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Scaled boundary finite element method for band gap analysis of one-dimensional phononic crystal solids

H.V. Cantanhêde[1] * **, E.J.P. Miranda Jr.**[2]**, J.M.C.Dos Santos**[1]

[1] University of Campinas, UNICAMP-FEM-DMC, Rua Mendeleyev, 200, CEP 13083-970, Campinas, SP, Brazil.

[2] Federal Institute of Maranhão, IFMA-EIB-DE, Rua Afonso Pena, 174, CEP 65010-030 São Luís, MA, Brazil.

* heliocs@fem.unicamp.br, edson.jansen@ifma.edu.br, zema@fem.unicamp.br

Abstract: In this paper, the wave propagation in one-dimensional (1D) phononic crystal solids is analysed. When composite materials and structures consist of two or more different materials periodically, there will be stop band characteristic, in which there are not elastic/acoustic propagating waves. These periodic structures are called phononic crystals. The main purpose is to study the properties of the periodicity on the band structure (also known as dispersion diagram), and especially, the location and width of band gaps. For this analysis, it is used the semi-analytical numerical method called scaled boundary finite element method (SBFEM), which models the domain by a small number of large-sized subdomains and discretises subdomain boundaries only. It uses the concept of transfer matrix method to calculate the wave propagation behaviour along 1D periodic systems. For comparison purpose, the band structure obtained from SBFEM is compared to the band structure calculated by plane wave expansion (PWE) and wave finite element (WFE) methods. Band gaps generated by Bragg scattering are obtained with the SBFEM, WFE and PWE through different test cases. Results are presented in the form of dispersion diagrams and frequency response functions.

Keywords: Scaled boundary finite element method, periodic structures, phononic crystals, Bragg-type band gaps.

INTRODUCTION

Phononic crystals (PCs), referring to composite structures consisting of periodic arrays of acoustic or elastic inclusions in a matrix system, have attracted extensive attention from researchers. Based on the band gap characteristic that sound wave cannot propagate in some frequency ranges, PCs have potential applications in the design of acoustic filters, waveguides, vibration isolators and noise suppressors. Several methods have already been developed to compute the elastic band gaps, for example, plane wave expansion (PWE) (Sigalas and Economou, 1992; Kushwaha and Halevi, 1994), finite difference time domain (FDTD) (Tanaka and Tomoyasu, 2000) and finite element (FE) (Axmann and Kuchment, 1999) methods. These methods present several drawbacks from the computational point of view. For instance, the plane wave expansion converges slowly because this method requires a large number of terms in the Fourier series in order to obtain accurate numerical results. The multiple scaterring theory (MST) method, which is based on the Bessel function or Hankel function expansions, can only be applied in the composite systems with spherical or elliptical scatterers. The FDTD method solves the elastic wave equation by discretizing time and space, but it involves quite large matrices and is time-consuming. The main objective of this paper is to extend the scaled boundary finite element method (SBFEM) to the elastic band gap analysis of one-dimensional (1D) phononic crystal solids.

METHODOLOGY

Plane wave expansion method

PWE is the most used approach in order to calculate the band structure of periodic systems, such as photonic crystals (El-Naggar, 2012), phononic crystals (Kushwaha *et al*., 1994; Sigalas and Economou, 1992), sonic crystals (Kushwaha and Halevi, 1994) and mechanical metamaterials (Xiao *et al*., 2012). PWE uses the system periodicity and Floquet-Bloch's theorem (Floquet, 1883; Bloch, 1928) to solve the wave equation, obtaining and eigenvalue problem $\omega(\mathbf{k})$. This eigenvalue problem must be solved for each Bloch wave vector, $\mathbf{k}$, also known as wave number, into the first Brillouin zone (FBZ) (Brillouin, 1946). One of the main advantages of using PWE is its facility of being implemented. PWE is regarded as a semi-analytical method, because Fourier series expansion in reciprocal space must be truncated. This method has also some drawbacks, for instance, when there is high geometry or material mismatch PWE presents slow convergence and improved plane wave expansion (IPWE) method must be used. PWE formulation and results will be provided in the final version of this article, since there is the number of pages limitation in this extended abstract.

Wave finite element method

Originally, the wave finite element (WFE) method has been developed to describe the wave propagation along 1D periodic structures (Zhong and Williams, 1995). The procedure is nothing else but a transfer-matrix approach and the use of the Bloch's theorem (Mead, 1973) for expressing the so-called wave modes, *i.e.*, waves which propagate from

substructure to substructure along the right and left directions of a periodic structure. WFE involves the modelling of only one unit cell by means of FE. Thus, the model size is considerably reduced compared to the full FE model.

Scaled boundary finite element method

The SBFEM has been applied to many fields of engineering (Wolf and Song, 1996). This method is based entirely on finite elements, but with the discretization only of the boundary. It combines the advantages of boundary element (BE) and FE methods. No fundamental solution is required, the spatial dimension is reduced by one, the radiation condition is satisfied exactly at infinity, and it yields a symmetric dynamic-stiffness matrix. A scaling centre O, located inside the domain, is chosen in a position from which the whole boundary must be reached. Only the boundary S needs to be discretized with line finite elements. The geometry of an element is interpolated using the shape functions $[N(\eta)]$. A radial coordinate ξ, from the scaling center O to the boundary surface. The geometry transformation from Cartesian coordinates $\hat{x}$, $\hat{y}$ to scaled boundary coordinates η , ξ is performed, which is equivalent to represent the surface in polar coordinates where the value of the radial coordinate at the boundary is constant. For more details about SBFEM see (Wolf and Song, 1997).

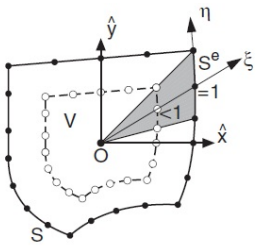

Figure 1 – Scaled boundary coordinate system for a 2D bounded domain (Song, 2009).

NUMERICAL RESULTS

Two cases of unit cells were simulated, a cell with two materials shown in Fig.3(a) and the other case with a cell of a single material shown in Fig.3(b). In case 1 the configuration of material is: $Carbon + Epoxy + Carbon$, in the case 2 the cell is all carbon. The PC parameters and material properties are listed in Table 1. The structural damping, η_A, η_B, also known as loss factors, are included as a complex Young's modulus, $E_A = \tilde{E}_A(1+i\eta_A)$, $E_B = \tilde{E}_B(1+i\eta_B)$, $i = \sqrt{-1}$. In Figure.4(a) is shown the mesh obtained through the FEM and Fig.4(b) is shows the mesh obtained through substructures from SBFEM where eight substructures were used to assemble the slice.

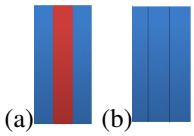

Figure 2 – (a) Case 1 (b) Case 2.

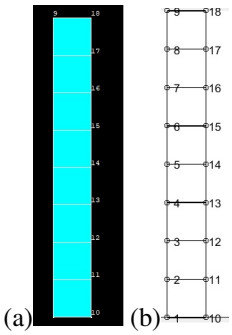

Figure 3 – (a) Finite element mesh of the slice (b) scaled boundary finite element mesh of the slice using substructure.

Table 1 – Materials used in the formation of the unit cell. The cell is composed of carbon (A) and epoxy (B).

Properties	Carbon (A)	Epoxy (B)
Young's modulus (GPa)	130	6
Density (kg/m^3)	1800	120
Poisson's ratio	0.40	0.35
Loss factor	0.01	0.01
Percentage of material	67%	33%

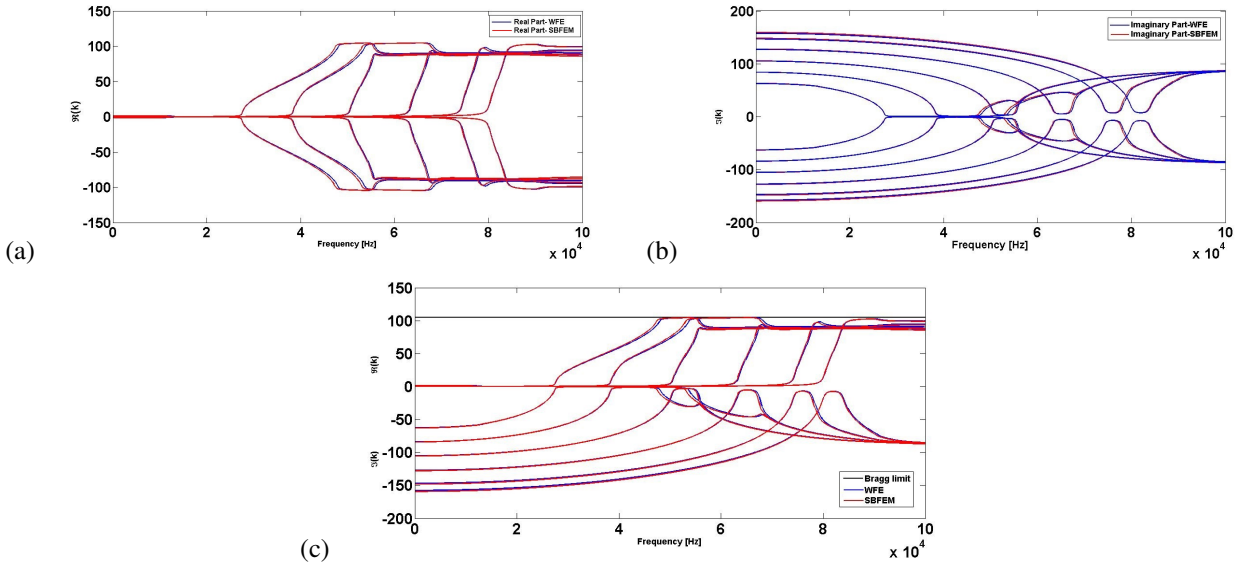

Figure 4 – (a) Real part of the dispersion relation for case 1 (b) Imaginary part of the dispersion relation for case 1 (c) Real and Imaginary part of the dispersion relation for case1.

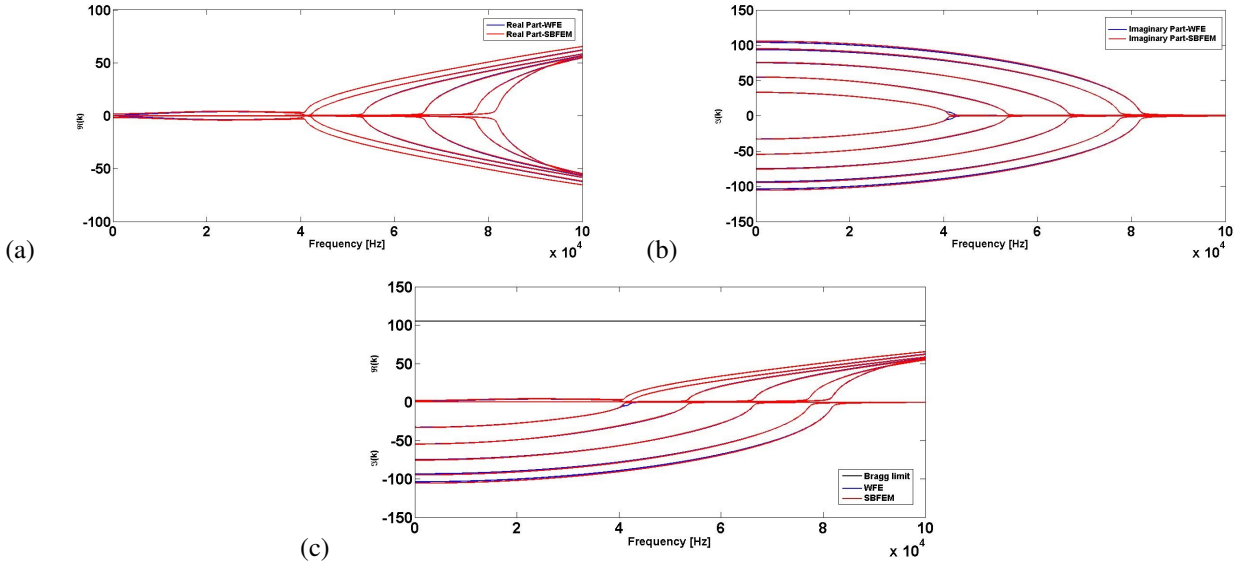

Figure 5 – (a) Real part of the dispersion relation for case 2 (b) Imaginary part of the dispersion relation for case 2 (c) Real and Imaginary part of the dispersion relation for case2.

For WFE analysis, the mass and stiffness matrices were obtained through the ANSYS software version 14.5 and the obtained slice mesh is shown in Fig.4, using the element PLANE 182 four-node element with two degrees of freedom per node, to the total, 8 quadrangular elements were used. The slice has a width of 0.01 m and length of 0.5 m. Figure 5 illustrates the dispersion relation for case 1 calculated by SBFEM (red line) and WFE (blue line) . The formation of band gaps is easily observed by the analysis of the dispersion relations for case 1. The Bragg-type band gaps can be observed

in Fig.5. The black line indicates the Bragg Limit. It is noted that the obtained results do not exceed this limit as expected. Figure 6 illustrates the dispersion relation for case 2 where only one material is considered. It can be noticed a slightly different behavior when compared to case 1 It is observed that the non-inclusion of a material in the cell causes that the band gaps do not appear and a good convergence between the SBFEM and WFE methods was observed for the two cases analyzed in this study. The results of the PWE and IPWE as well as the FRFs will be placed in the final presentation of the work.

CONCLUSION

In this paper, band gaps in 1D PC solids were obtained by SBFEM and WFE method. A numerical methodology based on a combination of SBFEM and Floquet Bloch's theorem,was proposed as an engineering tool to calculate the band gaps in 1D PC solids. Two different cases were analyzed, in the first the cell was composed of two different materials and in the second the cell was composed of a material only. It was noticed the formation of band gaps for case 1.

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