Position Stabilization Control of Flexible Joint Manipulator using Feedback Linearization and Gaussian Process Regression

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Abstract: Flexible joint model for robotic arms is suitable in the context of imperfections in the connection between the link and its base. The development of accurate control systems for these systems relies on the adequate compensation for uncertain dynamics. In this work, a new control scheme is presented for a flexible joint manipulator. Feedback Linearization and Gaussian Process Regression are combined to solve the position stabilization problem. Numerical results confirm the stronger improved performance of the proposed control scheme.

Keywords: Flexible joint, Feedback Linearization, Gaussian Process Regression, position stabilization.

INTRODUCTION

In the articulated robots, the use of transmission systems, such as harmonic reducers and devices with hydraulic drive, induces flexibility in your joints (Liu, Li and Wang, 2017). This flexibity is related to the clearance originated from the manufacturing process or characteristics of assembly or operation whose size exceeds the desired or allowed value (Erkaya, 2018). In robot arms (manipulators) with this joint type is necessary a robust control able to improve performance in trajectory tracking or position stabilization, especially if there is interaction of these mechanisms with people (Liu et al., 2018). In this case, a flexible joint model is applied to these mechanisms, as used by Liu, Li and Wang (2017), Erkaya (2018), Liu et al. (2018) and Kim (2018).

A large number of control strategies have been applied to problem of the flexible joint manipulators. For example, Ahmad, Tumari and Nasir (2013) applied Fuzzy-Logic in a PID approach to reduce the vibrations of the system and Xu et al. (2018) who used Sliding Mode Control (SMC) with a adaptative strategy in a trajectory tracking task.

Therefore, a different approach is suggested in this work. Instead of trying to represent the rigidity and damping effects, Gaussian Process Regression (GPR) is used to estimate modeling uncertainties and to compensate for them. A Gaussian process may be understood as an expansion of the Gaussian probability distribution (Rasmussen and Williams, 2006). Hence, GPR can be adopted as non-parametric model to represent samples from a multivariate Gaussian distribution, in order to estimate both structured and unstructured uncertainties related to plant dynamics (Doerr et al., 2018). Although GPR alone is not able to provide an efficient feedback controller, it could be conveniently associated with conventional control schemes. Here, a Feedback Linearization (FL) control scheme is adopted in this way.

In this work, a GPR is coupled to the FL control law to model the uncertain dynamics of a flexible joint robot arm with adoption of an online learning approach applied to position stabilization problem.

POSITION STABILIZATION CONTROL FOR THE FLEXIBLE JOINT MANIPULATOR

Consider the representation for the one-link flexible joint presented in the Fig. 1.

The dynamic of the system can be modeled by the following differential equation, considering that the damping is of the viscous type:

$$J\ddot{\theta} = -k_t(\theta - \theta_0) - c_t\dot{\theta} + \tau \tag{1}$$

where J are the moment of inertia, known with certain accuracy, and k_t and c_t are, respectively, the coefficients of stiffness and damping torsional whose knowledge about them is not accurate.

The Eq. 1 can be rewritten as follows:

$$\ddot{\theta} = J^{-1}(\tau - f) \tag{2}$$

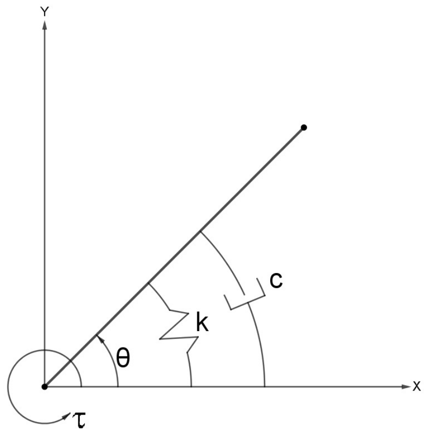


Figure 1: One-link flexible joint manipulator.

where $f = k_t(\theta - \theta_0) + c_t\dot{\theta}$ concentrates all the uncertain dynamics that depend on the states $\boldsymbol{\theta} = [\theta - \theta_0,\, \dot{\theta}]^\top$.

Assumption 1. The dynamics f is unknown, but bounded, i.e. $|f| \leq \delta$.

The control law for position stabilization task using the FL method is given by:

$$\tau = J(\ddot{\theta}_d - 2\lambda\dot{\tilde{\theta}} - \lambda^2\tilde{\theta}) + \hat{f} \tag{3}$$

where $\hat{f}$ represent the estimate of dynamic f and λ is a strictly positive constant.

Considering that Gaussian process regressor can be used as as non-parametric model to describe a distribution over functions, GPR is adopted to estimate uncertain dynamics.

Therefore, following Rasmussen and Williams (2006) and assuming noisy observations, it can be possible to use GPR to estimate f

$$\bar{f} = f(\boldsymbol{\theta}) + \varepsilon, \qquad \varepsilon \sim \mathcal{N}(0, \sigma^2) \tag{4}$$

A GP can be defined as a probability distribution over the space of functions f whose restriction to any finite number of function values is jointly Gaussian (Rasmussen and Williams, 2006). A GP is specified through its prior mean function μ and covariance function k:

$$f(\boldsymbol{\theta}) \sim \mathcal{GP}(\mu(\boldsymbol{\theta}), k(\boldsymbol{\theta}, \boldsymbol{\theta}')) \tag{5}$$

with $\mu(\boldsymbol{\theta}) = \mathbb{E}[f(\boldsymbol{\theta})]$ being the expected function value and $k(\boldsymbol{\theta}, \boldsymbol{\theta}') = \mathbb{V}(\boldsymbol{\theta}, \boldsymbol{\theta}')$ the covariance function of $f(\boldsymbol{\theta})$ and $f(\boldsymbol{\theta})'$.

Remark 1. Although Assumption 1 does not formally hold within the GP model, it should not pose any problem in practice. Considering that the Gaussian distribution has infinite support, in theory, it will always be possible for a value of $f(\boldsymbol{\theta})$ to arise outside the bounds of confidence for a sufficiently large data set. However, it should be first noted that, in real world applications, unlimited disturbances do not usually occur.

In GPR, learning a function amounts to predicting the (normal) distribution of function values $f(\boldsymbol{\theta}^*)$ at arbitrary inputs $\boldsymbol{\theta}^*$ based on previous evaluations. Given N data points $\mathcal{D}_N = \{\boldsymbol{\theta}_i, f_i\}_{i=1}^N$, the posterior mean and variance of $f(\boldsymbol{\theta}^*)$ can be stated as

$$\mathbb{E}(f(\boldsymbol{\theta})|\mathcal{D}_N) = \mu(\boldsymbol{\theta}^*) + k_N^T(\boldsymbol{\theta}^*)(K_N + \sigma^2 I)^{-1}\tilde{f}_N \tag{6}$$

$$\mathbb{V}(f(\boldsymbol{\theta})|\mathcal{D}_N) = k(\boldsymbol{\theta}^*, \boldsymbol{\theta}^*) - k_N^T(\boldsymbol{\theta}^*)(K_N + \sigma^2 I)^{-1}k_N \tag{7}$$

where $k_N := [k(\boldsymbol{\theta}^*, \boldsymbol{\theta}_1) \ldots k(\boldsymbol{\theta}^*, \boldsymbol{\theta}_N)]^T$, $\tilde{f}_N := [\bar{f}_1 - \mu(\boldsymbol{\theta}_1) \ldots \bar{f}_1 - \mu(\boldsymbol{\theta}_N)]$ and K_N is a Gramian matrix.

If the GPR can approximate the unknown dynamics f with certain accuracy, i.e, $|\tilde{f}| \leq \phi$, where $\tilde{f} = f - \hat{f}$, so it is can possible to proof that the control law (3) converges the error to a limited region $\Phi = \{(\tilde{\theta}, \dot{\tilde{\theta}}) \in \mathbb{R}^2 \mid |\tilde{\theta}| \leq \phi\lambda^{-2}$ and $|\dot{\tilde{\theta}}| \leq 2\phi\lambda^{-1}\}$ (Tanaka, Fernandes and Bessa, 2013).

Here, it is defined that the predictive mean (Eq. 6) is sufficient to estimate $\hat{f}$. The proposed control scheme is presented in Fig. 2 and in the Algorithm 1.

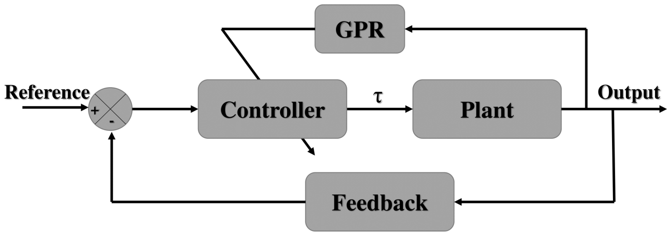


Figure 2: Proposed control strategy.

Algorithm 1: Feedback Linearization and GPR.

Define control parameters
Especify GP *prior* (mean μ, kernel k)
Define initial states: $\boldsymbol{\theta}_0$
Initialize data set: $\mathcal{D}_0$
while $t \leq t_{fim}$ **do**
    Evaluate desired trajectory: $\boldsymbol{\theta}_d$
    Compute tracking error: $\tilde{\boldsymbol{\theta}} = \boldsymbol{\theta} - \boldsymbol{\theta}_d$
    $\hat{f} \leftarrow \mathbb{E}(h(\boldsymbol{\theta}^*) \mid \mathcal{D}_{i-1})$
    $\tau \leftarrow J(\ddot{\theta}_d - 2\lambda\dot{\tilde{\theta}} - \lambda^2\tilde{\theta}) + \hat{f}$
    Apply τ in the system
    Update the states: $\boldsymbol{\theta}$
    Update GP *posterior*: $\mathbb{E}(h(\boldsymbol{\theta}^*) \mid \mathcal{D}_i), \mathbb{V}(h(\boldsymbol{\theta}^*) \mid \mathcal{D}_i)$
    $t \leftarrow t + \Delta t$
end

NUMERICAL SIMULATION

The proposed control scheme is now evaluated by means of numerical simulations. The fourth order Runge-Kutta method is employed and sampling rates of 250 Hz for the controller and 500 Hz for system dynamics are assumed.

In the dynamic model (Eq. 1) was considered that $J = 0.07$ kgm^2, k_t = 10 Nm and $c_t = 0.0001$ kgm^2/s. Guassian noise with $\sigma = 0.5$ is added to the f in order to emulate noisy signal. And the controller parameter was chosen $\lambda = 2.5$.

Squared exponential kernel is adopted as the covariance function for the Gaussian process regressor

$$k(\boldsymbol{\theta}, \boldsymbol{\theta}') = \exp\left(-\frac{-||\boldsymbol{\theta} - \boldsymbol{\theta}'||^2}{2\gamma^2}\right) \tag{8}$$

The length-scale γ was modified to evaluate the convergence behavior in each case. The values chosen for this parameter were 0.8, 1.0 and 1.2. The controller was set to stabilize in the angular position of $\theta_0 = \pi/2$ rad. The results are showed in the Fig. 3.

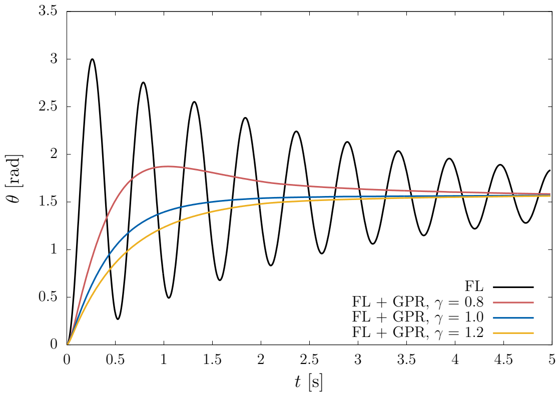
As observed in Fig. 3, even in the presence of modeling inaccuracies, the proposed control scheme is able to provide the position stabilization in a short time for each length-scale. The $\gamma = 1.0$ provides the faster convergence to the desired position, but the $\gamma = 1.2$ besides allowing the position stabilization it provides a softer convergence than the other cases.

On the other hand, the inclusion of a GPR compensation increase the input torque. Because of this an actuator more powerful is necessary to control the system.

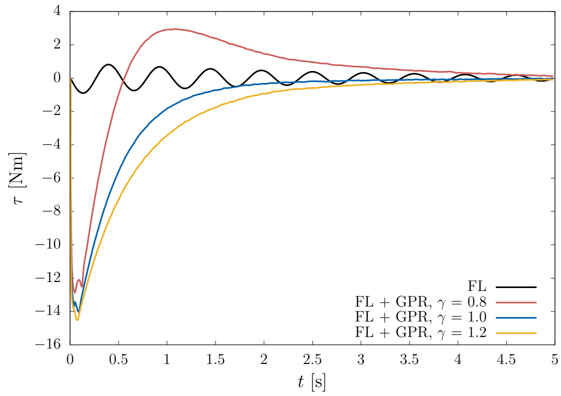
The efficacy of GPR can be clearly ascertained by comparing the phase portrait of the stabilization error obtained with both conventional and proposed control strategies, respectively, Fig. 3c and Fig. 3d, for $\gamma = 1.2$. The difference is accentuated when compared the convergence velocity error between the two cases.

CONCLUSIONS

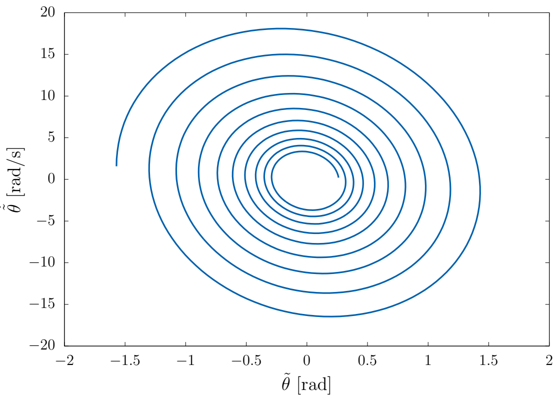
In this paper, a feedback linearization controller is combined with Gaussian process regression to deal with the position stabilization of a joint flexible manipulator. GPR is used as a non-parametric model to estimate the uncertain dynamics. By means of numerical simulations, the improved performance of the proposed scheme over the conventional feedback linearization controller is demonstrated.



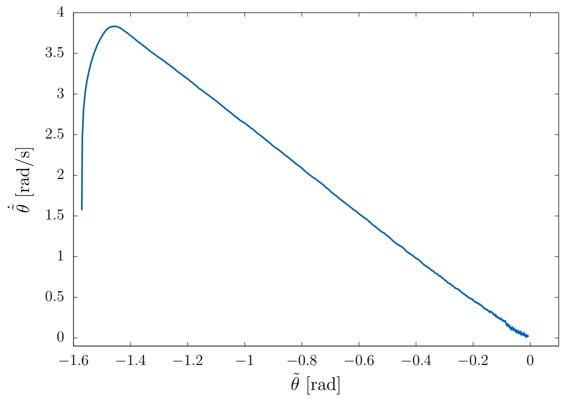
(a) Position stabilization.



(b) Input torque.



(c) Phase portrait with LR.



(d) Phase portrait with LR + GPR and $\gamma = 1.2$.

Figure 3: Position stabilization with conventional (FL) and proposed (FL + GPR) control approaches.

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