Decoupled Design of Repetitive Control for Fluid Power Systems

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Abstract: There are a lot of precesses that are cyclic in their behaviour. This ranges from manufacturing processes, to rotating machines, such as combustion engines, hydraulic and electric machines, etc. With the introduction of active components there is a need to control these components, at a high rate. By using the cyclic, repetitive nature of the operation, where a feedback from the previous cycle can be used, the response can be gradually improved over the cycles. This also applies to systems with repetitive disturbances, such as for cancellation of noise and vibrations. In this paper a simple approach to decouple design of the quasi static cyclic behaviour and transient response to changes in period time or input signal shape.

Keywords: Repetetive control, fluid power, control system

INTRODUCTION

The basic principle in repetitive control systems are that the input signal is periodic. It was introduced by [3] and subsequently developed in e.g. [9], [1], [2], [8]. In [5], [6], [7] the aspect of time variable period time is also considered. In [4] repetitive control was used to control a measurement set up for obtaining the dynamic characteristics of a hydraulic valve using a frequency sweep. This assumption is used by feeding back the error in the previous period to the current period. In this way very high gain at the periodic frequency and its harmonics is obtained. See Fig. 1.

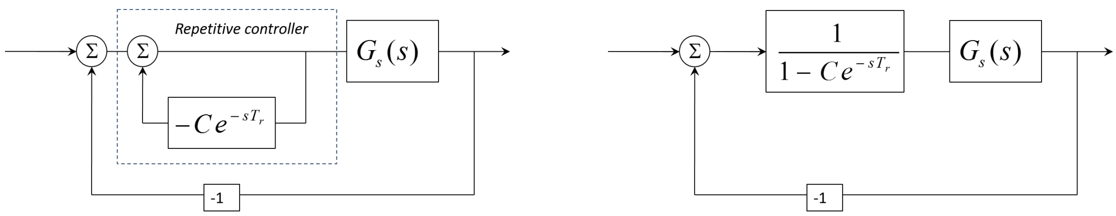

Figure 1 – Closed loop system with a repetitive controler

Here it is possible to define the transfer function $G_r(s)$ of the repetitive regulator as:

$$G_r(s) = \frac{1}{1 - C(s)e^{sT_r}} \tag{1}$$

Let the transfer function of the unregulated system be $G_s(s)$. The open loop transfer function with the repetitive controller will then be

$$G_o(s) = G_r(s)G_s(s) \tag{2}$$

The stability margins of the system can be obtained from the Bode diagram of the open loop system. If it is assumed that $C(s) = c_0$ and that the system is a hydraulic position servo where $G_s(s)$ can be expressed as:

$$G_s(s) = \frac{K_v}{s(\frac{s^2}{\omega_h^2} + 2\frac{\delta_h}{\omega_h} + 1)} \tag{3}$$

A useful representation for stability analysis of this system is to use the Nyquist diagram. Then normal way of doing this would be to plot the open loop transfer function $G_o(s)$ and see if it encircles -1. However, another way of using it is to plot one part of the transfer function, in this case $G_s(s)$ and see if it encircles any part of $-1/G_r(s)$ This approach is described for repetitive control in [3]. For the case that $C(s)$ is a constant c_0, $-1/G_r(s)$ can be written

$$-1/G_r(s) = c_0 e^{-sT_p} \tag{4}$$

This is a circle with radius c_0 with a centre at -1.

The corresponding Nyquist diagram is shown to the left in Fig. 2.

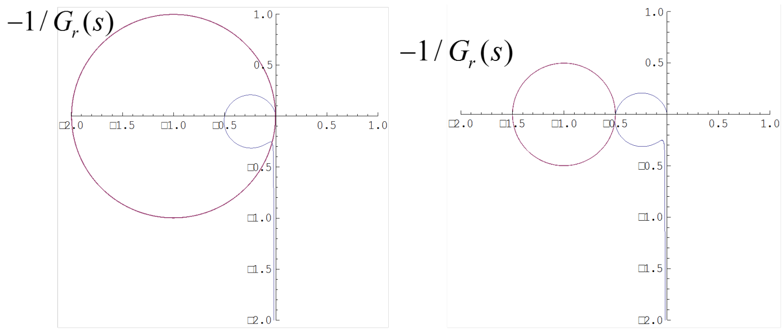

Figure 2 – Nyquist diagram. The transfer function $\mathbf{G_s(s)}$ should not encircle the circle represented by $\mathbf{1/G_r(s)}$.

Here it is very easy to decide weather the system is stable or not. Furthermore, this is a conservative measure of stability since a system that is determined to be stable in this way is stable for all period times T_p, since the radius of the circle will be the same. An observation that can be made is that it is very difficult to obtain a stable system if $c_0 = 1$. In the right in Fig. 2 the Nyquist diagram for $c_0 = 0.5$. However this greatly reduce the effectiveness of the regulator.

There is also the possibility to introduce to introduce some dynamics in $C(s)$ to enhance stability. If a low pass filter is introduced such that

$$C(s) = \frac{C_0}{\frac{s}{\omega_c} + 1} \tag{5}$$

The $1/G_r(s)$ function have a radius that is diminishing with frequency can thereby avoid being encircled.

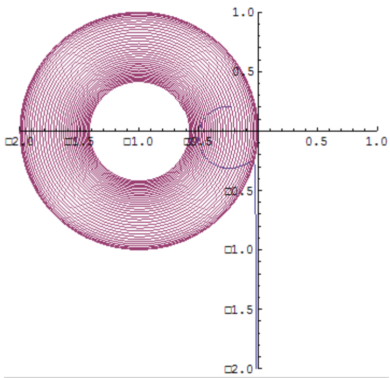

Figure 3 – Nyquist diagram with a low pass filter (only plotted up to a frequency slightly above ω_c).

Although efficient expressions for $C(s)$ can be found in e.g. [9] and [8] another different but very simple approach will be discussed here.

CYCLIC AVERAGING

In order to avoid the problem of instability indicated by Fig 2 when unit feedback gain is used, some kind of filtering of the delayed signal can be used. This has been proposed in ref [4] where it was implemented successfully for the control of an acceleration waveform generator.

Another type of filter is obtained if the delayed signal is a weighted average of previous signals at the same point in the load cycle. One example of implementation is given by equations (6)-(7) that represents exponential weighting of values of previous periods in such a way that old values are weighted less than newer ones.

$$x_{cf}(t) = \alpha x_{cf}(t - T_p) + (1 - \alpha) x_c(t) \tag{6}$$

Laplace transforming yields

$$X_{cf}(s) = \frac{1 - \alpha}{1 - \alpha e^{-sT_p}} X_c(s) \tag{7}$$

Here an α value close to zero means that x_{cf} depends very little on old periods this means that the x_{cf} can adapt quickly to a new input signal. An α value close to on the other hand gives a slow adaptation but makes the compensation much less sensitive to noise and, as will be demonstrated below, a more stable system.

The compensator transfer function $C(s)$ is thus

$$C(s) = \frac{1-\alpha}{1-\alpha e^{-sT_p}} \tag{8}$$

(9) in (1) yields

$$G_r(s) = \frac{1-\alpha e^{sT_p}}{(1-\alpha e^{-sT_p})-(1-\alpha)e^{-sT_r}} \tag{9}$$

If $-1/G_r(s)$ is plotted it can be seen that the circle in Fig 2 has been turned into a spiral that is spiralling away from origo as the frequency increase. In this way the circle is away from the Nyquist curve for the rest of the system as it is closing in on origo.

APPARENT TRANSFER FUNCTION

When describing the behaviour of the closed loop system, one way of presenting it is by evaluating the closed loop transfer function in the frequency domain. In this way features such as bandwidth can be estimated.

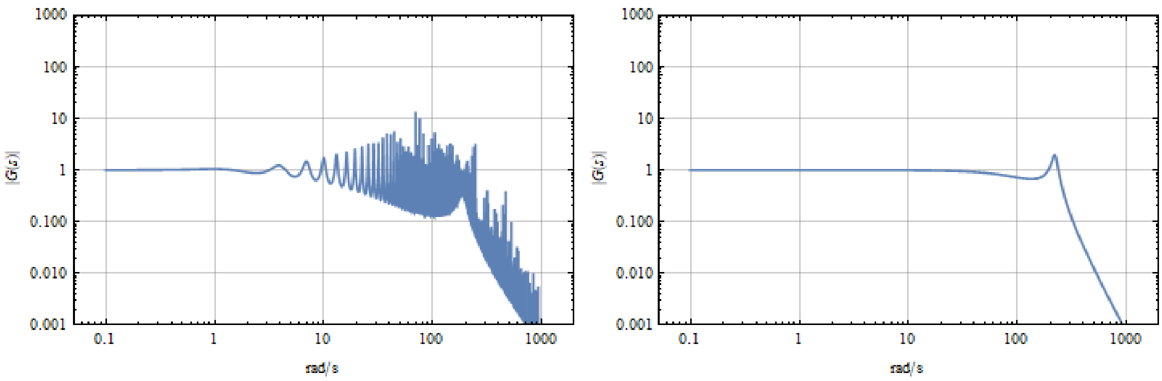

Figure 4 – Closed loop transfer function of repetitive controller, left and apparent closed loop transfer function (ACTF) to the right.

However, the closed loop transfer function as shown to the left in Fig. 4 is hardly a good description of the behaviour of the system subjected to repetitive load cycle. For this case it is much more informative to evaluate an *apparent closed loop transfer function* (ACTF). Here it is assumed that all signals are periodic with the period time T_p, and we can thus use the signal $u(t)$ instead of $u(t-T_r)$. (T_r is the delay time used in the controller) since they are identical. This corresponds to the response of a no-repetitive controller with the same response for the periodic input. A peculiarity of the system is that it is a-causal, since the repetitive controller can use information about future states fo the system under the assumption that all periods are identical. The resulting transfer function is shown to the right in Fig. 4 where te apparent closed loop transfer function (ACTF) is drawn for two slightly different period times. The controller, however, has the same delay T_r. It can be noted that there is a dramatic difference in the ACTF where the longer period time gives a much more damped response. In that case the delay time in the controller T_r is shorter than the input signal period time T_p the difference is T_c so that.

$$T_c = T_p - T_r \tag{10}$$

This means that a feedback from a signal $u(t-T_r)$ corresponds to an acausal feedback $u(t+T_c)$ which should provide a positive phase shift if it could be implemented. Note, since only the input signal has been changed (slightly) there is no change in the stability of the system. This example does, however, demonstrate the advantage with this control concept. A very well behaved system can be obtained by adjusting T_c.

One observation is that the ACTF is only a function of the time displacement T_c independent of the $C(s)$ transfer function.

SIMULATION RESULTS

A fully non-linear simulation was made in the Hopsan simulation package. The result is shown in Fig. 5. The transient can be seen as the simulation starts with the cylinder in the zero position and then oscillated around 0.1 m with a square wave with an amplitude of 0.05m.

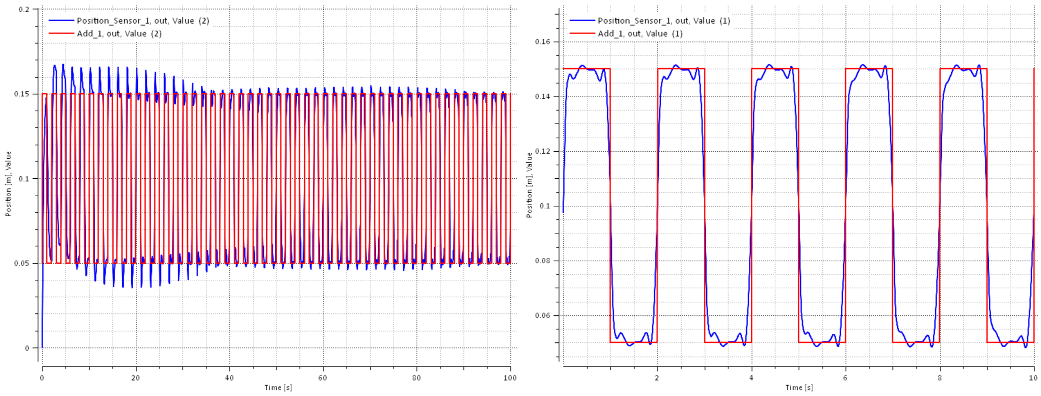

Figure 5 – Simulation results of hydraulic servo with repetetive control. To the right the last ten seconds of the simulation is shown in more detail.

CONCLUSIONS

Perhaps the most attractive feature of repetitive control is that it is very easy to tune. In the form presented here there are only two parameters, the forgetting factor α and the time offset T_c. Of these T_c alone is responsible for the behaviour of the apparent closed loop transfer function. In addition there is a low pass filter that is set to a quite high bandwidth and where the system performance is not very sensitive to variations in this. Repetitive control resembles an acausal regulator since it can use knowledge from the previous cycle as for approximations of future states in the next cycle. In this way the repetitive control concept can be easier to use for regulator design in general in system with cyclic behaviour than e.g. pole-placement, LQG-design or other methods.

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