Nonlinear Dynamics of an Origami-Wheel Robot

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Abstract: Self-folding structures have an increasing importance in different areas that include medicine, as the devices employed for minimally invasive surgery; architecture, as the luminosity control through foldable panels; aerospace engineering, as solar panel arrays; and robotic, which includes wheels. Smart material elements, such as shape memory alloys (SMAs) and piezoelectric, provide actuation that increases the range of applicability of origami structures, allowing the design self-folding adaptive systems. This paper deals with the dynamical analysis of an origami-wheel robot actuated by SMAs. The yaw rotation, that promotes the route change, is provided by the radius variation of the wheels. Dynamical aspects must be considered during both the design and operation of the wheel-robot. A 4-degree of freedom reduced order model (4-DOF) is proposed to represent the car dynamical behavior. This model is achieved considering simplifications based on mechanism analysis and symmetric behaviors. Numerical simulations investigate operational conditions, considering thermal and mechanical loads.

Keywords: Self-folding, origami, smart structure, smart materials.

INTRODUCTION

Autonomous mobile robots have several applications related to different fields as aerospace, oil industry, defense and security. A relevant issue associated with the design of these robots lies on the actuation and control. An alternative for path control is the use of two-wheel drive with differential steering and a free-wheeling wheel (caster). The independent control of each wheel allows these robots to have good maneuvers and interesting responses (Malu & Majumdar, 2014).

Another interesting improvement is the replacement of conventional wheels for deformable wheels that allows the car to cross obstacles, from small cracks to step-up (Lee et al., 2013). In addition, the path control is done by the direct actuation of wheels, using a small number of actuators, promoting a significant reduction of the robot weight.

Origami is an alternative to produce self-folding wheels. The use of SMAs can provide configuration changes, promoting the necessary actuation. This paper deals with the dynamical analysis of an origami-wheel robot. Kinematics analysis allows one to present a four-degree of freedom reduced order model. Numerical simulations bring robot paths promoted by temperature variations.

KINEMATICS ANALYSIS

The origami-wheel robot is composed by a central system that includes the control of the car (car body, with center of gravity, G), two origami-wheels (with center of gravity, A and B), two axes to attach the wheels and a roller to avoid a pendulum movement of the car body (Fig. 1-a). There are basically two ways of attaching the origami wheel to the shaft: one can attach the shaft to one of the acrylic plates of the origami-wheel in such a way that the origami wheel center of gravity slides on the shaft; the other possibility considers that the origami can be secured by a holder on the shaft, which is attached to the spring. This paper considers the second approach and therefore, the system is designed in such a way that the distance between the center of gravity of each wheel and the car body is constant and equal to *d* (Fig. 1-b).

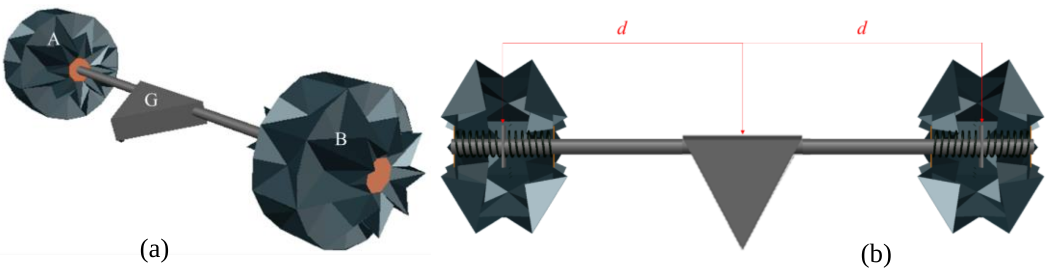

Figure 1 – Schematic pictures of the origami-wheel robot (a); Origami-wheel fixed to the axis (b).

The robot position at an time instant, relative to a fixed observer in space, is fully described by the positioning of the center of gravity of the car body (*x*,*y*,*z*) and by the inclination of the axis (*Φ*). Some of these variables, however, can be rewritten as a function of (or replaced by) R_A and R_B, the origami-wheel radius. Figure 2 shows the systems and variables employed to obtain the kinematic model of the origami-wheel robot. Note that the system is described by eight variables, as follows: robot center of gravity position (*x*,*y*), robot yaw (*Φ*), robot row (*θ*), wheels radii (R_A, R_B) and wheels spin (ϕ_A, ϕ_B).

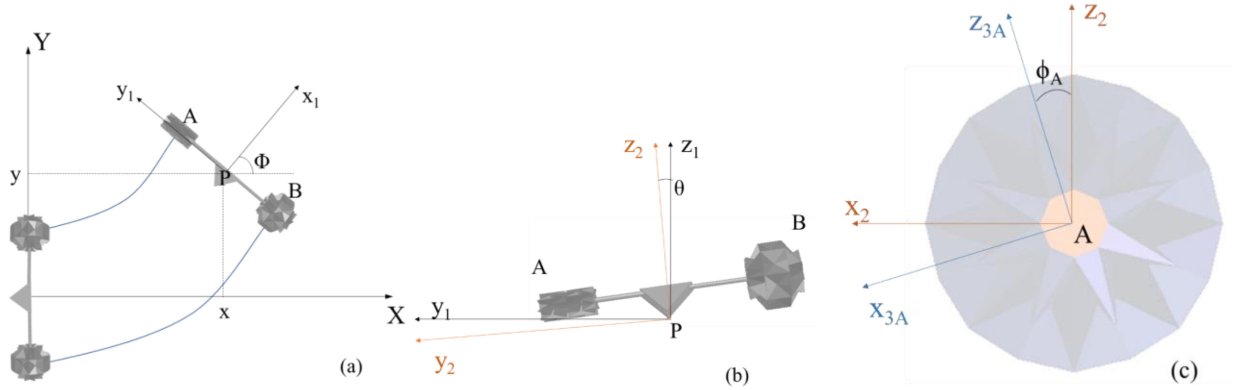

Figure 2 – Reference systems and notations used for the development of the numerical model of the origami-wheel robot. (a) Movement of the car seen by a fixed observer in RS F (XYZ) with yaw motion (*Φ*); (b) Roll motion, described by rotation *θ*; (c) RS accompanying the movement of wheel A (analogous for wheel B).

Assuming that there is no slip on the ground, the differential motions of the generalized coordinates have the geometric relationships in Eq. 1 for the wheel A and in Eq. 2 for the wheel B. These restrictions are obtained from the RSs W_A and W_B, respectively, and $D = \frac{R_B - R_A}{2}$.

$$C1: \dot{x}\cos(\Phi) + \dot{y}\sin(\Phi) - \dot{\Phi}[d\cos(\theta) - D\sin(\theta)] = \dot{\phi}_A R_A \quad (1)$$

$$C2: \dot{x}\cos(\Phi) + \dot{y}\sin(\Phi) + \dot{\Phi}[d\cos(\theta) - D\sin(\theta)] = \dot{\phi}_B R_B \quad (2)$$

The other two restrictions are related to the last non-slip condition of the wheels (Eq. 3), looking to the RS R or C, and the non-slip condition of the roller (Eq. 4).

$$C3: \ -\dot{\theta}[d\cos(\theta) - D\sin(\theta)] - \dot{D}\cos(\theta) = 0 \quad (3)$$

$$C4: \dot{x}\sin(\Phi) - \dot{y}\cos(\Phi) = 0 \quad (4)$$

Note that radius variation equations depend on the origami geometrical description. A waterbomb tessellation folding pattern is used as the origami-wheel, as can be seen in Fig. 3. The actuation is provided by 8 circumferentially distributed SMAs springs (Fig. 3-a), promoting the structure opening-closure. The deformable wheels are designed based on the origami waterbomb tessellation, a traditional fold described by 6 creases (4 valley fold and 2 mountain folds – Fig. 3-b). Waterbomb pattern is typically a rigid foldable origami and, hence, the unitary cell can be described by a 6R-linkage mechanism (Chiang, 2000; Chen *et al.*, 2016), with at least three degrees of freedom (Fig. 3-c). Besides, each wheel is symmetrically actuated, and external efforts are symmetrically distributed by the structure. Thus, it is possible to describe the unitary cell fold process by a reduced order geometric model, represented by a one degree of freedom system.

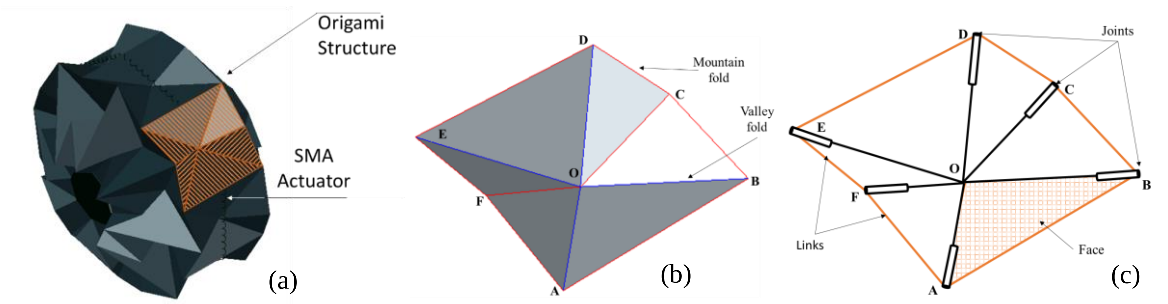

Figure 3 – Origami-wheel with unitary-cell highlighted (a); Waterbomb unitary cell (b); Equivalent mechanism (c).

Waterbomb pattern has a region of rigid-foldability that depends on the structure configuration, related to the total origami displacement. These rigid-foldability paths are related to uniform expansion/contraction of the origami and bending between layers (Chen *et al.*, 2016). Fang *et al.* (2017) developed a kinematic study of a 3×8 origami-wheel structure folding process, identifying the rigid-foldability region. If external forces are radially distributed on the origami-wheel, folding process has rotational and reflectional symmetry. Nevertheless, folding process is well represented by a single DOF model since the involved efforts are radially applied during contraction and axially applied during expansion. Therefore, all cells have the same behavior during expansion/contraction process. Based on that, a single cell is representative of the general origami-wheel behavior. Thus, the opening/closure process of the wheel is described by a geometric relation between the SMA actuators length and the half-length of the elastic passive spring, represented at Fig. 4.

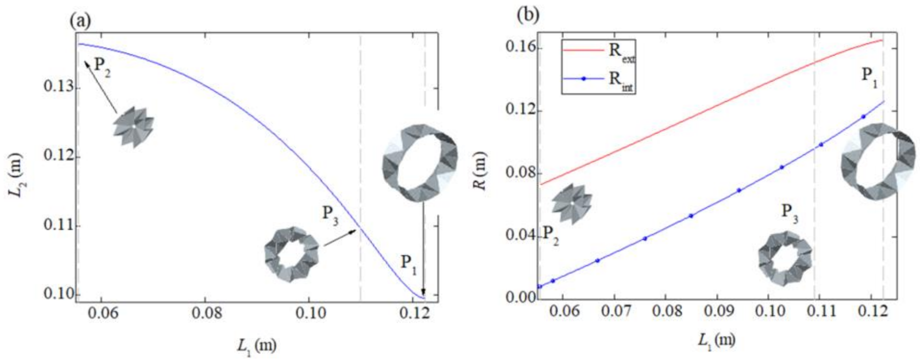

Figure 4 – Half-length of the elastic passive spring (L_2) (a) and external (R_{ext}) and internal (R_{int}) radius of the wheel (b) as a function of the SMA length (L_1).

DYNAMICAL ANALYSIS

By considering the Lagrangian modeling approach, it is necessary to express the kinetic energy and potential energy of the three bodies as the functions of generalized coordinates. Both the translational (Eq. 5) and the rotational (Eq. 6) kinetic energies have the contribution of both origami-wheels (with origami and acrylic plates masses) and the robot body (with center of gravity G). The potential energy is given by the contribution of the SMAs, the elastic passive spring and the potential gravitational energy.

$$^{TRA}E_C = \frac{m_t}{2}\left\{\dot{x}^2 + \dot{y}^2 + \dot{R}^2 + \dot{\Phi}^2 R^2 \sin^2(\theta) + \dot{\theta}^2 R^2 - 2\dot{\Phi} R \sin(\theta)\left[\dot{x}\cos(\Phi) + \dot{y}\sin(\Phi)\right] + \left[2\dot{\theta} R \cos(\theta) + 2\dot{R}\sin(\theta)\right]\left[\dot{y}\cos(\Phi) - \dot{x}\sin(\Phi)\right]\right\} + \frac{m_t - m_G}{2}\left[\dot{\Phi}^2 d^2 \cos^2(\theta) + \dot{\theta}^2 d^2\right] + \frac{M}{2}\left[2\dot{\Phi}^2 (f_A^2 + f_B^2)\cos^2(\theta) + 2\dot{\theta}^2 (f_A^2 + f_B^2) + 2(f_A^2 + f_B^2)\right] \quad (5)$$

$$^{ROT}E_C = \frac{\dot{\theta}^2}{2}(I_{A1} + I_{B1} + J_1) + \frac{\Phi^2}{2}\left[(I_{A1} + I_{B1} + J_3)\cos^2(\theta) + (I_{A2} + I_{B2} + J_2)\cos^2(\theta)\right] - \dot{\Phi}\sin(\theta)\left(\frac{\dot{\phi}_A I_{A2}}{2} + \frac{\dot{\phi}_B I_{B2}}{2}\right) \quad (6)$$

where $R = \frac{R_A + R_B}{2}$

The system is described by eight variables and has four constraints and, therefore, the robot motion is completely described by a 4-DOF system (Eq. 7), where $\boldsymbol{M}$ is the inertia matrix, $\boldsymbol{C}$ is the centrifugal and Coriolis matrix, $\boldsymbol{D}$ is the damping matrix, $\boldsymbol{G}$ is the gravity vector and $\boldsymbol{F}$ is the input vector. Besides, the generalized coordinates vector q has the form: $q = [x, \Phi, R_A, R_B\,]^T$.

$$\boldsymbol{M}(q)\ddot{q} + \boldsymbol{C}(\dot{q}, q)\dot{q} + \boldsymbol{D}(q)\dot{q} + G(q) = F \quad (7)$$

NUMERICAL SIMULATIONS

By considering equations of motion, the constraints and the geometrical relations that describe the origami opening/closure process, it is possible to describe the robot motion with a reduced order 4 degree of freedom model. By heating the SMAs on one of the wheels, it is possible to make a curve to the left (by reducing A-wheel radius) or to the right (by reducing B-wheel radius). Figure 5 shows the heating/cooling loads at the SMAs and the robot response represented by its trajectory.

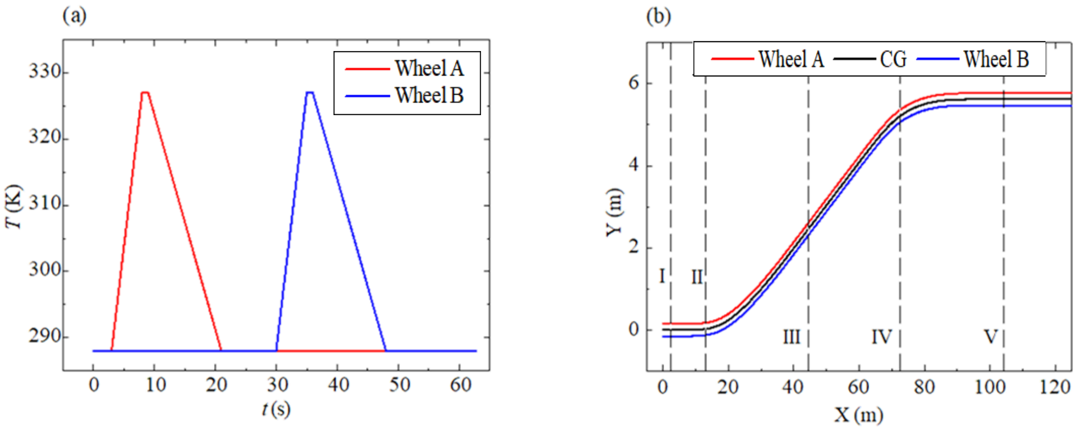

Figure 5 – Origami-wheel robot trajectory. (a) Thermal load applied to actuators at both wheels; (b) Trajectory followed by the robot.

External efforts such as soil irregularities, represented by mechanical external forces to the system, can change the robot response, promoting deviations at the trajectory. Figure 6-a shows different robot trajectories considering different forcing conditions. Note that, depending on the external force magnitude and frequency, the dynamical response can change from a periodic one to a chaotic-like response, changing dramatically the robot trajectory.

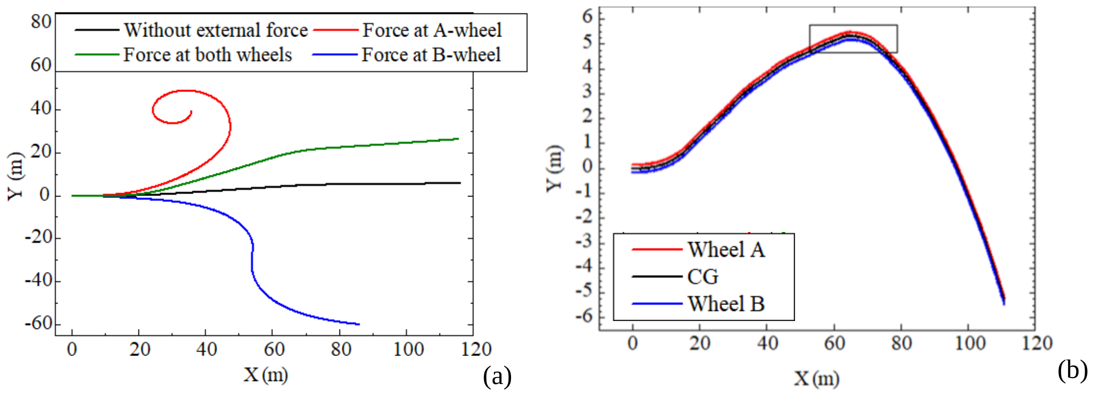

Figure 6 – (a) Origami-wheel robot CG trajectory for different mechanical loads; (b) Origami-wheel robot trajectory for a chaotic-like response induced by a perturbation around the mechanical load applied in bth wheels at figure (a).

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