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Two coupling mechanisms compared by their Lagrangians

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Abstract: Electromechanical systems are very common. The importance of constructing the dynamical equations of motors coupled with mechanical systems suggests a new strategy. In the literature, often, the derivation of the dynamical equations is wrong. One thinks that the standard derivations of the dynamical equations of purely mechanical systems can be mimicked to electromechanical systems. Unfortunately, it cannot. The main reason is that in electromechanical systems one deals with the presence of electromagnetic fields, continuous entities. This fields store electrical and mechanical energies. In purely mechanical systems the conservative mechanical energy is stored as elastic or gravitational energy , and the nonconservative terms enter the equation as nonconservative forces. This cannot be done in electromagnetic systems. This paper shows the right way to derive the dynamical equations applying the results for two systems. Both systems are formed by a motor, the electromagnetic subsystem; a coupling mechanism; and a mechanical subsystem. In one case the coupling is the scotch-yoke mechanism and the mechanical part is a cart, modeled as a particle. In the other the coupling mechanism is a slider-crank mechanism and the mechanical system is a sliding mass, again a particle. To explain clearly the ideas, the dynamical equations are derived in a different way, putting in evidence the common errors. The examples were taken from the recent literature, but the mistakes are older.

Keywords: Electromechanical systems, Lagrangians, Coupled systems, Nonlinear dynamics, Coupling mechanisms

INTRODUCTION

Electromechanical systems are characterized by a mutual influence between mechanical and electrical parts (Dantas et al., 2014, 2016). One particular aspect of systems with mutual interactions (Jeltsema and Scherpen, 2009) is a feature called coupling. In this case, an electromechanical coupling. A very important tool when it comes to formulate an extended Lagrangian (Haas et al., 2000). Although this is not a new subject (Wells, 1967), it needs to be discussed. Basically, this paper analyses two electromechanical systems. Both have a DC motor coupled to a mechanical part. Their difference lies in the choice of the mechanical coupling mechanism. The mechanical coupling of the first system is made by a mechanism called *slider-crank*. The second uses a mechanism called *scotch-yoke*. These systems have a feature: they work as a sort of master-slave condition. The motor rotates and the mechanical part translates. The focus is to formulate an extended Lagrangian for both systems. This paper also points out a mistake commonly made in deriving the dynamical equations of electromechanical systems. This procedures gives a wrong interpretation of coupled systems. It formulates a Lagrangian only for the mechanical part, disregarding the electrical subsystem. When one commits this first mistake is then led to commit a second unjustified simplification, which has even more serious consequences. This second mistake is to say that a torque provided by the motor to the mechanical system is a function of the generalized coordinates. But, of course, the torque is not a function of only the coordinates, it also depends on the initial conditions. Therefore, it must not be added as a generalized force to the description of the mechanical system.

DYNAMICS OF ELECTROMECHANICAL SYSTEMS

Slider-crank mechanism

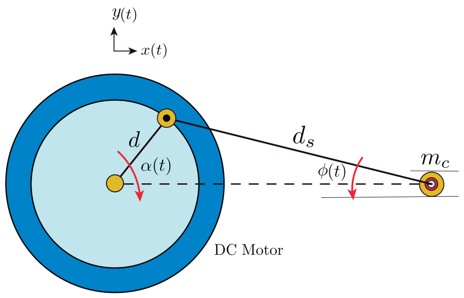

Figure 1 – Electromechanical system with a crank-slider mechanism.

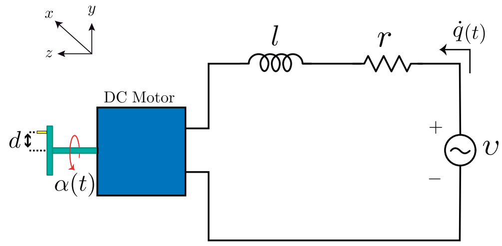

Figure 2 – DC motor

One of the systems analysed in this paper consists of a slider-crank mechanism linked to a sliding mass m_c. This system has a DC motor. There is viscous friction in the disk coupled to the motor (b_m). The distance related to the eccentricity of the crank is d. The length of the shaft is d_s. The angles in the crank and in the shaft are given by $\alpha(t)$ and $\phi(t)$, respectively. In figure 2 l is the electric inductance, r is the electric resistance, υ is the constat voltage, $\dot{q}$ is the current, representing the time derivative of the electric charge q. This system is a simplified model of the system in (Avanço et al., 2018). The same system without a pendulum and considering the sliding particle with mass m_c.

[Step 1: Identifying the problem]

This system has four types of motion involved. The rotational motion of the disk, the rotational motion of the shaft and the moving charges in the electromagnetic subsystem. It has one constraint due to the slider-crank mechanism. It has two degrees of freedom. The electromagnetic subsystem is described by the charge $q(t)$ and the mechanical part by the angular displacements $\alpha(t)$.

[Step 2: Electromechanical coupling]

For the electromechanical coupling, as it stores magnetic energy in the coupling terminal (Woodson and Melcher, 1968), it is interesting to analyse the equation

$$\frac{d\mathscr{W}_m^*}{d\alpha} = \tau_e. \tag{1}$$

In most DC motors, the torque τ_e is proportional to the armature current $\dot{q}$ and the strength of the magnetic field denoted by k_e, which is the motor electromagnetic force constant: $\tau_e = k_e\dot{q}$. See (Hughes and Drury, 2013). Substituting in equation 1 gives for $\mathscr{W}_m^* = \mathscr{W}^*$:

$$\mathscr{W}^* = k_e\dot{q}\alpha. \tag{2}$$

[Step 3: Kinetic coenergy and potential energy]

First, we begin by writing the position of the mass and the relation between the angles α and ϕ :

$$x = dcos(\alpha) + d_s cos(\phi), \qquad cos(\phi) = \left(1 - \frac{d^2}{d_s^2} sin(\alpha)^2\right)^{\frac{1}{2}}. \tag{3}$$

Substituting in the equation and deriving in time gives

$$\dot{x} = -d\dot{\alpha}\left(sin(\alpha) + \frac{dsin(\alpha)cos(\alpha)}{d_s\left(1 - \frac{d^2}{d_s^2}sin(\alpha)^2\right)^{\frac{1}{2}}}\right), \qquad \mathscr{K}(\alpha) = sin(\alpha) + \frac{dsin(\alpha)cos(\alpha)}{d_s\left(1 - \frac{d^2}{d_s^2}sin(\alpha)^2\right)^{\frac{1}{2}}}. \tag{4}$$

As a result, it is possible to simplify $\dot{x}$ with a function $\mathscr{K}$. For the horizontal velocity of the mass:

$$\dot{x} = -d\dot{\alpha}\mathscr{K}, \tag{5}$$

Writing the kinetic coenergy and the potential energy:

$$T = \frac{j_m\dot{\alpha}^2}{2} + \frac{m_c}{2}(-d\dot{\alpha}\mathscr{K})^2, \qquad V = 0. \tag{6}$$

[Step 4: Magnetic coenergy and electric energy]

For the magnetic coenergy of the system:

$$\mathscr{W}_m = \frac{l\dot{q}^2}{2} + \mathscr{W}^*, \qquad \mathscr{W}_m = \frac{l\dot{q}^2}{2} + k_e\dot{q}\alpha. \tag{7}$$

For the electric potential energy of the system:

$$\mathscr{W}_e = 0. \tag{8}$$

[Step 5: Lagrangian for electromechanical systems]

A formulation for electromechanical system can be developed (Bishop, 2008):

$$\mathscr{L} = T - V + \mathscr{W}_m - \mathscr{W}_e, \tag{9}$$

$$\mathscr{L} = \frac{j_m \dot{\alpha}^2}{2} + \frac{m_c}{2}(-d\dot{\alpha}\mathscr{K})^2 + \frac{l\dot{q}^2}{2} + k_e \dot{q}\alpha. \tag{10}$$

[Step 6: Generalized forces]

For the mechanical coordinate α and the electrical coordinate q:

$$Q_1 = -b_m \dot{\alpha}, \qquad Q_2 = \upsilon - r\dot{q}. \tag{11}$$

[Step 7: Lagrange's equations]

$$\frac{d}{dt}\left(\frac{\partial \mathscr{L}}{\partial \dot{\alpha}}\right) - \frac{\partial \mathscr{L}}{\partial \alpha} = Q_1, \tag{12}$$

$$\frac{d}{dt}\left(\frac{\partial \mathscr{L}}{\partial \dot{q}}\right) - \frac{\partial \mathscr{L}}{\partial q} = Q_2. \tag{13}$$

For equations 12 and 13, respectively:

$$\ddot{\alpha}[j_m + m_c d^2 \mathscr{K}^2] + \dot{\alpha}[b_m + m_c d^2 \mathscr{K}\dot{\mathscr{K}}] - k_e \dot{q} = 0 \tag{14}$$

$$l\ddot{q} + r\dot{q} + k_e \dot{\alpha} = \upsilon. \tag{15}$$

Scotch-yoke mechanism

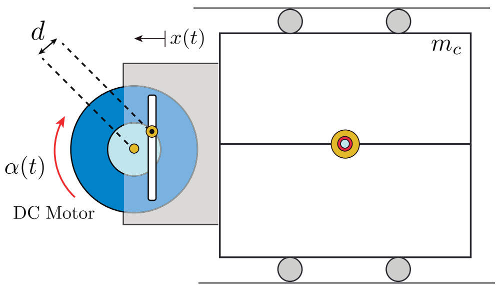

Figure 3 – Electromechanical system with a scotch-yoke mechanism.

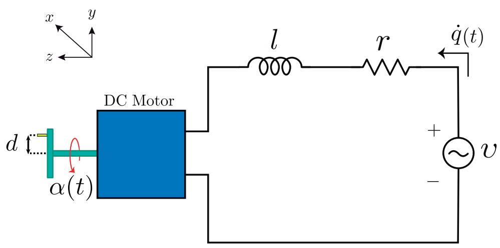

Figure 4 – DC motor

For the system coupled by a scotch-yoke mechanism, the equations are almost the same. This system differs from the slider-crank in the choice of coupling mechanism. It has been an object of study to analyze the behavior of electromechanical systems, see (Lima and Sampaio, 2016, 2012). They have the same electromechanical coupling. By changing the function $\mathscr{K}$ to

$$\mathscr{K} = sin(\alpha), \tag{16}$$

the equations for the scotch-yoke system are obtained from the Lagrangian:

$$\mathscr{L} = \frac{j_m \dot{\alpha}^2}{2} + \frac{m_c}{2}(-d\dot{\alpha}sin(\alpha))^2 + \frac{l\dot{q}^2}{2} + k_e \dot{q}\alpha. \tag{17}$$

The equations are:

$$\ddot{\alpha}[j_m + m_c d^2 sin^2(\alpha)] + \dot{\alpha}[b_m + m_c d^2 sin(\alpha)cos(\alpha)\dot{\alpha}] - k_e \dot{q} = 0, \tag{18}$$

$$l\ddot{q} + r\dot{q} + k_e \dot{\alpha} = \upsilon. \tag{19}$$

CONCLUSIONS

Many references claim to write a proper Lagrangian for electromechanical systems, please see (Avanço et al., 2017; Cveticanin et al., 2018) by adding a torque, said to be imposed, to the generalized forces in the Lagrangian's equations. This paper shows this is not correct and explains the mistake. The torque added wrongly in the equations is arbitrary, it depends only on the coordinates of the system and it must not be added as a generalized force. The standard derivations of the dynamical equation of purely mechanical systems do not apply in this situation. Also, the derivation of a Lagrangian that only provides mechanical terms of an electromechanical system is already an error. There must be only one Lagrangian to describe the whole system. As an example of a wrong method, take this solution proposed by the recent literature (Avanço et al., 2018) for the electromechanical system with a slider-crank mechanism analysed in this paper:

$$Q_1 = M_{motor}(\dot{q}, \dot{\alpha}), \qquad M_{motor}(\dot{q}, \dot{\alpha}) = \tau_e(\dot{q}) - b_m \dot{\alpha}. \tag{20}$$

In (Avanço et al., 2018), the torque M_{motor} is given by equations (20) and (21) (the equation numbers in this paragraph correspond to (Avanço et al., 2018)), so as a solution of a coupled system because (20) cannot be solved alone. With the hypothesis made, one gets (22) and (23) uncoupling the system. Now M_{motor} is only function of the coordinates and the initial condition associated to (20) is lost. We stress the main point, when the hypothesis that the torque is only function of the generalized coordinates is made the initial condition is lost, the electrical system is uncoupled, and now depends of the mechanical system. This mistake is a common one and, unfortunately, it pervades the literature. The mistake has a bonus, if one uncouples the system it is easy to do a lot of *computations* since now one has only an ode to solve. The reference (Lima and Sampaio, 2018) shows the mistake clearly. Finally, note that the two Lagrangians are equal, with exception of the form of the $\mathscr{K}$ term, that amounts to the description of the geometry of coupling. When one discretises the two problems appears a gyroscopic term, a non-symmetric operator multiplying the velocity term, identical in the two problems. This means there is an exchange of energy between the subsystems and the non-existence of normal modes in the linearized equations. This point will be treated in detail in the paper.

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REFERENCES

Avanço R.H., Tusset A.M., Balthazar J.M., Nabarrete A., and Navarro H.A. On nonlinear dynamics behavior of an electro-mechanical pendulum excited by a nonideal motor and a chaos control taking into account parametric errors. Journal of the Brazilian Society of Mechanical Sciences and Engineering, 40:23, 2018.

Bishop R.H. Mechatronic systems sensors and actuators-fundamentals and modelling, The mechatronics handbook. Boca Raton, FL: CRC Press Inc, 2008.

Cveticanin L., Zukovic M., and Balthazar J.M. Dynamics of mechanical systems with non-ideal excitation. Springer, 2018.

Dantas M., Sampaio R., and Lima R. Asymptotically stable periodic orbits of a coupled electromechanical system. Nonlinear Dynamics, 78(1):29-35, 2014.

Dantas M.J.H., Sampaio R., and Lima R. Existence and asymptotic stability of periodic orbits for a class of electromechanical systems: a perturbation theory approach. Zeitschrift für angewandte Mathematik und Physik, 67(1):2, 2016.

Haas W., Schlacher K., and Gahleitener R. Modeling of electromechanical systems. Johannes Kepler University Linz, Department of Automatic Control and Control Systems Technology, Preliminary Version, 13, 2000.

Hughes A. and Drury W. Electric motors and drives: fundamentals, types and applications. Newnes, 2013.

Jeltsema D. and Scherpen J.M. Multidomain modeling of nonlinear networks and systems. IEEE Control Systems, 29(4), 2009.

Lima R. and Sampaio R. Stochastic analysis of an electromechanical coupled system with embarked mass. Mecânica Computacional, 31:2709-2733, 2012.

Lima R. and Sampaio R. Two parametric excited nonlinear systems due to electromechanical coupling. Journal of the Brazilian Society of Mechanical Sciences and Engineering, 38(3):931-943, 2016.

Lima R. and Sampaio R. Pitfalls in the dynamics of coupled electromechanical systems. Technical Report, PUC-Rio, 2018.

Wells D.A. Schaum's outline of theory and problems of Lagrangian dynamics: with a treatment of Euler's equations of motion, Hamilton's equations and Hamilton's principle. McGrawHill, 1967.