

Flexible Multibody Simulations with Global Shape Functions Obtained from Isogeometric Finite Element Models

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Abstract:

Dynamic mechanisms, whose bodies undergo both large nonlinear rigid body motions and elastic deformations can be modeled and analyzed using the method of flexible multibody systems. In case the deformations are small and elastic the floating frame of reference formulation is often the most efficient modeling approach. The deformations are here approximated using a set of global shape functions. In this work it is shown how the global shape functions can be determined from isogeometric finite element models by modal reduction. As application example a planar flexible slider-crank mechanisms is used. Isoparametric and isogeometric finite element models are created for the flexible piston rod. The resulting eigenfrequencies and the results of the flexible multibody simulations are compared.

Keywords: *floating frame of reference, isogeometric finite element models, modal reduction*

INTRODUCTION

The method of flexible multibody systems allows the modeling of bodies, which undergo both large nonlinear rigid body motions and deformations. If the deformations are small and elastic the floating frame of reference formulation is often the most efficient modeling approach [3]. The key idea in this formulation is that the large nonlinear motion is described by a reference frame, while the small deformations are approximated using a set of global shape functions which are multiplied by time-dependent elastic coordinates.

A general way to determine an appropriate set of global shape functions is to firstly create a finite element model of the flexible body. Then in a second step the global shape functions can be derived from the finite element model applying model order reduction techniques and computing a set of standard data [2,4]. While this procedure is well-described, tested, and standardized for isoparametric finite element models there are only few results in which the global shape functions are derived from isogeometric finite element models. However, isogeometric analysis is a promising approach, which aims to unify the two worlds of geometric model representation and finite element analysis [1]. Besides the use in statics also the application in the modeling and analysis of dynamic flexible multibody systems is of great interest.

Therefore, using the example of a planar slider-crank mechanisms, it is shown in this work how global shape functions can be determined from isogeometric finite element models and be used in the multibody simulation to model a flexible piston rod. The results of the modal analysis and the flexible multibody simulations are compared with a classical isoparametric finite element model.

Floating Frame of Reference Formulation

In the floating frame of reference formulation, the overall motion of the flexible bodies is described by two sets of coordinates, see Fig. 1. The first set contains the position coordinates \mathbf{r}_{IR} and rotating parameters $\boldsymbol{\beta}_{IR}$, which give the absolute position and orientation of a reference frame K_R . The second set includes n_e elastic coordinates \mathbf{q}_e . They are multiplied by a set of global shape functions Φ to approximate the deformation \mathbf{u}_P of point P, which is located at \mathbf{c}_{RP} in the undeformed configuration. Therefore, the absolute position \mathbf{r}_{IP} of point P can be displayed as

$$\mathbf{r}_{IP} = \mathbf{r}_{IR} + \mathbf{c}_{RP} + \mathbf{u} = \mathbf{r}_{IR} + \mathbf{c}_{RP} + \Phi \mathbf{q}_e. \quad (1)$$

It is worth mentioning that all position vectors in Eq. (1) are given in the reference frame K_R .

The position, velocity, and acceleration of every point P can be expressed in terms of the position coordinates \mathbf{z}_I and velocity coordinates \mathbf{z}_{II} , which are defined as

$$\mathbf{z}_I = [\mathbf{r}_{IR} \quad \boldsymbol{\beta}_{IR} \quad \mathbf{q}_e]^T \quad \text{and} \quad \mathbf{z}_{II} = [\mathbf{v}_{IR} \quad \boldsymbol{\omega}_{IR} \quad \dot{\mathbf{q}}_e]^T. \quad (2)$$

Thereby, \mathbf{v}_{IR} , $\boldsymbol{\omega}_{IR}$, and $\dot{\mathbf{q}}_e$ are the velocity and the angular velocity of the reference frame, and the time derivative of the elastic coordinates.

Deriving the kinematics, i.e. deriving the velocity and acceleration of each point P of the flexible body, and employing a principle of mechanics the equations of motion of a single flexible body can be formulated. The equations of motion of

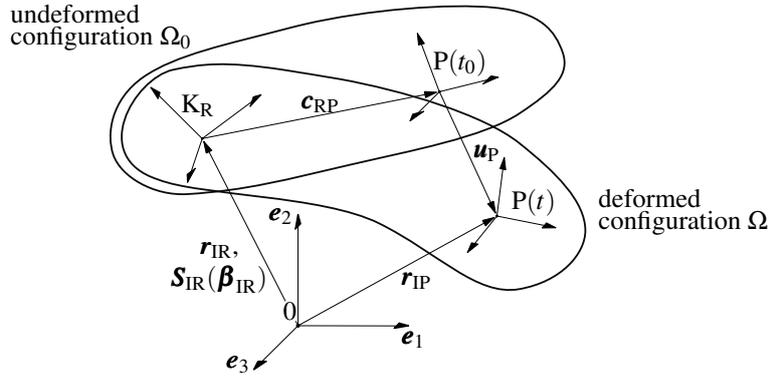


Figure 1 – Kinematics in the floating frame of reference formulation

all bodies together with the kinematic relation between the time derivatives of the position coordinates $\dot{\mathbf{z}}_I$ and the velocity coordinates \mathbf{z}_{II} and the constraint equations $\mathbf{c} \in \mathbb{R}^{n_c}$ form a system of differential-algebraic equations (DAE)

$$\begin{aligned} \dot{\mathbf{z}}_I &= \mathbf{Z}\mathbf{z}_{II}, \\ \mathbf{M}\dot{\mathbf{z}}_{II} - \mathbf{C}^T \boldsymbol{\lambda} &= \mathbf{f}(\mathbf{z}_I, \mathbf{z}_{II}, t), \\ \mathbf{c}(\mathbf{z}_I, t) &= \mathbf{0}. \end{aligned} \quad (3)$$

In Eq. (3) \mathbf{Z} is the kinematic matrix, \mathbf{M} is the global mass matrix, \mathbf{C} is the Jacobian of the constraint equations \mathbf{c} , $\boldsymbol{\lambda}$ is a set of Lagrange multipliers and \mathbf{f} is the right-hand-side vector, that comprises all inner forces, generalized inertia forces and applied loads. For the solution of Eq. (3) the index of the DAE is firstly reduced from three to one by differentiating the constraint equations twice. Then the resulting index 1 system can be solved by the *ode23t* MATLAB solver.

Isogeometric Analysis and Global Shape Functions

A major question using the floating frame of reference formulation is how to obtain the global shape function Φ , which are used to approximate the deformation of the flexible body. On the one hand, the number of global shape function should be as small as possible. On the other hand the selected shape functions must be able to precisely describe the actual deformations. A general way to determine Φ is to generate a finite element model of the flexible body and then to identify the global shape functions from the finite element model using model reduction techniques.

Using the example of the planar flexible piston rod the generation of the isogeometric finite element model is briefly explained in the following. Thereby, a single *patch* is used to model the piston rod. The NURBS shape functions, which are used to describe the geometry and to perform the analysis, are defined on the parameter space, see Fig. 2. The 2D parameter space is divided into *elements* by the *knot vectors*

$$\boldsymbol{\Xi} = [\xi_1 \quad \xi_2 \quad \dots \quad \xi_{n+p+1}] \quad \text{and} \quad \mathbf{H} = [\eta_1 \quad \eta_2 \quad \dots \quad \eta_{m+q+1}]. \quad (4)$$

Thereby, p and q are the polynomial orders and n and m are the numbers of the basis functions $N_{i,p}$ and $M_{j,q}$ which are used to define the basis splines. The basis functions $N_{i,p}$ and $M_{j,q}$ can be computed recursively using the Cox-de Boor algorithm

$$\begin{aligned} p = 0: \quad N_{i,0}(\xi) &= \begin{cases} 1 & \text{if } \xi_i \leq \xi < \xi_{i+1} \\ 0 & \text{otherwise} \end{cases} \\ p > 0: \quad N_{i,p}(\xi) &= \frac{\xi - \xi_i}{\xi_{i+p} - \xi_i} N_{i,p-1}(\xi) + \frac{\xi_{i+p+1} - \xi}{\xi_{i+p+1} - \xi_{i+1}} N_{i+1,p-1}(\xi). \end{aligned} \quad (5)$$

In addition to the basis functions a control net, which is formed from control points $\mathbf{B}_{i,j}$ and a set of weights $w_{i,j}$ are needed to evaluate the NURBS surface

$$\mathbf{s} = \sum_{i=1}^n \sum_{j=1}^m R_{i,j}^{p,q}(\xi, \eta) \mathbf{B}_{i,j} \quad \text{with} \quad R_{i,j}^{p,q}(\xi, \eta) = \frac{N_{i,p}(\xi) M_{j,q}(\eta) w_{i,j}}{\sum_k \sum_l N_{k,p} M_{l,q} w_{k,l}}. \quad (6)$$

In the isogeometric analysis, the displacements of the control points $\mathbf{d}_{i,j} = \mathbf{B}_{i,j} - \mathbf{B}_{i,j}^0$ are chosen as degrees of freedom (DoF), whereby $\mathbf{B}_{i,j}$ and $\mathbf{B}_{i,j}^0$ are the positions of the control points in the undeformed and deformed configuration. The deformation of the NURBS surface is given by

$$\mathbf{u}_P = \sum_{i=1}^n \sum_{j=1}^m R_{i,j}^{p,q}(\xi, \eta) \mathbf{d}_{i,j}. \quad (7)$$

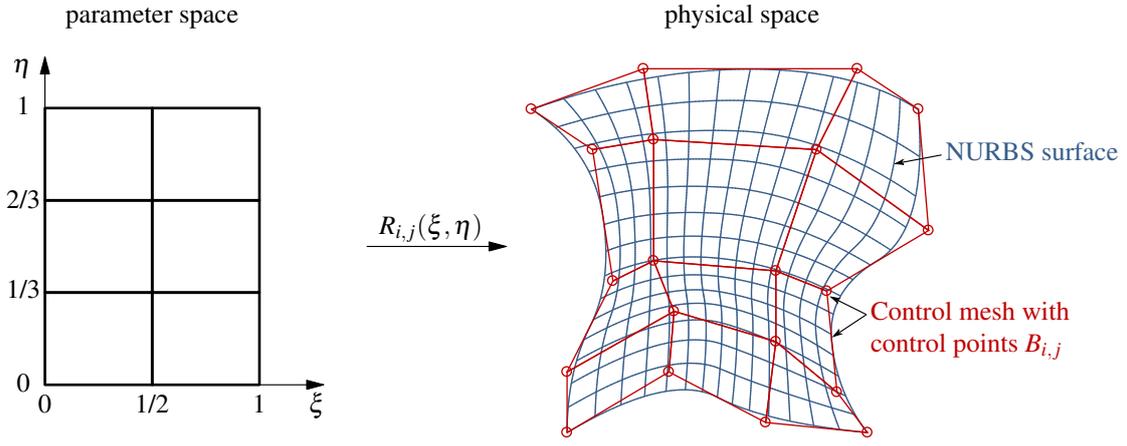


Figure 2 – Parameter and physical space in isogeometric analysis

Alternatively, using matrix notation instead of the summation notation Eq. (7) yields.

$$\mathbf{u} = \begin{bmatrix} R_{1,1}^{p,q} & 0 & R_{1,2}^{p,q} & 0 & \dots & 0 \\ 0 & R_{1,1}^{p,q} & 0 & R_{1,2}^{p,q} & \dots & R_{p,q}^{p,q} \end{bmatrix} \begin{bmatrix} d_{11,x} \\ d_{11,y} \\ d_{12,x} \\ d_{12,y} \\ \vdots \\ d_{pq,y} \end{bmatrix} = \mathbf{N}\mathbf{d} \quad (8)$$

The remaining steps in the derivation of the finite element model resembles closely the classical isoparametric approach. Incorporating the kinematics $\dot{\mathbf{u}} = \mathbf{N}\dot{\mathbf{d}}$ and the virtual displacements $\delta\mathbf{u} = \mathbf{N}\delta\mathbf{d}$ in, for instance, d'Alembert's principle the linear equations of motion

$$\mathbf{M}_{IGA}\ddot{\mathbf{d}} + \mathbf{K}_{IGA}\mathbf{d} = \mathbf{f}_{IGA} \quad (9)$$

of the unconstrained NURBS surface can be determined. However, care must be taken in the assembling of the mass matrix \mathbf{M}_{IGA} , the stiffness matrix \mathbf{K}_{IGA} and the right-hand-side vector \mathbf{f}_{IGA} , because due to the higher order of the polynomials, the numerical integration is not limited to single elements but groups of elements in the parameter space.

In order to determine the global shape functions Φ from the isogeometric finite element model (9), first boundary conditions have to be applied which yields the constrained finite element model

$$\bar{\mathbf{M}}_{IGA}\ddot{\bar{\mathbf{d}}} + \bar{\mathbf{K}}_{IGA}\bar{\mathbf{d}} = \bar{\mathbf{f}}_{IGA}. \quad (10)$$

Secondly a model reduction of the constrained system (10) is performed. In this work, simple modal truncation is used. Therefore, the eigenvalue problem

$$(\bar{\mathbf{K}}_{IGA} - \omega_i^2 \bar{\mathbf{M}}_{IGA}) \phi_i = \bar{\mathbf{f}}_{IGA}, \quad (11)$$

is solved and then the first n_e eigenshapes are gathered in the shape function matrix $\Phi = [\phi_1 \dots \phi_{n_e}]$.

APPLICATION EXAMPLE

The geometry and material parameters of the investigated slider-crank mechanism are summarized in Fig. 3. As can be seen the height h of the flexible piston rod is not constant by changes quadratically along the x -axis as

$$h(x) = 2\ell_y \left[\left(x - \frac{\ell_x}{2} \right)^2 + \left(\frac{\ell_x}{2} \right)^2 \right]. \quad (12)$$

Two finite element models are created to analyze the piston rod. On the one hand, an ANSYS model is set up with 150×10 bilinear planar elements. With two DoF per node, the total number of DoF is 3322. On the other hand, an isogeometric model is created. It possesses with $n = 25$ and $m = 8$ less local shape function, but the polynomial orders are $p = 7$ and $q = 4$ and, thus, significantly higher than in the ANSYS model. The total number of DoF is here 400. For both models the DoF along the left edge are constrained by a rigid clamping.

First of all, a modal analysis is performed for both finite element models and the resulting eigenfrequencies and eigenshapes are studied. In Tab. 1, it can be seen that the first five eigenfrequencies of the ANSYS model and the isogeometric analysis (IGA) model are in very good agreement. It should be noted that the first four eigenmodes are bending modes, while the fifth is a longitudinal mode.

crank	length	0.1 m
piston rod	max. dimensions	$(1.0 \times 0.06 \times 0.01)$ m
	density	7850 kg/m^3
	Young's modulus	$2.1 \cdot 10^{11} \text{ N/m}^2$
	Poisson's ratio	0.3
	damping parameters	$\alpha = 3.86,$ $\beta = 2.25 \cdot 10^{-5}$
piston	mass	5 kg

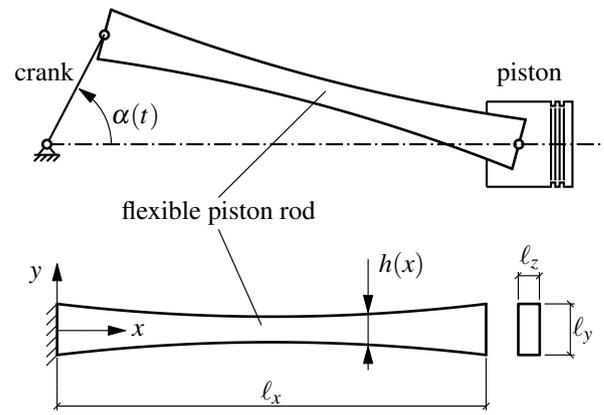


Figure 3 – Geometrical and material properties of the flexible slider-crank mechanism

i	1	2	3	4	5
ANSYS f_i Hz	31.8	200.6	560.6	1082.3	1254.9
IGA f_i Hz	31.7	201.7	564.5	1089.3	1255.5

Table 1 – Comparison of eigenfrequencies

These first five eigenshapes are then kept in a modal truncation. The SID are computed and a time simulation is performed. In the time simulation the flexible slider-crank mechanism is accelerated with 2 second from a rest position and rotates for another second with a constant angular velocity Ω^1 . The motion is imposed by a rheonomic constraint for α

$$\alpha(t) = \begin{cases} \sum_{i=0}^7 a_i t^i, & 0 \leq t \leq 2 \text{ s} \\ \Omega^1 t + \alpha^1, & 2 \text{ s} < t \leq 3 \text{ s} \end{cases} \quad \text{with} \quad \begin{cases} t = 0 \text{ s} : \alpha = \dot{\alpha} = \ddot{\alpha} = \ddot{\alpha} = 0 \\ t = 2 \text{ s} : \alpha = 12\pi \text{ rad}, \dot{\alpha} = 12\pi \text{ Hz}, \ddot{\alpha} = \ddot{\alpha} = 0 \end{cases} \quad (13)$$

The time simulation results of the two flexible multibody simulations are in very good agreement, as can be seen from the trajectories of the piston and the first elastic coordinate in Fig. 4. Therefore, it can be concluded that it is possible to obtain the global shape functions for flexible multibody systems from isogeometric finite element models. This procedure can now be used, for instance, in the structural optimization of flexible multibody systems, in which an accurate sensitivity analysis is needed. Further applications can be flexible multibody systems, in which the topology of the flexible bodies change during the time simulation.

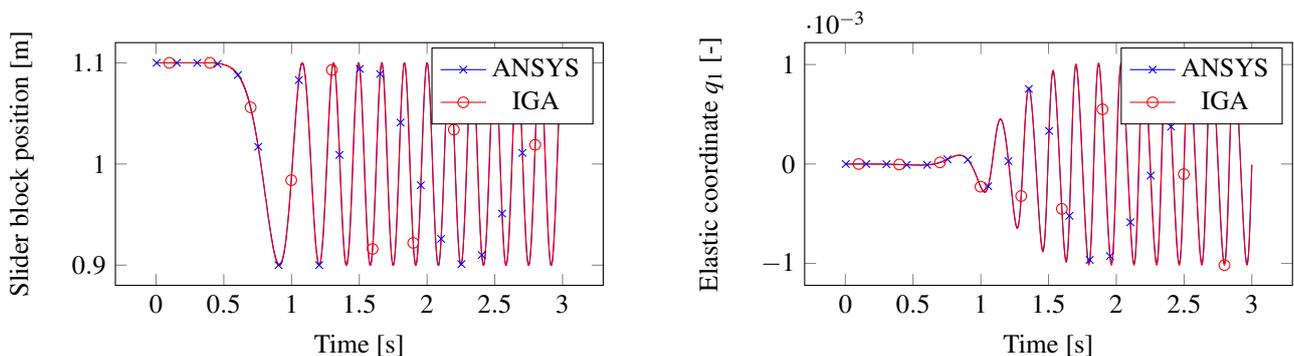


Figure 4 – Verification of flexible multibody simulation

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