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Flexural Wave Band Gaps in Phononic Crystal Thick Plates with Defects

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Abstract: Much attention has been paid to mechanical wave propagation in phononic crystals. Phononic crystals are artificial composites consisting of a periodic array of inclusions embedded in a matrix. Due to its periodicity, Bragg-type band gaps are opened up and there are no propagating mechanical waves in these ranges of frequency. There are only evanescent waves in these ranges of frequency. We investigate the band structure of flexural waves propagating in an elastic phononic crystal thick plate with defects. We consider point defect and straight, bending and branching linear defects. The classical Mindlin-Reissner theory of thick plates is assumed. The phononic crystal thick plate consists of an epoxy matrix with Al_2O_3 inclusions in a square lattice. We also study the influence of inclusion geometry – circular, hollow circular, square and rotated square. The improved plane wave expansion method with supercell technique is used to calculate the phononic band structure. Three defect bands are created when the radius of the point defect becomes close to zero. The straight linear defect can be applied as high efficiency waveguide, whereas bending and branching linear defects act as waveguides or filters depending on the range of frequency.

Keywords: phononic crystal, Mindlin-Reissner theory, Bragg-type band gaps, improved plane wave expansion, supercell technique, waveguide, filter.

INTRODUCTION

Phononic crystals (PnCs) are artificial periodic composites designed to exhibit mechanical band gaps, where mechanical (elastic or acoustic) wave propagation is forbidden and there are only evanescent waves. They have been quite studied in recent years (Huang, Zhang and Chen, 2014; Lu et al., 2017; Cheng, Shi and Mo, 2018). Phononic band gaps are created by the periodically mismatch between the constituent materials. This mismatch can be considered to arise either from difference of material properties or geometry (continuum-scale theory), or from interatomic force constants and masses (atomic-scale theory).

The ability of creating phononic band gaps is similar to the electronic and photonic band gaps in semiconductors and photonic crystals, respectively. The physical origin of phononic and photonic band gaps can be understood at micro-scale using the classical wave theory to describe the Bragg and Mie resonances based on the scattering of mechanical and electromagnetic waves propagating within the crystal (Olsson III and El-Kady, 2009).

PnCs have many applications, such as vibration isolation technology (Jensen, 2003; Wang, Wen and Wen, 2005; Casadei et al., 2012), acoustic barriers/filters (Ho et al., 2003; Qiu et al., 2005; Yang et al., 2010), noise suppression devices (Casadei et al., 2010; Xiao, Wen and Wen, 2012) and surface acoustic devices (Benchabane et al., 2006).

Yao et al. (2009) studied the band structures of bending waves in a PnC thin plate with a point of defect using the improved plane wave expansion (IPWE) method combined with the supercell technique. The bending waves are highly localized at or near the defect, resulting in defect modes. They observed that the frequency and number of the defect modes are strongly dependent on the filling fraction and the size of the point defect. Feng et al. (2015) proposed a methodology for continuous tuning of confined line defect modes in a two-dimensional (2D) PnC. The line defect was considered as a free vibrating plate, and separated from the surrounding phononic structure. They illustrated that different resonant modes, i.e. flexural and extensional modes, have different relationships with the width of the line defect. Furthermore, the unwanted resonant mode could be excluded from the phononic band gaps by carefully selecting the defect width.

The main purpose of this study is to investigate the band structure, also known as dispersion diagram, of a PnC thick plate with defects. We regard point defect and straight, bending and branching linear defects, similar to the studies of Yao et al. (2009) and Yao et al. (2010), respectively. However, Yao et al. (2009) and Yao et al. (2010) assumed the simple Kirchhoff-Love (Kirchhoff, 1850; Love, 1888) thin plate theory, whereas we regarded Mindlin-Reissner (Mindlin, 1951a, Reissner, 1945) thick plate theory. The PnC thick plate is composed by Al_2O_3 inclusions distributed in a square lattice. We also study the influence of inclusion geometry – circular, hollow circular, square and rotated square. The IPWE approach with supercell technique is used to calculate the band structure.

PHONONIC CRYSTAL THICK PLATE MODELING

Figure 1 (a) sketches the cross section of the PnC thick plate composed by Al_2O_3 inclusions with arbitrary inclusion geometry embedded in an epoxy matrix taking into account square lattice. The figures of the PnC thick plate with defects will be presented on the complete paper, since there is the limitation of the number of pages in this extended abstract. Figure 1 (b) represents the first irreducible Brillouin (Brillouin, 1946) zone (FIBZ) for square lattice. We consider four types of Al_2O_3 inclusion, i.e., circular, hollow circular, square and rotated square with a 45° angle of rotation with respect to the *x*, *y* axes.

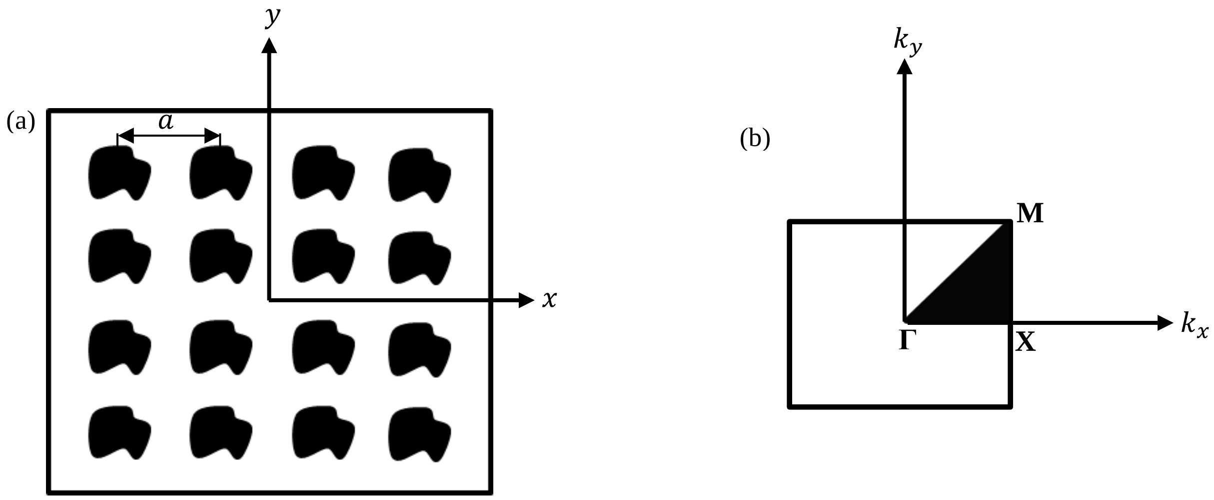

Figure 1 – Transverse cross-section of the phononic crystal thick plate: an array of inclusions (Al_2O_3) periodically distributed in an epoxy matrix for square lattice (a). First irreducible Brillouin zone in shaded region for square lattice (b).

The points of the FIBZ for square lattice in Fig. 1 (b) are Γ (0,0), X $\left(\frac{\pi}{a}, 0\right)$ and M $\left(\frac{\pi}{a}, \frac{\pi}{a}\right)$, where a is the lattice parameter. From Mindlin-Reissner theory (Mindlin, 1951a, Reissner, 1945), considering the isotropic case, the plate equations of motion are (Mindlin, 1951b):

$$-\alpha\left(\Psi_x + \frac{\partial w}{\partial x}\right) + \frac{\partial}{\partial x}\left(D\frac{\partial \Psi_x}{\partial x} + \beta\frac{\partial \Psi_y}{\partial y}\right) + \frac{\partial}{\partial y}\left(\gamma\frac{\partial \Psi_y}{\partial x} + \gamma\frac{\partial \Psi_x}{\partial y}\right) = \delta\frac{\partial^2 \Psi_x}{\partial t^2}, \tag{1a}$$

$$-\alpha\left(\Psi_y + \frac{\partial w}{\partial y}\right) + \frac{\partial}{\partial x}\left(\gamma\frac{\partial \Psi_y}{\partial x} + \gamma\frac{\partial \Psi_x}{\partial y}\right) + \frac{\partial}{\partial y}\left(D\frac{\partial \Psi_y}{\partial y} + \beta\frac{\partial \Psi_x}{\partial x}\right) = \delta\frac{\partial^2 \Psi_y}{\partial t^2}, \tag{1b}$$

$$\frac{\partial}{\partial x}\left[\alpha\left(\Psi_x + \frac{\partial w}{\partial x}\right)\right] + \frac{\partial}{\partial y}\left[\alpha\left(\Psi_y + \frac{\partial w}{\partial y}\right)\right] = \tau\frac{\partial^2 w}{\partial t^2}, \tag{1c}$$

where $\alpha = \alpha(\mathbf{r}) = \kappa^2\mu h$, $\mathbf{r} = x\mathbf{e}_1 + y\mathbf{e}_2$ is the two-dimensional spatial vector, $\mathbf{e}_i$ $(i = 1,2)$ are the basis vectors in the real space, $\kappa^2 = \frac{\pi^2}{12}$, μ is the shear modulus, h is the plate thickness, Ψ_x=$\Psi_x(\mathbf{r}, t)$ is the local rotation on the x direction, $D = D(\mathbf{r}) = \frac{Eh^3}{12(1-\nu^2)}$, E is the Young's modulus, ν is the Poisson's ratio, $\beta = \beta(\mathbf{r}) = D\nu$, Ψ_y=$\Psi_y(\mathbf{r}, t)$ is the local rotation on the y direction, $\gamma = \gamma(\mathbf{r}) = \frac{D(1-\nu)}{2}$, $\delta = \delta(\mathbf{r}) = \frac{\rho h^3}{12}$, ρ is the density, $\tau = \tau(\mathbf{r}) = \rho h$ and w=$w(\mathbf{r}, t)$ is the displacement on z direction.

IPWE method is applied to Eq. (1) and a generalized eigenvalue problem of $\omega^2(\mathbf{k})$ should be solved for each $\mathbf{k}$ into the FIBZ. This mathematical formulation is not discussed because the limitation of the number of pages. However, this formulation has also been reported by Miranda Jr. and Dos Santos (2016).

SIMULATED EXAMPLES

We present in this section only the previous results of the elastic band structure of the PnC thick plate without defects. We calculate the first ten bands of the elastic band structure assuming a fixed filling fraction of 0.283, thickness ($h = a$) and lattice parameter ($a = 0.022$ m) for the four inclusions considering square lattice. We use 289 plane waves for the Fourier series expansion on the reciprocal space. This resulted in a good convergence.

Figure 2 (a-d) shows the elastic band structures of the PnC thick plate with square lattice regarding the four inclusion geometries. We plot the elastic band structure in the three principal symmetry directions of the FIBZ. The plots are given in terms of frequency in Hz versus the reduced Bloch wave vector $\bar{\mathbf{k}} = \mathbf{k}a/2\pi$.

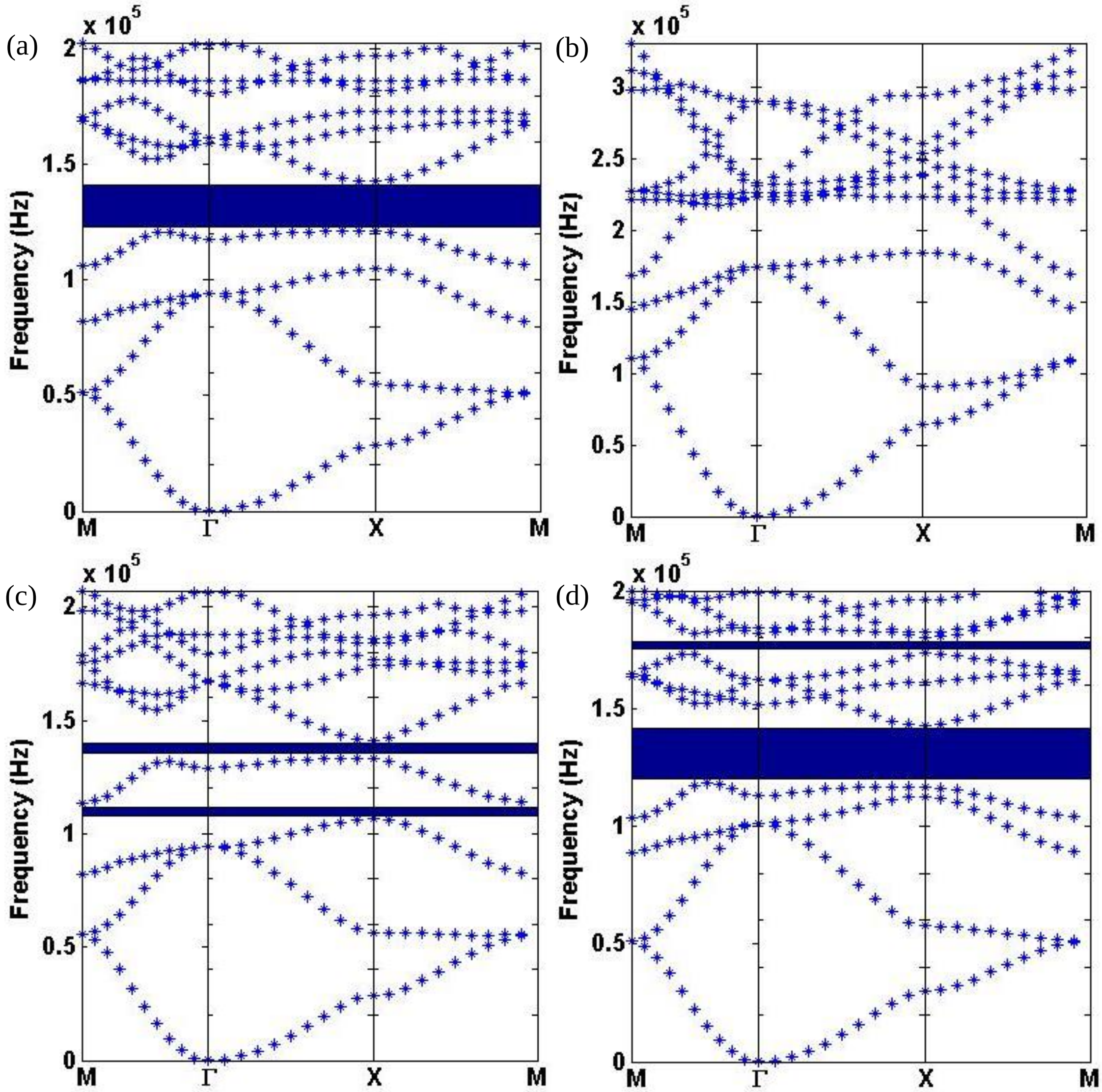

Figure 1 – Elastic band structures of a phononic crystal thick plate with square lattice. The following types of inclusions are considered: (a) circular; (b) hollow circular; (c) square; (d) rotated square with a 45° angle of rotation with respect to the *x*, *y* axes.

CONCLUSIONS

We obtain relatively broad band gaps for a PnC thick plate without defects, considering circular, hollow circular, square and rotated square inclusions in a square lattice. The rotated square inclusion presents the broadest band gap. The other relevant results not shown here because of the number of pages limitation will be discussed when the point and linear defects are included. Defect bands (up to three) can be created when the radius of the point defect varies. The linear defects can be applied as high efficiency waveguide or filters.

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REFERENCES

Benchabane, S., Khelif, A., Robert, L., Rauch, J.Y., Pastureaud, T. and Laude, V., 2006, “Elastic Band Gaps for Surface Modes in an Ultrasonic Lithium Niobate Phononic Crystal”, Proceedings SPIE, Vol.6182, No.618216, pp. 1-13.

Brillouin, L., 1946, “Wave Propagation in Periodic Structures”, Dover Publications, New York, USA.

Casadei, F., Dozio, L., Ruzzene, M. and Cunefare, K.A., 2010, “Periodic Shunts Arrays for the Control of Noise Radiation in an Enclosure, Journal of Sound and Vibration”, Vol.329, pp. 3632-3646.

Casadei, F., Beck, B.S., Cunefare, K.A. and Ruzzene, M., 2012, "Vibration Control of Plates through Hybrid Configurations of Periodic Piezoeletric Shunts", Journal of Intelligent Material Systems and Structures, Vol.23, No.10, pp. 1169-1177.

Cheng, Z.B., Shi, Z.F. and Mo, Y.L., 2018, "Complex Dispersion Relations and Evanescent Waves in Periodic Beams via the Extended Differential Quadrature Method", Composites Structures, Vol.187, pp. 122-136.

Feng, D., Xu, D., Xiong, B. and Wang, Y., "Continuous Tuning of Line Defect Modes in Silicon Two-Dimensional Phononic Crystal", Journal of Physics D: Applied Physics, Vol.48, No.225102, pp. 1-7.

Ho, K.M., Cheng, C.K., Yang, Z., Zhang, X.X. and Sheng, P., 2003, "Broadband Locally Resonant Sonic Shields", Applied Physics Letters, Vol.83, No.26, pp. 5566-5568.

Huang, Y., Zhang, C.L. and Chen, W.Q., 2014, "Elastic Wave Band Structures and Defect States in a Periodically Corrugated Piezoelectric Plate", Journal of Applied Mechanics, Vol.81, No.081005, pp. 1-6.

Jensen, J.S., 2003, "Phononic Band Gaps and Vibrations in One- and Two-Dimensional Mass-Spring Structures", Journal of Sound and Vibration, Vol.266, pp. 1053-1078.

Kirchhoff, G., 1850, "Über das Gleichgewicht und die Bewegung einer Elastischen Scheibe", Journal für die reine und Angewandte Mathematik, Vol.40, pp. 51-88.

Love, A.E.H., 1888, "The Small Free Vibrations and Deformation of a Thin Elastic Shell", Philosophical Transactions of the Royal Society A, Vol.179, pp. 491-546.

Lu, Y., Yang, Y., Guest, J.K. and Srivastava, A., 2017, "3-D Phononic Crystals with Ultra-Wide Band Gaps", Scientific Reports, Vol.7, No.43407, pp. 1-13.

Mindlin, R.D., 1951a, "Influence of Rotatory Inertia and Shear on Flexural Motions of Isotropic Elastic Plates", Journal of Applied Mechanics, Vol.18, pp. 31-38.

Mindlin, R.D., 1951b, "Thickness-Shear and Flexural Vibrations of Crystal Plates", Journal of Applied Physics, Vol.22, pp. 316-323.

Miranda Jr., E.J.P. and Dos Santos, J.M.C., 2016, "Flexural Wave Band Gaps in Al_2O_3/Epoxy Composite Rectangular Plate Using Mindlin Theory ", Proceedings of the 3rd Brazilian Conference on Composite Materials, Gramado, Brazil, pp. 1-8.

Olsson III, R.H. and El-Kady, I., 2009, "Microfabricated Phononic Devices and Applications", Measurement Science and Technology, Vol.20, No.012002, pp. 1-13.

Qiu, C.Y., Liu, Z.Y., Mei, J. and Shi, J., 2005, "Mode-Selecting Acoustic Filter by Using Resonant Tunneling of Two-Dimensional Double Phononic Crystals", Applied Physics Letters, Vol.87, No.104101, pp. 1-3.

Reissner, E., 1945, "The Effect of Transverse Shear Deformation on the Bending of Elastic Plates", Journal of Applied Mechanics, Vol.12, pp. A68-A77.

Wang, G., Wen, J.H. and Wen, X.S, 2005, "Quasi-One-Dimensional Phononic Crystals Studied Using the Improved Lumped-Mass Method: Application to Locally Resonant Beams with Flexural Wave Band Gap", Physical Review B, Vol.71, No.104302, pp. 1-5.

Xiao, Y., Wen, J. and Wen, X, 2012, "Sound Transmission Loss of Metamaterial-Based Thin Plates with Multiple Subwavelength Arrays of Attached Resonators", Journal of Sound and Vibration, Vol.331, pp. 5408-5423.

Yang, Z., Dai, H.M., Chan, N.H. and Ma, G.C, 2010, "Acoustic Metamaterial Panels for Sound Attenuation in the 50-1000 Hz Regime", Applied Physics Letters, Vol.96, No.041906, pp. 1-3.

Yao, Z.-J., Yu, G.-L., Wang, Y.-S. and Shi, Z.-F. 2009, "Propagation of Bending Waves in Phononic Crystal Thin Plates with a Point Defect", International Journal of Solids and Structures, Vol.46, pp. 2571-2576.

Yao, Z.-J., Yu, G.-L., Wang, Y.-S., Shi, Z.-F. and Li, J.-B., 2010, "Propagation of Flexural Waves in Phononic Crystal Thin Plates with Linear Defects", Journal of Zhejiang University-Science A (Applied Physics & Engineering), Vol.11, No.10, pp. 827-834.

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