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Exploring the nonlinear dynamics of bistable energy harvester

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EXTENTED ABSTRACT

Recent technology advances and increasing power demands have been encouraging renewable energies research, among which are the energy harvesting technologies (Pfenniger et al., 2014), (Spies, Pollak and Mateu, 2015). Energy harvesting devices are able to support low power applications such as boarded electrical circuits and sensors, medical electrical implants (e.g. pace-markers) and general low-power-consumption electrical equipment far from power supply, for example. Its concept consists of an electromechanical device able to capture environmental energy, available from natural or artificial sources as heat, pressure differences, vibration, etc, and converting it into an electrical potential. As nonlinearities may improve energy harvesting processes gains (Cottone, Vocca and Gammaitoni, 2009), this subject has been the object of study of recent works by Leite et al (2016), Peterson, Lopes and Cunha (2017), Lopes, Peterson and Cunha (2017) and Lopes, Peterson and Cunha (2018).

Motivated by necessity of investigate in deep the nonlinear phenomena underlying new energy harvesting systems, this work aims to characterize in detail, by means of bifurcation diagrams and attraction basins, a nonlinear bistable dynamical system associated to the electromechanical oscillator presented in Erturk, Hoffmann and Inman (2009). This characterization is necessary not only to have a better understanding of the basic physics underlying the energy harvesting system, but also to allow the optimization of its performance.

The energy harvesting system addressed in this work is the piezo-magneto-elastic beam schematically represented in Figure 1, proposed by Erturk, Hoffmann and Inman (2009). It consists on a cantilever slim ferromagnetic beam, attached to a rigid base driven by a harmonic forcing, which vibrates in a nonlinear regime due to the presence of a pair of magnets at the bottom of the base. On the top of this beam there is a piezoelectric transducer, which produces electrical potential when activated by mechanical pressure, that couples this mechanical structure to a resistive electrical circuit.

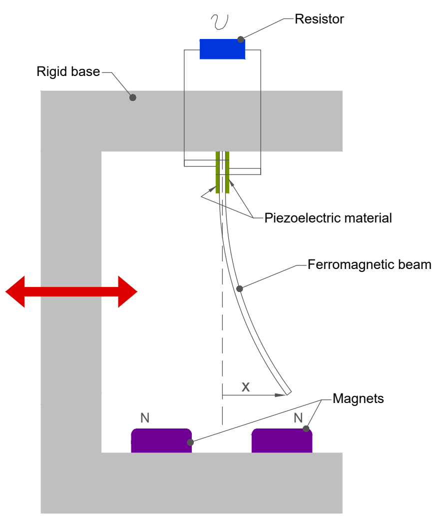

Figure 1 – Schematic representation of the piezo-magneto-elastic energy harvesting system.

The behavior of this dynamical system model evolves according to

$$\ddot{x} + 2\,\xi\,\dot{x} - 0.5\,x\,(1 - x^2) - \chi\,v = f\,\cos(\Omega t), \tag{1}$$

$$\dot{v} + \lambda\,v + \kappa\dot{x} = 0, \tag{2}$$

where f and Ω represent harmonic forcing amplitude and frequency, respectively, ξ and κ are the mechanical and electrical piezo-electric coupling terms, ksi is the damping ration and $lambda$ is the time reciprocal constant. T beam tip displacement and velocity are given respectively by x and $\dot{x}$, while output voltage is denoted as v.

Bifurcation diagrams are presented on Figure 2. Slices represents diagrams for system dynamics with excitation amplitudes uniformly incremented of 0.019, from 0.019 to 0.275. Frequencies on each slice are regularly sampled between 0.1 and 1.4, both on forward and backward modes. Forward one considers Ω incremental sampling and is depicted by blue colors scale, while backward, a decremental, been shown on red colors scale. Chaotic dynamics are depicted by smudgy diagrams regions, more evident on higher amplitudes specially for frequencies from 0.1 to 0.6, but also present on mean amplitudes about the second half of frequency control interval. Multi-periods regions also may be identified close to chaotic areas both after and before blurred regions or even between those non-regularities, what is featured in detail with the slice for an excitation amplitude of 0.147, shown on the right. A noisy region, filled with discontinuities both on forward and backward diagrams is also noted for lower frequency values, comprised between 0.1 and 0.4.

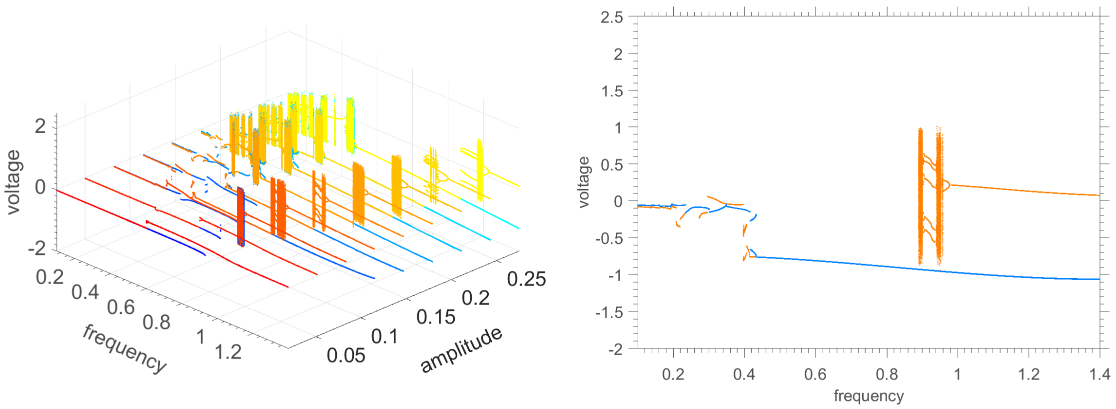

Figure 2 – Forward (blue scale) and backward (red scale) voltage bifurcation diagrams, in the left, with $0.1 \le \Omega \le 1.4$ and f from 0.019 to 0.275. The diagram that corresponds to $f = 0.147$ is shown in the right.

A closer investigation of this discontinuities region by means of voltage time series, with excitation frequencies $\Omega = 0.2941$ and $\Omega = 0.3711$, reveals a regular dynamical behavior such as shown in Figure 3. Non-chaotic oscillation period is clearly identified both on forward and on backward time series, unlike what may be seen for $\Omega = 0.9002$, presented on the same figure. In this conditions, backward analysis shows a non-periodic oscillation.

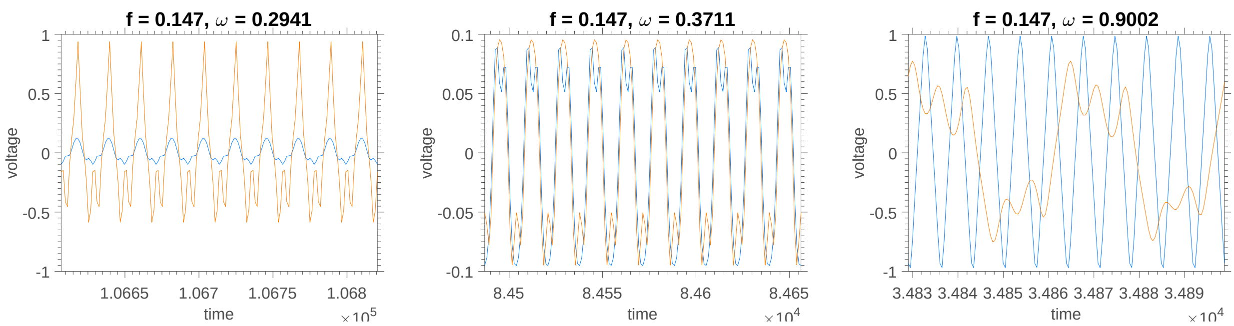

Figure 3 – Different time series of voltage associated to the forward (blue) and backward (orange) bifurcation diagrams with $f = 0.147$; $\Omega = 0.2941$ (left), $\Omega = 0.3711$ (center) and $\Omega = 0.9002$ (right).

In Figure 4, the basin of attraction corresponding to $\Omega = 0.8$ and $f = 0.051$ and their respective attractors are shown. It is possible to see the presence of six attractors, depicted in green, blue, cyan, red, magenta and a chaotic one in grey. The sensitivity to initial conditions is clear, although there are some zones where the behavior is well delimited, as in the green, red and blue zones. Is important to note that the green attractor is the most energetic, as it has the wider range both in displacement and velocity.

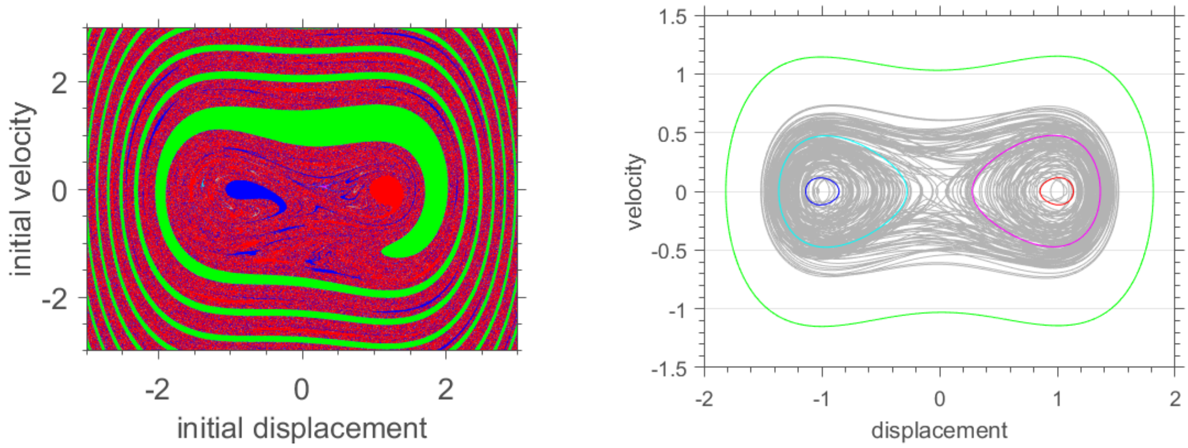

Figure 4 – Basin of attraction corresponding to $\Omega = 0.8$ and $f = 0.051$ (left) and their respective attractors (right).

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