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Stick-slip oscillations or couple-decouple oscillations?

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Abstract: This work analyzes the nonlinear dynamics of an electromechanical system with dry-friction. The system is composed of a cart, whose motion is excited by a DC motor. It is a coupled system, the mechanical and the electromagnetic subsystems interact. The coupling between the motor and the cart is made by a mechanism called scotch yoke, so that the motor rotational motion is transformed into horizontal cart motion over a rail. It is considered the existence of dry-friction friction between the cart and the rail. Due to the friction, the resulting motion of the motor can be characterized by two qualitatively different modes, the stick- and slip-modes, with a non-smooth transition between them. The focus of the work is to show that the sequence of stick- and slip-modes can be seen as a sequence of coupled- and decouple-modes between electrical and mechanical parts of the system. Thus, for the system analyzed, stick-slip oscillations could be called as couple-decouple oscillations. Another objective of the work is to find the stick- and slip-mode parts of the trajectory for different values of the system parameters.

Keywords: Electromechanical system, dry friction, stick-slip oscillations, stick-duration

INTRODUCTION

Electromechanical systems present an interesting behavior characterized by the mutual influence between the electrical and mechanical parts of the system (Dantas, Sampaio and Lima, 2014, Dantas, Sampaio and Lima, 2016, Clerkin and Sampaio, 2017 and Manhães et al., 2018) Each part of the system affects the behavior of the other, i.e., they interact. The coupling varies with the coupling conditions, it is not a functional relation and depends on the initial conditions (Lima and Sampaio, 2015) The dynamics of the coupled system is given by an initial value problem comprising a set of coupled differential equations (Lima, Sampaio, and Hagedorn, 2018 and Lima and Sampaio, 2018). The problem becomes even more interesting if it is considered the existence of dry-friction in the electromechanical system. The nonlinearity provided by the friction can induce stick-slip oscillations on the system (Lima and Sampaio, 2017a and Lima and Sampaio, 2017b). Depending on the values of the system parameters, the response of the system can be composed of a sequence alternating stick and slip-modes.

DYNAMICS OF THE ELECTROMECHANIAL SYSTEM WITH DRY-FRICTION

The system analyzed in this paper is composed by a cart whose motion is driven by a DC motor. The motor is coupled to the cart through a pin that slides into a slot machined on a plexiglas plate that is part of the cart, as shown in Fig. 1. The pin hole is drilled off-center on a disk fixed in the axis of the motor, so that the motor rotational motion is transformed into horizontal cart motion over a rail. The dynamics of a DC motor is given by the following initial value problem (IVP).

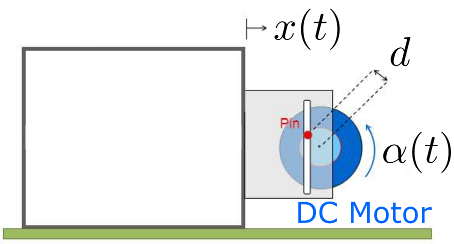

Figure 1 – First Mechanical System.

Given a source voltage v, find (α, c) such that, for all $t > 0$,

$$l\dot{c}(t) + r\,c(t) + k_e\dot{\alpha}(t) = v(t)\,, \tag{1}$$

$$j_m\ddot{\alpha}(t) + b_m\dot{\alpha}(t) - k_e\,c(t) = -\tau(t)\,, \tag{2}$$

with the initial conditions

$$\dot{\alpha}(0) = \dot{\alpha}_0 \quad , \quad \alpha(0) = \alpha_0 \quad , \quad c(0) = c_0\,, \tag{3}$$

where t is the time, v is the source voltage, c is the electric current, $\dot{\alpha}$ is the angular speed of the motor, l is the electric inductance, j_m is the motor moment of inertia, b_m is the damping ratio in the transmission of the torque generated by the motor to drive the coupled mechanical system, k_e is the motor electromagnetic force constant and r is the electrical resistance. The module of the available torque to the coupled mechanical system is τ. The source voltage is considered to be

$$v(t) = v_0 + v_1 \sin(\omega_v t)\,. \tag{4}$$

The mass of the mechanical system is m and the horizontal cart displacement is represented by x. It is considered that the cart is not allowed to move in the vertical direction. Due to the problem geometry, and noting $||\boldsymbol{d}|| = d$, the horizontal motion of the cart and the angular displacement α of the motor are related by the constraint

$$x(t) = d\,\cos(\alpha(t))\,. \tag{5}$$

In the model of the coupling between the motor and the mechanical system, it is assumed that the motor shaft is rigid. Thus, the available torque to the coupled mechanical system, $\boldsymbol{\tau}$, can be written as

$$\boldsymbol{\tau}(t) = \boldsymbol{d}(t) \times \boldsymbol{f}(t)\,, \tag{6}$$

where $\boldsymbol{d}$ is the eccentricity of the pin of the motor and $\boldsymbol{f}$ is the coupling force between the DC motor and the cart. The component of $\boldsymbol{d}$, which is perpendicular to the plane of the cart movement, is always zero and, the others horizontal and vertical components can be calculated from the angular displacement α of the motor. Assuming that there is no friction between the pin and the slot machined on an acrylic plate, the vector $\boldsymbol{f}$ only has a horizontal component, called f, which is the horizontal force that the DC motor exerts in the cart. Thus, the module of $\boldsymbol{\tau}(t)$ is

$$\tau(t) = -f(t) d \sin\alpha(t)\,. \tag{7}$$

Since the cart is modeled as a particle, its movement in the horizontal direction satisfies the equation:

$$m\,\ddot{x}(t) = f(t) + f_r(t)\,, \tag{8}$$

where f_r is the dry-friction force between the cart and the rail. The initial value problem to the coupled motor-cart system with dry-friction is: given v, find $(\alpha,\, c)$ satisfying

$$l\dot{c}(t) + r\,c(t) + k_e\dot{\alpha}(t) = v_0 + v_1 \sin(\omega_v t)\,, \tag{9}$$

$$\ddot{\alpha}(t)[j_m + md^2(\sin(\alpha(t)))^2] + \dot{\alpha}(t)[b_m + md^2 \sin(\alpha(t))\cos(\alpha(t))] - k_e c(t) = -f_r(t)\,d\,\sin(\alpha(t))\,, \tag{10}$$

for given initial conditions of electric current, angular speed and position of the motor. The friction is modeled as Coulomb, which is a simple model. The non-smooth behavior of the dry-friction force can induce in the system stick-slip oscillations. Depending on the values of the system parameters, the response of the system can be composed of a sequence alternating stick and slip-modes.

During the stick-mode, the cart does not move, so that the angle of disc is constant. The frictional force and the current can vary. The stick mode occurs when $\dot{\alpha} = 0$ and when the frictional force, which satisfies

$$k_e\,c(t) = f_r(t)\,d\,\sin(\alpha(t))\,, \tag{11}$$

is in the interval $-f_{max} \le f_r \le f_{max}$, where $f_{max} = \mu m g$, g is the gravitational acceleration and μ is the friction coefficient between the cart and the rail. Equation (11) is obtained considering $\dot{\alpha} = 0$ and $\ddot{\alpha} = 0$ in Eq. (10). Remark that during the stick-mode, the frictional force varies and depends on the angular position of the motor. There is a functional relation between these two variables. Besides of this, the initial value problem that describes the dynamics of the coupled motor-cart system with dry-friction is reduced to just one differential equation given by

$$l\dot{c} + r\,c = v_0 + v_1 \sin(\omega_v t)\,, \tag{12}$$

where the initial condition of the current is the value of it in the beginning of the stick-mode. Observe that during the stick-mode, the sum of the forces that act over the cart is zero (it does not move). The horizontal coupling force between the DC motor and the cart, f, is balanced by the dry-friction force, f_r. This balance lasts until the frictional force, given in Eq. (11), reaches its maximum value, f_{max}. During the stick-mode, the dynamics of the system is governed only by the dynamics of the electrical circuit of the motor. Electrical and mechanical systems do not interact, the system is decoupled. During the slip-mode, the dry-friction force is

$$f_r(t) = -mg\mu\ \text{sgn}(\dot{x}(t)) = -mg\mu\ \text{sgn}(-\dot{\alpha}(t)\, d\, \sin(\alpha(t)))\,. \tag{13}$$

RESULTS

Defined a time interval for analysis, one of the variables of great interest in the analysis of systems with dry-friction is the total time of stick as function of the friction coefficient. For the electromechanical system analyzed in this work, this variable presents an interesting behavior, as shown in Fig. . When μ is lower then 0.57, there is no stick, when μ is greater then 2.84 the only mode is stick. For values of μ between 0.57 and 2.84, both can exist. For computation, duration t_a was chosen as 30 seconds. For the integration, it was used the function *ode45* of the *Matlab* software, which applies the Runge-Kutta 4th/5th-order method as time-integration scheme with a varying time-step algorithm. The maximal step size is equal to 10^{-4} seconds, and the relative and absolute tolerance are equal to 10^{-4}. The values of the parameters used in all simulations were $m = 5.0$ kg, $d = 0.01$ m, $v_0 = 4.0$ V and $v_1 = 4.0$ V. The motor parameters are given in Table 1 and the initial conditions of the system are $\alpha(0) = \pi/2$ rad, $\dot{\alpha}(0) = 0$ rad/s and $c(0) = v_0/r$ Amp.

Parameter	**Value**
l	1.880×10^{-4} H
j_m	1.210×10^{-4} Kg m^2
b_m	1.545×10^{-4} Nm/(rad/s)
r	$0.307\ \Omega$
k_e	5.330×10^{-2} V/(rad/s)

Table 1 – Values of the motor parameters used in simulations.

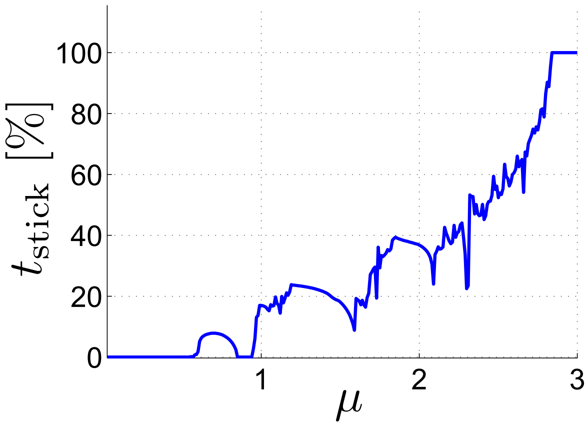

Figure 2 – Total time of stick as function of the friction coefficient.

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