provided a better understanding of the problem under study.

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