Lumped-parameter modular modeling of a pipe conveying fluid

Renato Maia Matarazzo Orsino[1] **and Renan Letizio Di Giulio**[1]

[1] Offshore Mechanics Laboratory, Mechanical Engineering Department, Escola Politécnica, University of São Paulo, Av. Professor Lucio Martins Rodrigues, Tv. 4, n. 434, São Paulo - SP, 05508-020, Brazil, reorsino@usp.br, renan.giulio@usp.br

Abstract: One of the most classical problems in Fluid-Structure Interaction, the pipe conveying fluid was modeled for the first time as a lumped-parameter articulated chain of rigid pipes. Even though high-hierarchy and reduced-order models have proved to be effective to analyze the dynamic response of this system, their derivation is more complex when compared to a lumped-parameter (LPM) one. This paper proposes a modular method for deriving a generic n-dof LPM for a pipe ejecting or aspirating fluid. By a modal analysis based on comparisons with a benchmark continuous model, the accuracy of a LPM in representing the dynamics of the system is analyzed as a function of n. Similar comparisons to reduced-order models of similar number of degrees of freedom are also performed.

Keywords: Fluid-Structure Interaction, Pipe Conveying Fluid, Modular Modeling Methodology, Lumped-parameter Modeling, Reduced Order Modeling

INTRODUCTION

The dynamics of a pipe conveying fluid is a classical Fluid-Structure Interaction problem which has been extensively studied over the past half-century, as documented in a treatise by Païdoussis (2014), involving theoretical, computational and experimental analyses. In most of these studies the pipe is modeled as a continuous one-dimensional flexible structure. However, the first mathematical model for this system, introduced by Benjamin (1961), consisted of a planar chain of articulated rigid pipes with one fixed-end and one free-end. Even with a small number of degrees of freedom, this model was able to represent the main characteristics of the dynamic response of this system. Altough this might seem to highlight an advantage of adopting a lumped-parameter model (LPM) for this system, there is no way to determine *a priori* the number of degrees of freedom to approximate the response obtained by a continuous model up to a given precision. Moreover, contrary to reduced-order modeling (ROM), the increase of the number of degrees of freedom is not even enough to ensure convergence when such a modeling strategy is adopted. This paper aims to propose an investigation on this issue.

Based on already sucessful applications of the Modular Modeling Methodology (Orsino, 2016; Orsino, 2017) to the derivation of non-linear finite element method and reduced-order models for a elastic cantilevered pipe ejecting fluid (Orsino and Pesce, 2018a; Orsino and Pesce, 2018b; Orsino, Pesce and Franzini, 2017), this paper presents a modular scheme for deriving a generic n-dof lumped-parameter model for a pipe ejecting or aspirating fluid. Applying numerical analyses the dynamical response of this model is explored and the convergence to results observed in the continuous model is verified.

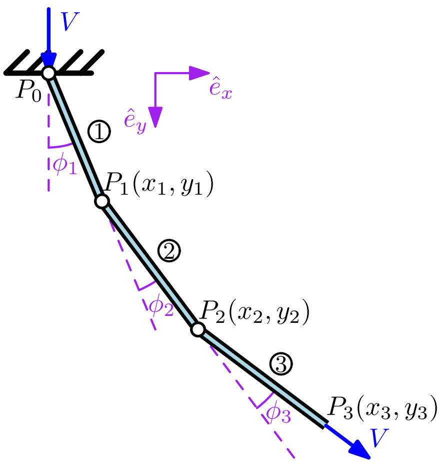

Figure 1: 3-dof lumped-parameter model of a cantilevered pipe ejecting fluid

LUMPED-PARAMETER MODEL (LPM)

A lumped-parameter model for a cantilevered pipe is proposed, following Benjamin (1961), by considering that it is composed by a finite number (n) of articulated rigid segments. Each revolute joint is assumed to be coupled with a linear torsional spring. Numbering the revolute joints from 1 to n, successively, starting from the one at the clamped end section of the pipe, denote by P_{j-1}, $j = 1,\ldots,n$, the center of the j-th joint. Also, let P_n denote the center of the free end section. The j-th segment of the pipe, delimited by the cross sections centered at P_{j-1} and P_j, is modeled as a planar rigid body conveying an internal plug flow with a relative velocity of constant magnitude V. Figure 1 illustrates a 3-dof model for a clamped-free pipe ejecting fluid.

The derivation of the mathematical model follows from the application of McIver's extended form of Hamilton's principle (Padoussis, 2014):

$$\delta \int_{t_1}^{t_2} L\,dt + \int_{t_1}^{t_2} (\delta W_{\mathrm{M}} + \delta W_{\mathrm{D}})\,dt = 0 \tag{1}$$

with L denoting the Lagrangian of the system, δW_{M} the virtual work associated to the flux of momentum due to the internal flow and δW_{D} the virtual work associated to dissipative drag forces. Adopt the following notation:

- $\vec{r}_j = x_j\hat{e}_x + y_j\hat{e}_y$ is the the position vector, with respect to P_0, of the point P_j;
- $\vec{r}^{\,*}_j$ is the position vector, with respect to P_0, of the center of mass G_j of the j-th segment of the pipe;
- $\hat{\tau}_j$ is the unit vector of the direction of the centerline $P_{j-1}P_j$;
- $\vec{\omega}_j$ is the angular velocity of the j-th segment of the pipe;
- l_j is the length of the segment $P_{j-1}P_j$;
- μ_{p} and μ_{f} are the masses per unit of length of the pipe and the internal fluid, respectively;
- η_j is the radius of gyration of the j-th segment of the pipe;
- k_j is the torsional stiffness of the linear spring at the j-th joint;
- $\vec{g} = g\hat{e}_y$ is the local acceleration of gravity;
- d_{i} and d_{i} are the external and internal diameters of the pipe;
- C_{D} is the drag coefficient of any segment of the pipe;
- ρ_{e} is the mean density of fluid in the external environment.

When the pipe ejects fluid, i.e. the free end corresponds to the outlet section of the pipe, the following expressions hold:

$$L = \sum_j \left[\frac{1}{2} l_j(\mu_{\mathrm{p}}+\mu_{\mathrm{f}})\left(\frac{\mu_{\mathrm{p}}}{\mu_{\mathrm{p}}+\mu_{\mathrm{f}}}\dot{\vec{r}}^{\,*2}_j + \frac{\mu_{\mathrm{f}}}{\mu_{\mathrm{p}}+\mu_{\mathrm{f}}}(\dot{\vec{r}}^{\,*}_j + V\hat{\tau}_j)^2 + \eta_j^2\vec{\omega}_j^2 + 2\vec{g}\cdot\vec{r}^{\,*}_j\right) - \frac{1}{2}k_j\phi_j^2\right] \tag{2}$$

$$\delta W_{\mathrm{M}} = -\mu_{\mathrm{f}} V(\dot{\vec{r}}_n + V\hat{\tau}_n)\cdot\delta\vec{r}_n \tag{3}$$

Denoting by χ_j the ratio between the lenght of the segments $P_{j-1}G_j$ and $P_{j-1}P_j$, it can be stated that:

$$\vec{r}^{\,*}_j = (1-\chi_j)\vec{r}_{j-1} + \chi_j\vec{r}_j, \qquad \hat{\tau}_j = \frac{\vec{r}_j - \vec{r}_{j-1}}{l_j} \qquad \text{and} \qquad \vec{\omega}_j^2 = \frac{(\dot{\vec{r}}_j - \dot{\vec{r}}_{j-1})^2}{l_j^2} \tag{4}$$

On the other hand, when the pipe is aspirating fluid, in which case the free end is the inlet section of the pipe, the following expressions must be used in the derivations:

$$L = \sum_j \left[\frac{1}{2} l_j(\mu_{\mathrm{p}}+\mu_{\mathrm{f}})\left(\frac{\mu_{\mathrm{p}}}{\mu_{\mathrm{p}}+\mu_{\mathrm{f}}}\dot{\vec{r}}^{\,*2}_j + \frac{\mu_{\mathrm{f}}}{\mu_{\mathrm{p}}+\mu_{\mathrm{f}}}(\dot{\vec{r}}^{\,*}_j - V\hat{\tau}_j)^2 + \eta_j^2\vec{\omega}_j^2 + 2\vec{g}\cdot\vec{r}^{\,*}_j\right) - \frac{1}{2}k_j\phi_j^2\right] \tag{5}$$

$$\delta W_{\mathrm{M}} = \mu_{\mathrm{f}} V(\dot{\vec{r}}_n - \alpha V\hat{\tau}_n)\cdot\delta\vec{r}_n \tag{6}$$

In this latter expression, α stands for a non-dimensional parameter associated to the velocity profile of the incoming flow in the inlet section, which, in the aspirating case, might not be modelled as a plug flow, as it is inside the pipe. Indeed, let V_{i} be a function that evaluates, for each point in the inlet surface $\mathrm{S_i}$ of the aspirating pipe, the component, orthogonal to the surface, of relative velocity of the incoming flow. Assuming that the fluid is incompressible, from continuity it can be stated that:

$$\iint_{S_i} V_i \, dS = V \frac{\pi d_i^2}{4} \tag{7}$$

A similar relation, based on the integral over the inlet surface of V_i^2 is adopted as a definition for α:

$$\iint_{S_i} V_i^2 \, dS = \alpha V^2 \frac{\pi d_i^2}{4} \tag{8}$$

If the flow at the inlet section comes close to an uniform profile, then α approaches the unity. Even though, considering the definition provided by Eq. (8), α might be a function of the state of the system, it can be assumed, at least in a first approximation, as a constant representing a time average value of this function whenever the dynamics of the system shows some regular behavior.

Dissipative effects due to drag can be approximated by choosing representative points P_j° at each segment $P_{j-1}P_j$, so that $P_{j-1}P_j^\circ$ represents a fraction ξ_j of the lenght l_j of the segment. Let $\vec{r}_j^\circ$ be the position vector of P_j° with respect to P_0, so that $\vec{r}_j^\circ = (1-\xi_j)\vec{r}_{j-1} + \xi_j \vec{r}_j$. In the absence of any external free-stream flow, the total virtual work due to drag effects can be approximated by the following expression:

$$\delta W_D = \sum_j \left[\frac{1}{2} \rho_e C_D d_e l_j |\vec{u}_j| \vec{u}_j \right] \cdot \left[(1-\xi_j)\delta\vec{r}_{j-1} + \xi_j \delta\vec{r}_j \right] \qquad \text{with} \qquad \vec{u}_j = \hat{\tau}_j \times (\hat{\tau}_j \times \dot{\vec{r}}_j^\circ) \tag{9}$$

It is worth noting that the choice of the representative points P_j° affects the computation of generalized drag forces. However, increasing the number n of segments adopted in the discretization of the pipe makes this computation less sensitive on the values of ξ_j selected.

Therefore, a n-dof lumped-parameter model for this system can be expressed in terms of the coordinates (x_j, y_j, ϕ_j), $j = 1, \ldots, n$, provided that the following constraints among these variables are satisfied:

$$\cos\phi_1 = \frac{\vec{r}_1 \cdot \hat{e}_y}{l_1} \qquad \text{and} \qquad \sin\phi_1 = \frac{\vec{r}_1 \times \hat{e}_y \cdot \hat{e}_z}{l_1} \tag{10}$$

$$\cos\phi_j = \frac{(\vec{r}_j - \vec{r}_{j-1}) \cdot (\vec{r}_{j-1} - \vec{r}_{j-2})}{l_j l_{j-1}} \qquad \text{and} \qquad \sin\phi_j = \frac{(\vec{r}_j - \vec{r}_{j-1}) \times (\vec{r}_{j-1} - \vec{r}_{j-2}) \cdot \hat{e}_z}{l_j l_{j-1}} \qquad (j = 2, \ldots, n) \tag{11}$$

The Modular Modeling Methodology provides algorithms to enforce these constraints in the Hamiltonian formulation previously presented, leading to the equations of motion wanted for a generic n-dof lumped-parameter model of a pipe conveying fluid.

MODAL ANALYSIS

A modal analysis of the LPM can reveal how accurately it can approximate results obtained by a continuous model. The scenarios explored by Gregory and Padoussis (1966), in their modal analysis based on the use of a linear partial differential equation of motion for a cantilevered pipe ejecting fluid, are adopted as paradigms. The parameters of the LPM are adjusted to match the values of the non-dimensionals $\beta = \mu_f/(\mu_p + \mu_f)$ and $\gamma = gl^3(\mu_p + \mu_f)/EI$ (with $l = \sum_j l_j$) adopted in these reference scenarios; the non-dimensional $v = Vl\sqrt{\mu_f/EI}$, representing the magnitude of the internal plug flow relative velocity, is treated as a control parameter in terms of which *root loci* diagrams are produced for different n-dof articulated models. For the sake of comparison, these same diagrams are generated for the reduced-order model (ROM) proposed for this system by Orsino and Pesce (2018b), for different numbers m of projection functions (note that, the number of degrees of the ROM is m). Examples of these *root loci* diagrams (non-dimensional eigenvalues) for the paradigmatic case of a pipe ejecting fluid with $\beta = 0.2$ and $\gamma = 0$ are shown in Fig. 2. For a better visualization on the dependency of the eigenvalues λ on the control parameter v, the same results for the same scenarios are also plotted in Fig. 3, as Im(λ) vs. v charts, in which a color scale is used to indicate the magnitudes and signs of the real parts of the eigenvalues, highlighting the transitions to unstable response (critical velocities).

It is worth noting that once the generalized drag forces modeled involve only quadratic terms, they have no effect in the modal analysis presented in this section. Also, due to an assumption of homogeneity of the segments, for an n-dof articulated pipe: $l_j = l/n$, $\chi_j = 1/2$ and $\eta_j^2 = l_j^2/12$. Setting the torsional stiffnesses of the springs to match the paradigmatic model, however, is not a trivial procedure. The strategy adopted is to assume that all the $k_j = k$. Then, in the absence of internal flow, the shape of the first mode of the articulated pipe can be estimated by assuming that under the application of a generic orthogonal force at the free-end, the deflection curve described by the joints should be the same as the polynomial curve described by an equivalent elastic cantilever. Thus, the Rayleigh quotient so obtained provides an estimate of the frequency of the first mode as a function of k and n. Therefore, k is obtained by comparing this expression with the one given in the literature for the frequency of the first mode of a cantilever (expressed as a function of EI and l).

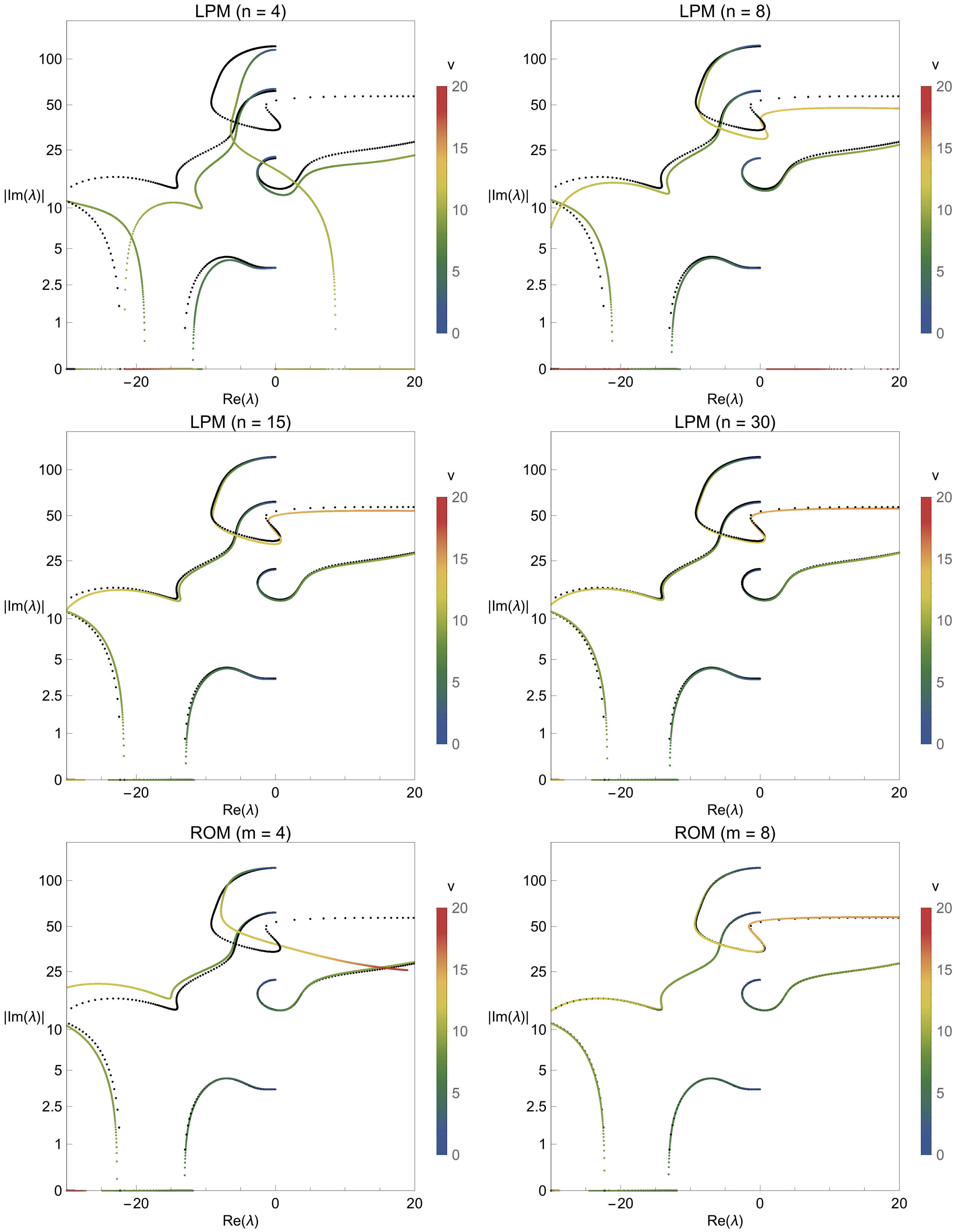

Figure 2: *Root loci* diagrams for discretized models of a pipe ejecting fluid with $\beta = 0.2$ and $\gamma = 0$. The magnitude of the non-dimensional internal flow relative speed is represented by the color scale. Black dots represent the paradigmatic results of the benchmark PDE model.

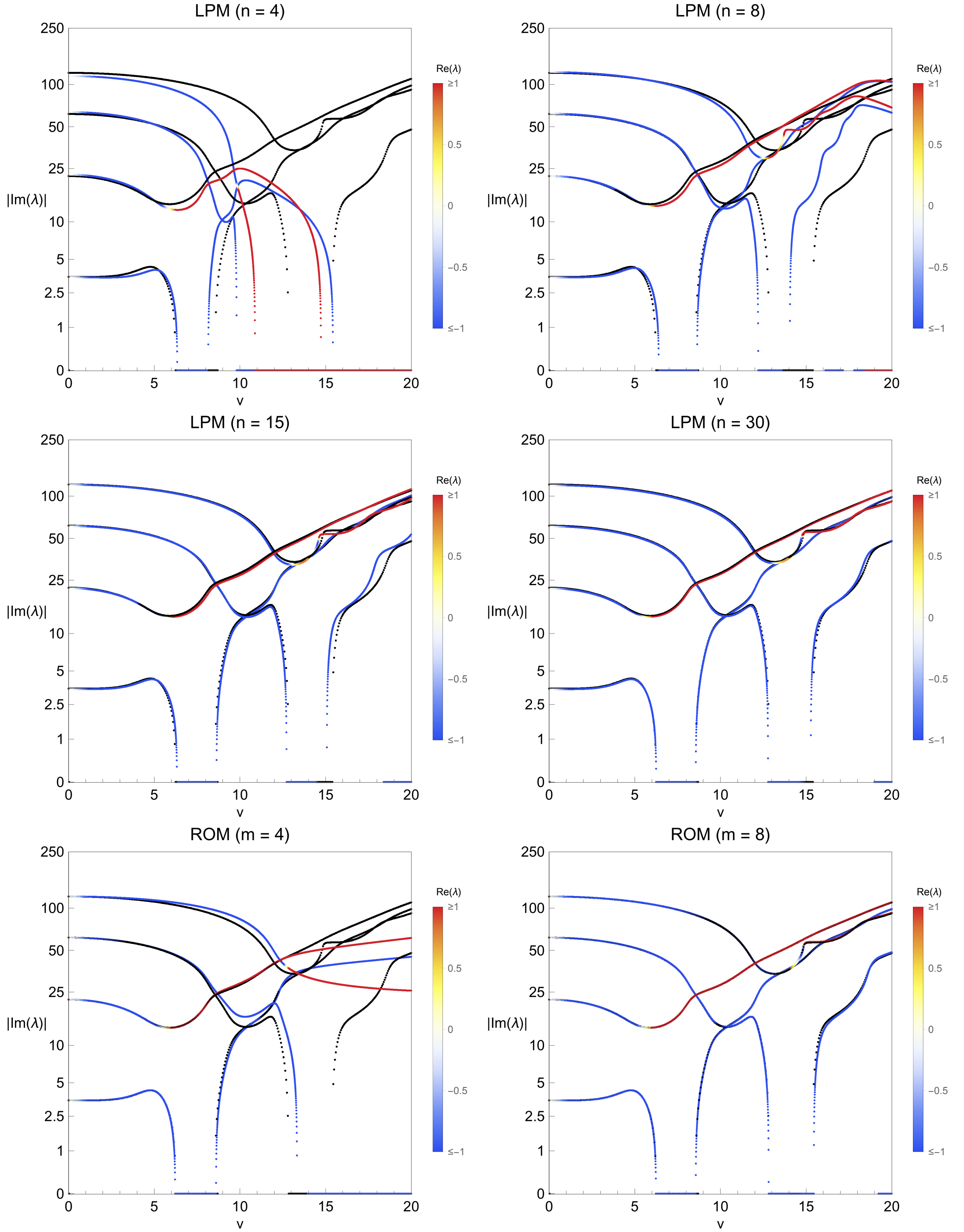

Figure 3: Absolute values of the imaginary parts of the non-dimensional eigenvalues as a function of the non-dimensional internal flow relative speed for discretized models of a pipe ejecting fluid with $\beta = 0.2$ and $\gamma = 0$. The associated real parts are represented by a temperature color scale. Black dots represent the paradigmatic results of the benchmark PDE model.

FINAL REMARKS

This paper presented a general formulation, based on the application of Analytical Mechanics and on the Modular Modeling Methodology for the derivation of non-linear lumped-parameter models for cantilevered pipes either ejecting or aspirating fluid, the latter case requiring some special considerations in modeling the flux of momentum in the inlet section.

Modal analyses revealed that, at least in linearized scenarios, the dynamics described by the lumped-parameter models of the ejecting pipe actually converge to the description obtained by the continous paradigmatic model even though requiring a greater number of degrees of freedom to attain a similar precision, when compared to reduced-order models. The modal analyses presented are also useful to identify bifurcation features of the model, being possible to predict a critical speed at the second natural mode in all the analyzed scenarios. A natural extension of this work would involve applications of non-linear dynamics analyses on lumped-parameter models with small numbers of degrees of freedom in order to acquire a better understanding on the global behavior of this system.

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