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A Comparative Study of a Vibro-Impact System with Linear and Non-linear Bit-Rock Interactions

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Abstract: This paper presents a comprehensive numerical study of a higher order drifting oscillator. This system has been applied to model vibro-impact drilling dynamics by Ajibose et al. (2010, 2012, 2015) and Liao et al. (2016, 2017). We focus on the study of the bit-rock interactions, for which both linear and nonlinear models of the drilled medium are considered. Based on path-following methods, two-parameter bifurcation analyses are implemented via a software package, COCO (Dankowicz and Schilder, 2013). The analyses considered the excitation frequency, amplitude of excitation and the static force as the main control parameters, while the rate of penetration (ROP) was chosen as the main system output so as to assess the performance of the system when linear and nonlinear bit-rock impact models are used.

Keywords: Vibro-impact drilling; Bit-rock interaction; Nonsmooth dynamical system; COCO

INTRODUCTION

The development of innovative drilling techniques has attracted much attention during the past years, with studies focusing on both mathematical modeling and experimental research. In particular, special attention has been given to the study of rock deformation under dynamic impact, which can significantly improve the drilling efficiency. A number of experiments revealed that the elastic deformation of the rock formation is sustained for a certain period, until the applied load exceeds a threshold, which is dependent on the indenter's shape and size (Ajibose, Wiercigroch and Akisanya, 2015). In addition, the resulting penetration rate-load curves showed a linear dependence, except for the beginning of the loading phase and the end of the unloading phase. In order to simulate elasto-plastic rock deformation, an approach is proposed by Wiercigroch's group (Ajibose et al. 2010,2012), they identified two clear regimes during the bit-rock interactions, namely, the loading and the unloading phases, which are characterized by different physical phenomena. In the loading phase, part of the input kinetic energy is dissipated due to friction, while the rest of the energy is stored as elastic strain energy. During the unloading phase, the stored elastic strain energy is released to recover a part of the rock indentation. The difference in the energy dissipation is reflected by the difference between the loading and unloading stiffness coefficients. Based on the previous work, we will focus on comparing the linear and nonlinear bit-rock interaction models in a vibro-impact drilling system.

MATHEMATICAL MODELING

Fig. 1 (a) presents the physical model of the vibro-impact drilling system (Mukthar, 2015) to be analyzed in the present work. The model consists of a mass m_2 that represents the drill-bit assembly is excited by a sinusoidal force with amplitude F_a and frequency Ω. This mass is connected to the mass m_1 which stands for the drill-string components above the drill-bit assembly. F_b accounts for the static force applied to m_1 in drilling direction, and m_1 and m_2 are coupled via a spring with stiffness k and a damper with coefficient c. X_1, X_2, and X_3 stand for the absolute positions of m_1, m_2, and the plate of the slider, respectively. G is the initial gap between the bit and the rock surface represented by the plate of the slider. The interactions between the drill-bit and the rock surface can be modeled using the linear (solid lines) or nonlinear (dash lines) interaction models as shown in Fig. 1 (b), where X_p, X_f, X^*, and F^* are the initial position of the drill bit when it contacts rock surface, the end position of the drill bit when it leaves rock surface, the maximal bit penetration in drilled medium, and the maximal bit-rock force, respectively. It should be noted that the end positions of the linear and nonlinear interaction models, X_f^{lin} and X_f^{nlin}, when the drill bit leaves the rock medium, are different.

In particular, we assume that the energy dissipations of both models for an entire interaction are equivalent, i.e. the light grey area shown in Fig. 1 (b) equals to the dark grey one. The energy dissipation of the linear bit-rock model can be calculated as,

$$E_{lin} = \frac{1}{2}k_l^{lin}(X^* - X_p)^2 - \frac{1}{2}k_u^{lin}(X^* - X_f^{lin})^2,$$

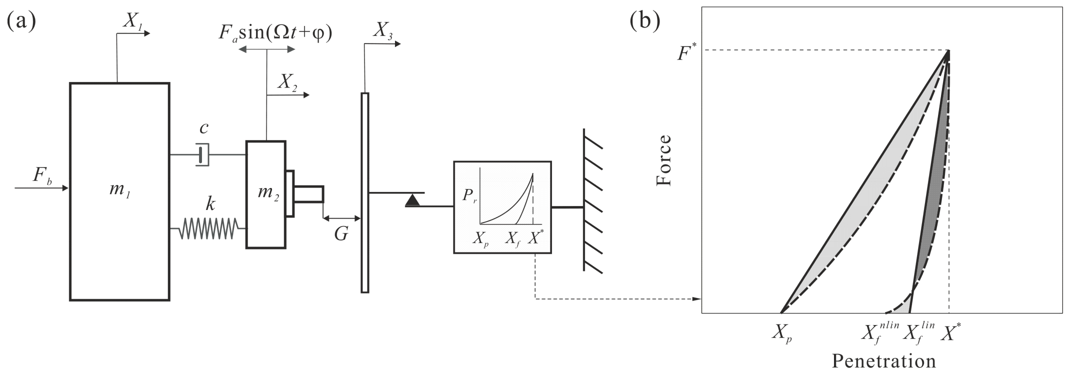

Figure 1 – (a) Physical model of the vibro-impact drilling system (Mukthar, 2015). (b) Linear (solid lines) and nonlinear (dash lines) bit-rock interaction models.

where k_l^{lin}, k_u^{lin} are the loading and unloading stiffnesses of the linear model. The energy dissipation for the nonlinear bit-rock model can be calculated as,

$$E_{nlin} = \int_{X_p}^{X^*} k_l^{nlin}(X - X_p)^{n_l}\,\mathrm{d}X - \int_{X_f^{nlin}}^{X^*} k_u^{nlin}(X - X_f^{nlin})^{n_u}\,\mathrm{d}X,$$

where k_l^{nlin}, k_u^{nlin} are the loading and unloading stiffnesses of the nonlinear model. For a complete bit-rock interaction, if the energy dissipations of these two models are equivalent, it gives

$$E_{lin} = E_{nlin}. \tag{1}$$

In Eq. (1), according to the material experiments introduced in (Liao et al. 2016), X_p, X_f^{nlin}, X^*, and F^* can be measured, k_l^{lin}, k_l^{nlin}, k_u^{nlin}, n_l, and n_u can be determined using experimental data, and X_f^{lin} and k_u^{lin} can be calculated using

$$F^* = k_u^{lin}(X^* - X_f^{lin}). \tag{2}$$

Therefore, by adopting this approach, all the bit-rock interactive parameters for the linear and nonlinear models can be obtained. A detailed mathmatical modeling of this system can be found in (Liao et al. 2016, 2017).

TWO-PARAMETER CONTINUATION

The two-parameter continuation should start from a confirmed codimension-one bifurcation, such as saddle-node bifurcation and period-doubling bifurcation. Generally, codimension-one bifurcations can be explored via numerical integration or one-parameter continuation. The relevant investigations for this high order drifting oscillator have been comprehensively studied (Liao et al. 2016, 2017). Therefore, in this work, we will omit the exploration process of codimension-one bifurcations, and directly focus on the two-parameter continuation of the drifting oscillator. By using COCO, the obtained traces of codimension-one bifurcations are extended in two-parameter spaces; as shown in Fig. 2 (a)-(d), the orange and green bifurcation curves are obtained by following the saddle-node and period-doubling bifurcations, respectively.

For the linear interaction model, the results of the two-parameter continuations with respect to (ω, a) and (ω, b) are shown in Fig. 2 (a) and (b), and the results for the nonlinear one are shown in Fig. 2 (c) and (d), where purple dots indicate the grazing points that terminate two-parameter continuations. It is worth noting that at these points, the bit-rock impact regime is changed between one impact and multiple impacts. As demonstrated in subplots (e) and (f), when the grazing points approach, the no-contact phase disappears gradually, and the bit experiences the loading and unloading phases iteratively.

As can be seen in Fig. 2, the structures of the corresponding two-parameter curves are similar. Comparing subplots (a) and (c), the two-parameter curve of saddle-node bifurcation (orange) for the nonlinear model exists in a larger parameter range than the curve obtained by the linear model. For the two-parameter curve of period-doubling bifurcation (green), the nonlinear model exists until $(\omega = 2.4343, a = 0.0420)$ as ω increases, while the one for the linear model terminates at $(\omega = 2.0530, a = 0.0310)$. When ω decreases, the green curve for the nonlinear model ends at $(\omega = 0.1276, a = 0.3970)$, but the one for the linear model can extend further. Comparing subplots (b) and (d), the saddle-node curve (orange) for the nonlinear model ends at $(\omega = 1.1585, a = 0.0841)$ as ω increases, while the curve for the linear model can extend further.

In addition, an isola of period-doubling bifurcations (David eta al. 1982) is observed in both Fig. 2 (b) and (d). There are two turning points (red dots) on the isola, which define the interval of existence of the isola. An illustration of the vibration conditions corresponding to the bottom and upper branches of the isola are shown in subplots (g) and (h), respectively. As presented in Fig. 2 (g), the bit experiences a long no-contact period followed by a short loading and unloading period. For the upper branch, a short no-contact period accompanying with a long loading and unloading period is recorded in Fig. 2 (h).

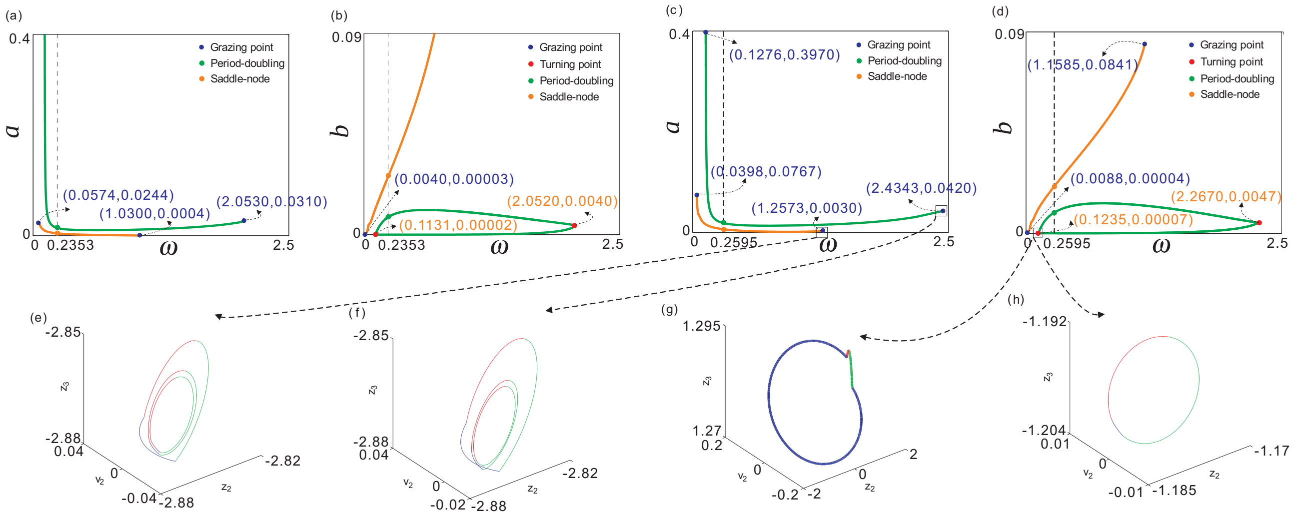

Figure 2 – Two-parameter bifurcation curves for (a, b) linear and (c, d) nonlinear models, the nondimensional parameters for subplots (a, b) are $\xi = 0.0172$, $\alpha = 0.0133$, $\beta = 0.0033$, $g = 0.02$, $n_1 = 1$, $n_u = 1$, and $\kappa = 1.1706$, and those for subplots (c, d) are $\xi = 0.0188$, $\alpha = 0.0133$, $\beta = 0.0040$, $g = 0.02$, $n_1 = 0.86$, $n_u = 1.31$, and $\kappa = 3.1815$.

RATE OF PENETRATION

The main purpose of studying the higher order drifting oscillator with the linear and nonlinear impact models is to investigate their rates of penetration for improving drilling efficiency. According to (Pavlovskaia, Hendry and Wiercigroch, 2015), the stable period-one response with one impact per period of excitation provides the best performance of vibro-impact drilling in terms of the rate of penetration (ROP), which is calculated as,

$$ROP = \frac{1}{T}[z_f - z_p],$$

where T is the excitation period.

Fig. 3 presents all the calculated ROPs for the higher order drifting oscillator with the linear and nonlinear impact models by varying different control parameters, ω, a, and b. As can be seen from Fig. 3 (a) and (b), the stable period-one response exists in $\omega \in [0.1449, 0.3064]$ for the linear model, and the stable period-one response for the nonlinear model exists in $\omega \in [0.1270, 0.3795]$. The maximum ROP, $ROP = 6.07 \times 10^{-4}$, for the linear model achieves at $\omega = 0.2$, and the maximum ROP for the nonlinear model obtained at $\omega = 0.1270$ is $ROP = 19.57 \times 10^{-4}$. As the amplitude of excitation increases, the ROPs for both models increase, and the maximum ROP is achieved at $a = 0.0157$ for the linear model and $a = 0.0213$ for the nonlinear one. The stable period-one responses are recorded for $b \in [0.0077, 0.0257]$ for the linear model and $b \in [0.0088, 0.0203]$ for the nonlinear model with the maximum ROP obtained at $b = 0.00257$ and $b = 0.0203$, respectively.

CONCLUDING REMARKS

In this presented work, our study presented the two-parameter continuation of period-doubling and saddle-node bifurcations. As could be observed from Fig. 2, the resulting two-parameter bifurcation picture showed clear similarities in qualitative terms. The precise numerical values and parameter ranges, however, present differences which are nevertheless not significant. A remarkable common feature of the bifurcation picture was the presence of an isola of period-doubling bifurcations of limit cycles in the ω-b plane, which indicates that this dynamical phenomenon is robust and has its roots in the main structure of the mathematical model. Similarly, the transition from periodic regimes with one impact per period to periodic motions with multiple impacts via grazing phenomena could be detected for both linear and nonlinear bit-rock interaction models.

Another feature that was investigated in the present work was the behavior of the ROP as the system parameters are perturbed. It was found that for the nonlinear model the ROP is more sensitive to variations of the frequency of excitation, as small changes in this parameter produced large fluctuations in the ROP, which can also lead to significant drops in the

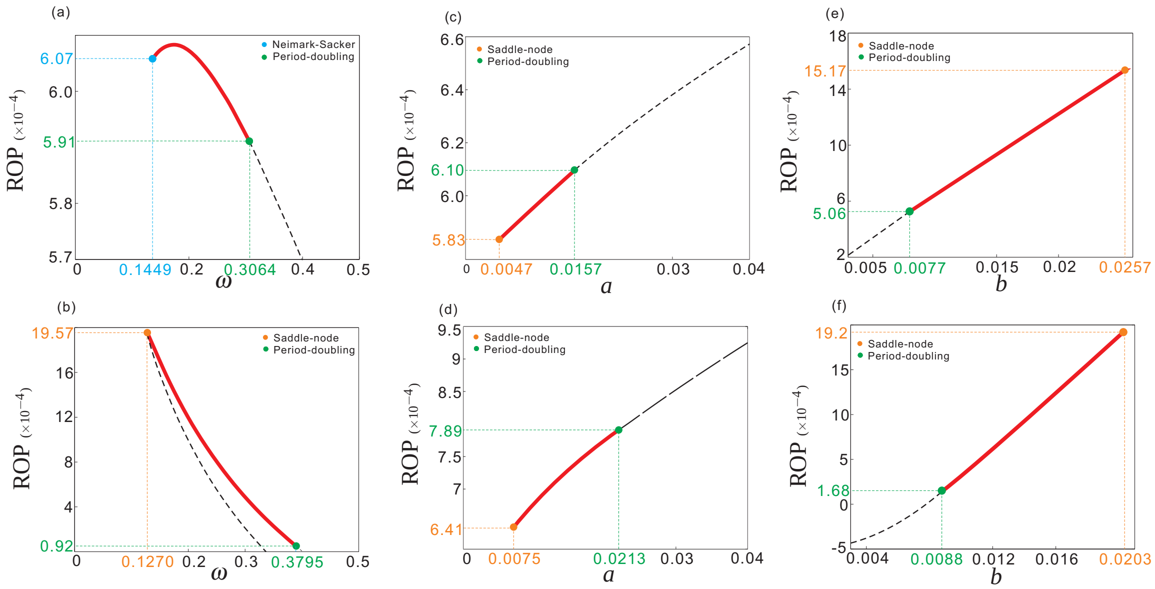

Figure 3 – Variations of penetration rates for period-one one-impact responses. Subplots (a), (c), (e) show the penetration rates of the linear model against frequency, amplitude, and static force, respectively. Subplots (b), (d), (f) show the penetration rates of the nonlinear model against frequency, amplitude, and static force, respectively. In subplots, the solid red segments represent the stable branches, while the dashed black segments show the obtained unstable branches.

drilling speed. Similarly, the amplitude of excitation and the static force presented a stronger influence on the ROP when the nonlinear law is used. Therefore, for the parameter ranges considered, our numerical investigation indicates that the linear and nonlinear bit-rock impact models can produce qualitatively similar system behavior, provided the higher order drifting oscillator operates under low frequencies of excitation, small excitation amplitude or small static forces. In these cases, no relevant differences can be observed during the system operation. However, if the parameter values do not meet these conditions, the dynamics of the higher order drifting oscillator can differ significantly, and further studies should be conducted in order to determine what type of bit-rock interaction model better describes the vibro-impact drilling process.

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