Modeling and fatigue analysis of an hybrid riser system subjected to vortex-induced motion

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Abstract: As a structure responsible to connect the platform to the wellhead in offshore operations, the riser is subjected to weathering. The fluid flow interacts with the structure and provides the vortex shedding behind the riser which may lead to the vortex-induced motion (VIM) and the vortex-induced vibration (VIV) phenomena. The VIM occurs usually in larger diameter structures such as the float support of the hybrid riser and the VIV occurs in slender structures such as the riser itself. When the frequency of vortex shedding approximates to the natural frequency of the riser, such phenomena appears increasing the amplitude of the motions. The latter is known as lock-in and may have a significant contribution in decreasing of the life time of the riser. In order to analyze the fluid flow influence on the life time of the riser, this work uses a two degrees of freedom mathematical model which describes the dynamics of the system coupled to the vortex shedding given by Van der Pol oscillators. The main objective of this work is to investigate the parameters which influence the life time of the riser and provide a knowledge of the phenomena involved. The float support diameter presented great influence on the fatigue of the riser, mainly at the lock-in state. Once the Strouhal number changes the frequency of vortex shedding, it affect directly the band of the lock-in and the life time of the structure. This study uses a well-known model to provide a better understanding of the dynamics and life time of the riser structures.

Keywords: Fatigue, vortex-induced motion, vortex-induced vibration, riser dynamics

INTRODUCTION

In the last years, the offshore oil exploration has sought more and more deeper oil wells in search of news sources (Torres *et al*, 2008). This seek has meant a big challenge about the riser structure, that is subject to higher pressures and environmental loads. These loads contribute to the rising of important phenomena such as vortex-induced motion (VIM) and vortex-induced vibration (VIV) (Tsukada, 2009). To model the hybrid riser system many studies uses finite elements, providing a good approximation of the real system, however this approach requires a computationally heavy processing. Another way to model a riser system is to apply the known dynamics of the riser and to analyze its dynamics when coupled with the Van der Pol oscillator, making it possible to predict the behavior of the hybrid riser for different situations of loads, flow type and allowing the study for different parameters of the system.

Such model has been suggested by (Facchinetti *et al*, 2004), with a 1 degree of freedom (DOF) model that describes the riser's motions in the transverse direction of the fluid flow. Recently, (Postnikov *et al*, 2017) added 1-DOF to the model proposed by (Facchinetti *et al*, 2004), to get a 2-DOF model. The proposed model describes the system with more precision and allows the study of the contribution of the motions in the flow direction due to the VIM and VIV phenomena.

Once the hybrid risers are subjected to the VIM arising from the buoy's motion, its motions' amplitude can be large and the stresses generated may be very prejudicial for the life time of its structure (Song *et al*, 2010). The understanding of these phenomena is essential for the correct estimation of the riser's life time, in order to avoid economical prejudice or even accidents. In order to estimate the hybrid riser fatigue lifetime, this work uses the two degrees of freedom model proposed by (Postnikov *et al*, 2017), that allows the study of the contribution of the VIM and VIV phenomena on the life span results and also provides a better knowledge of the influence of the different parameters involved.

THE DYNAMIC MODEL

The nature of the vortex shedding process behind cylindrical structure suggests that the forces acting on the structure from the fluid can be modeled by a nonlinear Van der Pol oscillator.

The equations of motion on a horizontal XY plane for a cylinder able to oscillate in cross-flow and in-line directions in terms of displacement x and y, are

$$m^* \ddot{x} + r_s \dot{x} + hx = F_x,$$
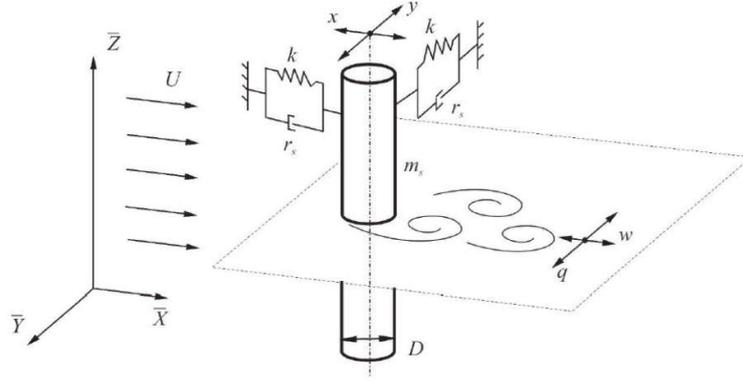$$m^* \ddot{y} + r_s \dot{y} + hy = F_y,$$

(1)

Figure 1 – Model of coupled structure and wake oscillators on 2 DOF Postnikov *et al.* (2017).

where

$$m^* = \frac{4\mu}{\pi} - C_M \tag{2}$$

and the total hydrodynamic force components in x and y directions are F_x and F_{y}. m^* denotes mass per unit length including the added mass, C_M is the constant-added mass coefficient; the stiffness and damping of the system are h and r_s, respectively. By developing the Eq. (1) and writing it in its non-dimensional form, the 2-DOF model proposed by Postnikov *et al.* (2017) is obtained and it is composed by the four second order coupled nonlinear differential equations as follows

$$
\begin{aligned}
&\ddot{x} + 2\xi\dot{x} + x = 8\pi^2 St^2 \sqrt{(\frac{U_r}{2\pi} - \dot{x})^2 + \dot{y}^2} \left(\frac{1}{2}M_L q\dot{y} + (M_D + \frac{1}{2}M_D^{fl}w)(\frac{U_r}{2\pi} - \dot{x})\right), \\
&\ddot{y} + 2\xi\dot{y} + y = 8\pi^2 St^2 \sqrt{(\frac{U_r}{2\pi} - \dot{x})^2 + \dot{y}^2} \left(\frac{1}{2}M_L q(\frac{U_r}{2\pi} - \dot{x}) - (M_D + \frac{1}{2}M_D^{fl}w)\dot{y}\right), \\
&\ddot{w} + \varepsilon_x 2\Omega(w^2 - 1)\dot{w} + 4\Omega^2 w = (\frac{A_x}{D})\ddot{x}, \\
&\ddot{q} + \varepsilon_y \Omega(q^2 - 1)\dot{q} + \Omega^2 q = (\frac{A_y}{D})\ddot{y},
\end{aligned}
\tag{3}
$$

where x and y are the in-line and cross-flow dimensionless displacements, respectively; ξ is the damping factor, U_r is the reduced velocity. ε_x, ε_y, A_x and A_y are the Van der Pol wake oscillators parameters in each direction, μ is the mass ratio, Ω is the frequency of the vortex shedding and m^* is the mass of structure, including the added mass, $(m_s + m_f)$. The dimensionless parameters may be written as

$$
\begin{aligned}
U_r &= \frac{2\pi U}{\omega_n D}, \\
\omega_n &= \sqrt{\frac{h}{m}}, \\
\Omega_f &= \frac{2\pi St U}{D}, \\
\Omega &= \frac{\Omega_f}{\omega_n}, \\
q &= \frac{2C_L}{C_{L0}}, \\
w &= \frac{2C_D^{fl}}{C_{D0}^{fl}},
\end{aligned}
\qquad
\begin{aligned}
M_L &= \frac{C_{L0}}{16\pi^2 ST^2\mu}; \\
M_D &= \frac{C_{D0}}{16\pi^2 ST^2\mu}; \\
M_D^{fl} &= \frac{C_{L0}}{16\pi^2 ST^2\mu}; \\
\mu &= \frac{(m_s + \pi C_M \rho_f D^2)}{\rho_f D^2}, \\
y &= \frac{Y}{D}, \\
x &= \frac{X}{D}.
\end{aligned}
\tag{4}
$$

To obtain the responses of the dynamic model, Eq. (3) is computationally implemented via Runge-Kutta methods and the results of displacement, as well as was the nondimensionalized drag and lift coefficient, are plotted in the dimensionless number associated t time $\tau = \omega_n t$.

FATIGUE LIFE TIME

By using the parameters values used for this work are presented in Tab. 1 and solving the Eq. 3 for the initials boundary conditions $y(0) = 0.1$, $y'(0) = 0$, $x(0) = 0$, $x'(0) = 0$, $q(0) = 0$, $q'(0) = 0$, $w(0) = 0$, $w'(0) = 0$ is possible to obtain the results below.

Table 1 – Base parameters for the model.

Parameters	Description	Value	Unit
D	Riser diameter	0.4064	m
L	Riser length	100	m
U	Velocity of fluid	0.2	m/s
D_b	Buoy diameter	5.5	m
U_r	Reduced velocity	6.68	$-$
St	Strouhal	0.2	$-$
A_x	Empiric parameter of oscillator	12	$-$
A_y	Empiric parameter of oscillator	12	$-$
ε_x	Empiric parameter of oscillator	0.3	$-$
ε_y	Empiric parameter of oscillator	0.3	$-$
C_{D0}	Zero-drag coefficient	1.2	$-$
C_{D0}^{fl}	Floating drag coefficient	0.2	$-$
C_{L0}	Zero-lift coefficient	0.3	$-$
C_M	Constant-added mass coefficient	1.0	$-$

Figure 2 represents the dynamic behavior of the system by showing the displacement in the in-line and cross-flow directions. It is possible to see that the cross-flow amplitude becomes bigger than in-line and the oscillation frequency are smaller than the in-line displacement. Though the fact of the oscillations in the X direction have a higher frequency when compared to the Y direction and impacts on the life time of the system, it is known that the amplitude of the cross displacement have a much more important impact on the life time. Based on this, all the fatigue analyses are made of Y direction.
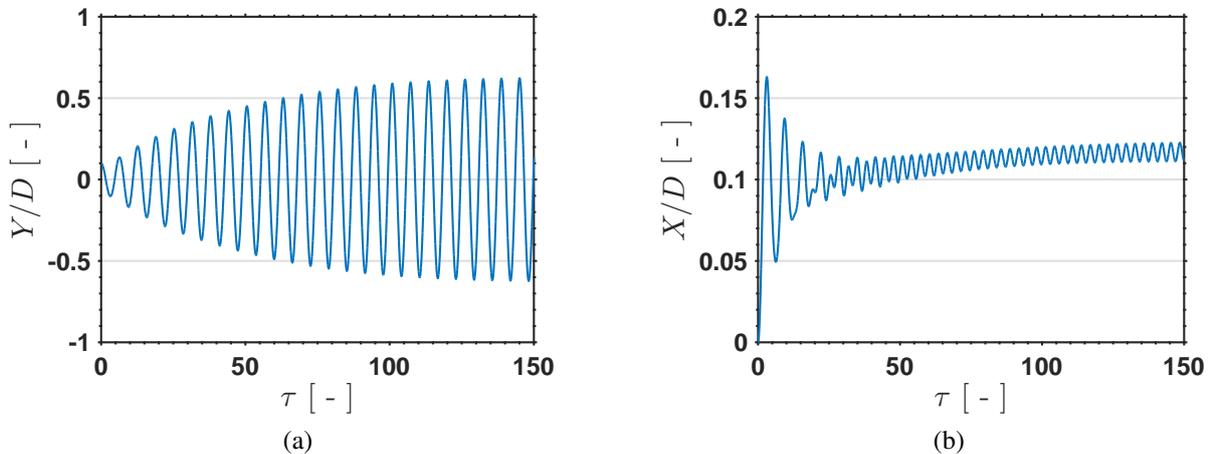

(a) (b)

Figure 2 – (a) Cross-flow displacement and (b) in-line displacement with $U_r = 6.68$.

The floater trajectory at top view show the motion amplitudes and reinforce that the cross line amplitude is bigger than in-line (see Fig. 3). The exact eight shape is obtained when in ideal case, the longitudinal frequency of oscillations are the double of the transversal.
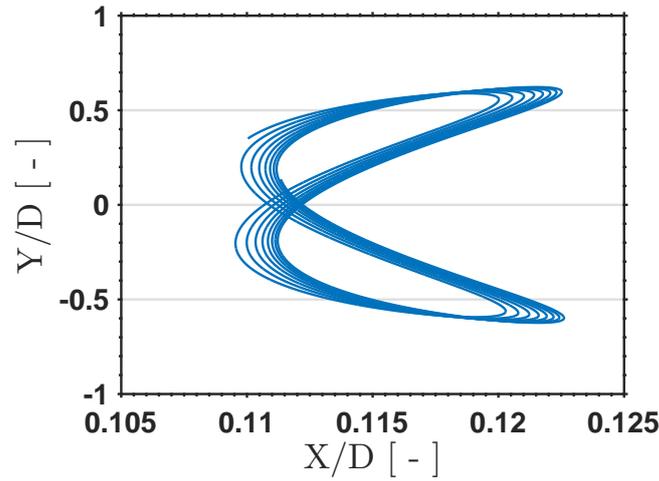
Figure 3 – Floater trajectory at top view with $U_r = 6.68$.

To obtain the life time of the riser, the system is modeled as an inverted pendulum shown in Fig. 4, that is clamped at the bottom and is free to move at the top, where the floater is placed. The riser is modeled as a beam of circular cross-section, such that the stiffness is known. Using the cross-flow displacement Y, we obtain the force P (Eq. 5), and also, the stress amplitude, from which one may calculate the life time of the structure using the S-N curve described per (DNV-RP-C203, 2005) norm.

$$P = \frac{3YEI}{L^3},$$

(5)

E and I denotes the modulus of Young and the second moment of area, respectively. The maximum and minimum stresses are calculated by the principle of superposition, considering the bending of the riser, the axial force exerted by the buoy and the axial force due to the mass of the riser in water.
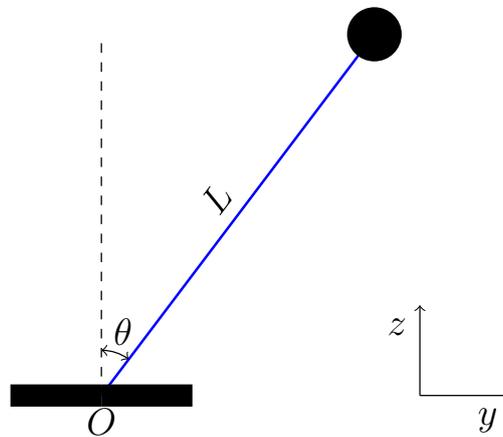

Figure 4 – Riser of lenght L modeled as an inverted pendulum.

To get the fatigue life time of the system, the total number of oscillations that the riser is able to carry out before failure is calculated. According to norm (DNV-RP-C203, 2005), the number (N) of oscillations is given by Eq. 6

$$log(N) = log(a) - mlog(\Delta\sigma),$$

(6)

where $\Delta\sigma$ is the amplitude of stress, m and $log(a)$ are given parameters that describes the S-N curves.

For the parameters shown in Tab. 1, the following results for the displacement amplitude and life time can be observed. Figure 5(a) show the maximum displacement amplitude in the cross-flow direction of each reduced velocity (U_r). As can be seen, between $U_r = 4$ and $U_r = 7$ occurs a rise of the displacement amplitude, this phenomena is known as lock-in and provides a great decrease of the life time of the structure (Fig. 5(b)), where the minimum life time observed was 5.8 years.
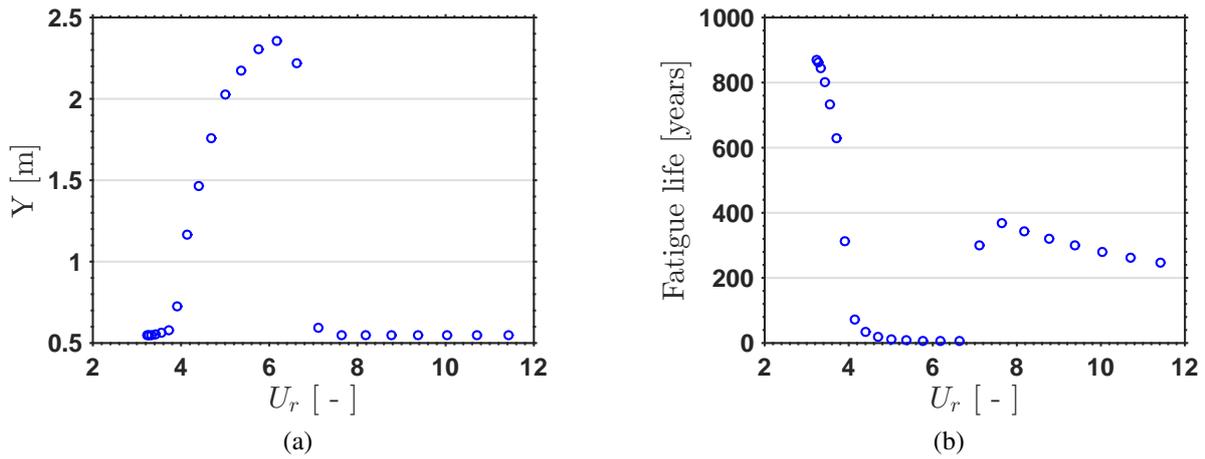
(a)

(b)

Figure 5 – (a) Cross-flow displacement and (b) Life time.

In order to investigate the influence of each parameters on the life time of the riser, the value of the parameters involved was modified and the obtained results could be analyzed. First addressed parameter was the buoy diameter (D_b). The buoy diameter has an important influence on the life time, once it is directly proportional to the cross-flow displacement amplitude (Fig. 6(a)). The increase of the buoy diameter generates a decrease of the life time as seen in Fig. 6(b). The results show that for the diameter of 4 m the maximum displacement is 1.71 m, which generates a fatigue life of 15.10 years and a diameter of 9 m generates a displacement of 3.85 m and a life time of 1.32 years. Then, a small increasing of this parameter can provide a big decrease of the life time of riser, showing itself as one of main important parameter to be studied.
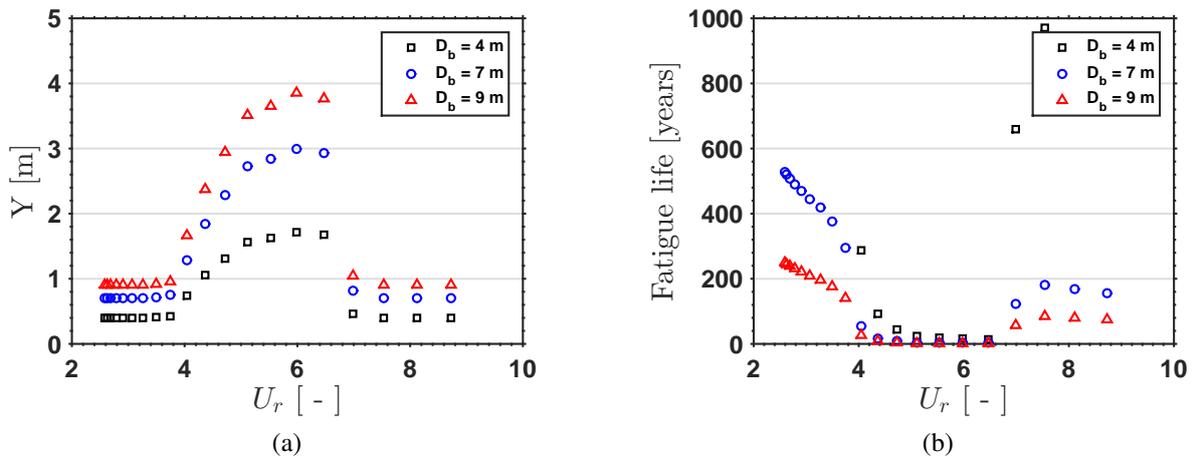

(a)

(b)

Figure 6 – (a) Cross-flow displacement and (b) Life time in function of D_b.

Another parameter analyzed was the Strouhal number (St), which is a very important variable in a flow and it is used to describes the oscillating vortex shedding. It is observed that further of change the cross-flow displacement, the Strouhal number modified the lock-in zone, as shown in Fig. 7. The maximum displacement obtained is 2.87 m for $St = 0.18$, which represents a life time of 3.02 years, while for $St = 0.26$, the displacement get is 1.44 m, resulting in a life time of 27.10 years. In practice, this parameter will depend on the environment of the local of installation of riser and usually doesn't suffers big variations, but is an important variable that provides the knowledge of the range of lock-in that the structure will have in operation.

The damping factor is the dimensionless number which denotes the amount of energy the system dissipates when the riser moves into the water, and depends basically of the fluids characteristics. By studying these parameter it can be noticed that the higher the damping, the more difficult become the displacement and thus, the lifetime become higher, as can become seen in Fig. 8. For $\xi = 0.001$, the maximum displacement is 2.77 m, which represents a lifetime of 3.54 years, for the higher damping of $\xi = 0.012$, a maximum of 1.94 m displacement is obtained, causing a lifetime of 10.25 years. Once known the ξ, it is not possible to modify this parameter, and its variations usually are very are Even though, a small variation of the damping may not affect the life time very much, as shown by the simulations.
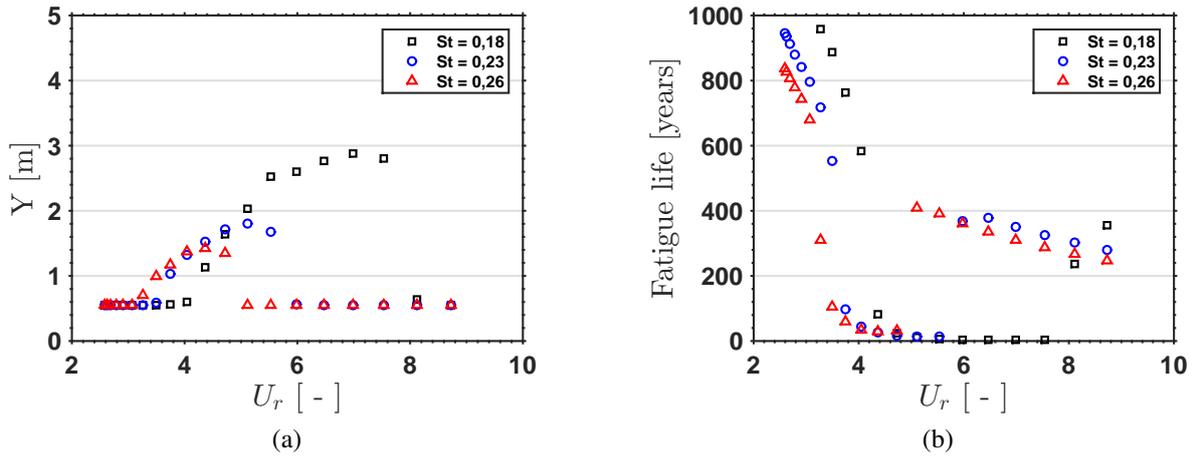
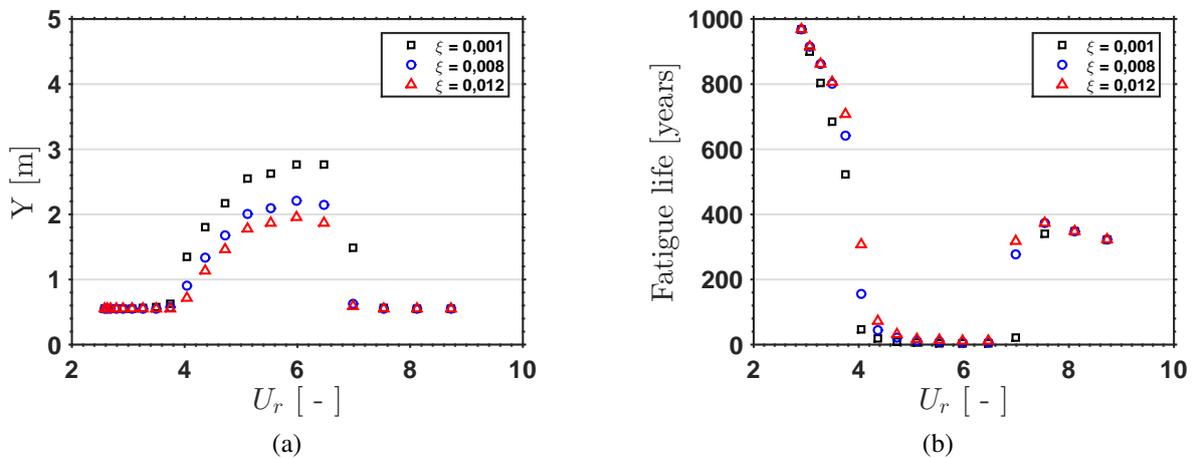**Figure 7 – (a) Cross-flow displacement and (b) Life time in function of St.**

Figure 8 – (a) Cross-flow displacement and (b) Life time in function of ξ

CONCLUSIONS

The complexity of the hybrid riser systems and its interactions with the surrounding fluid difficults the mathematical modeling of the dynamics of the system. In view of this complexity, the system is approached on a simplified form, considering the fluid velocity constant along the water depth, which is not a real case, when the fluid flow have a variation of velocity and direction. The Strouhal number used considers the worst case, when in the real case it varies according to the local of installation of the hybrid-riser system. Moreover the life time was obtained considering a constant current velocity associated to the maximum displacement of the buoy.

Although these simplifications are applied to the problem, it provides good results about the life time and allows a very interesting understanding of all the dynamics of the system, where the interference of each parameter in isolation on the life time and on the dynamic behavior of the model can be observed. In the intention of a better understanding of the system and to get more complete results more degrees of freedom could be considered, by adding more points of analysis along the length of the riser. Each point could be described by the same equations showed in this paper, thus allowing a variation in the flow velocity at these points, possibly leading to a more usefull representation of the behavior of the real riser system.

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