

A study on the dynamics of a periodic bending-shear-torsion coupled composite beam metamaterial

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Abstract: There has been a constant search for materials that present better mechanical characteristics and low-cost, and structural forms that have good behavior to mechanical vibrations. Thus, composite materials and periodic structures have been widely studied. A composite material consists of the combination of two or more materials which together produces desirable mechanical properties, whereas the periodic structure consists of substructures (or unity cell) positioned periodically in a principal structure. This cells produces a variation of the stiffness and in the range of frequency, consequently, in the dynamic behavior of the mechanical structure. The objective of this paper is to present a study of the structural behavior of a beam made of the composite laminated material, subdivided in small structures and assembled periodically. In this work was analyzing the response frequency to elements with 0° and 45° fiber orientations. The periodic structure is built coupling these elements. Through of the results, it was possible to notice that the periodic structure presented band gap for high frequency (6 at 9 kHz). It has observed that the structure behaves as if there were a torsional dis-coupling. Besides, it verified the relationship between the wavenumber and the fiber orientation for a composite material beam, and the effects of the variation these parameters and the periodic structure in the structural vibration.

Keywords: Composite laminated beam, periodic structure, mechanical vibration, bending-shear-torsion coupling, Timoshenko Beam

INTRODUCTION

Composite material consists in the combination of two or more materials which together produce desirable properties and better mechanical characteristics than conventional materials. They are commonly grouped into three different types: fibrous composites, particulate composites, and laminated composites. Every consists of a basic combination of a matrix material with a fiber material. The matrix material is responsible for keeping the fiber together, it acts as a load-transfer medium and protects them, whereas the fibers are the principal load-carrying members and it is used to improve properties through variation of the fiber direction (Reddy, 2003). On the other hand, the periodic structures also have been widely investigated how another manner to obtain the desired mechanical characteristic and, with this, better structural dynamic behavior.

Most studies related to mechanical vibrations in the composite material beam analyze the effects caused by the different fibers orientations in the frequencies and modes vibration. Teh and Huang (1980) analyzed the influence of the fiber orientation in the bending-torsion coupling beam. In this study, is possible to identify that there are ranges of fibers orientations that the vibration modes of the structure and their frequencies are more affected than others. Abarcar and Cunniff (1972) compared the experimental and numerical results of the effect of fiber orientation on the mode of vibration of cantilever beams of unidirectional fiber reinforced composite materials, they analyzed the interaction between bending and twisting for some fiber orientations.

In most studies of periodic structures has been characterized the effects due to the add some geometric discontinuity that repeat along of the structure. Some papers present the adding mass and stiffness in periodic regions. In this paper, the periodic structure will be presented by adding different fiber orientation for each substructure (or unity cell) (Fig. 5). This permit different combinations of stiffness. Besides, in this paper, it will be present the different fibers orientation effects in the wavenumber response.

For analysis, it will be used to spectral element method (SEM), since is a widely used to solving wave propagation problems in structures (Doyle, 1997), due to low computational cost, and accuracy in results, even though, it is used to solve simple problems due to difficulty in obtaining exact wave solutions for a multi-dimensional problem (Lee, 2009). The SEM is a semi-analytic method that used the exact dynamic stiffness matrices. The spectral element assembly is done using the same principle as finite element matrix. However, SEM is calculated in the frequency domain (Doyle, 1997). The formulation, it uses the superposition of wave modes via Discrete Fourier Transform Theory and FFT algorithm (Lee, 2009).

The purpose of this paper is to study the structural behavior of a beam made of a composite laminated material subdivided in small structures and assembled periodically. Each periodic structure may present different ply orientation for laminae. The composite material was chosen due to the fiber-reinforced composite materials have many advantages (high

strength-density ratio, weight reduction, corrosion and fatigue resistance) and industrial applications, e.g., Aeronautical and Automotive industry, Civil Construction, Wind turbine blades, Structural mechanics are only some applications (Gibson, 2011). In this case, it permits to obtain better mechanical behavior in the different combination of matrix and fibers materials.

Finally, the paper provides the necessities mechanisms to comprehend and study the mechanical behavior of periodic structures and analyze the dynamics effects of the beam made of the composite material assembled periodically. This paper considers the dynamics of axially loaded bending-shear-torsion coupled composite beams, based on the first shear deformation theory (FSDT), presented by Lee, U. (2009), which is the equivalent of the Timoshenko beam theory for composite beams. Therefore, this paper will discuss and illustrate the effect of structural dynamic that can be obtained by adding periodic structures with different plies orientation of composite laminated material in a Timoshenko beam.

1 THEORY

The lamina constitutive equation can be represented for a matrix corresponding to an orthotropic material (Fig. 1).

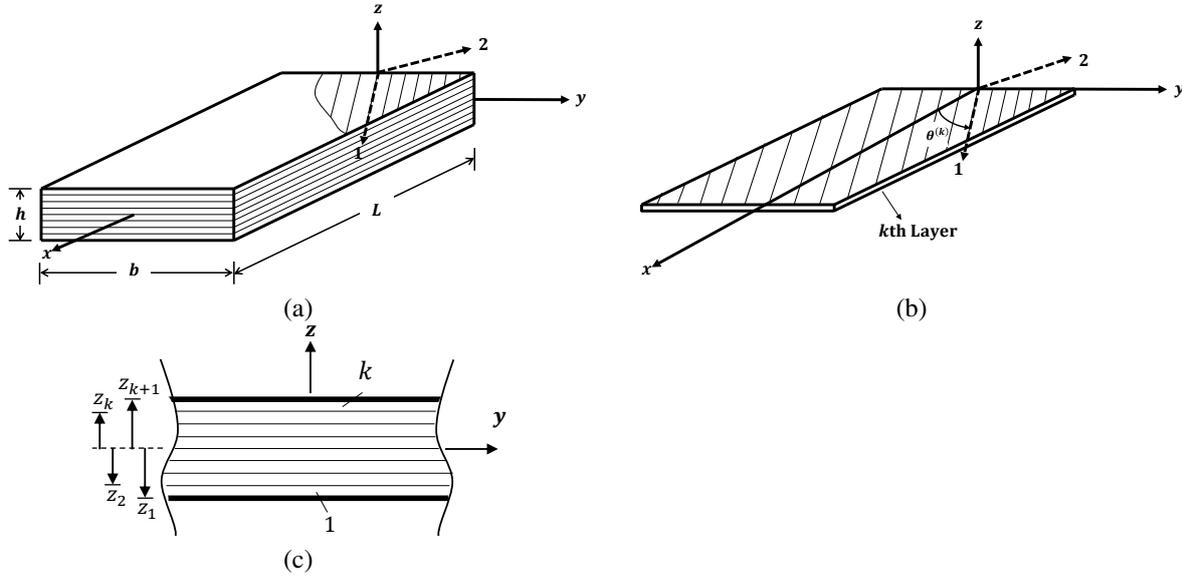


Figure 1 – Free body diagram.

In this case, for monoclinic material, it assumes that lamina is a continuum and it behaves as a linear elastic material. Equation 1 presents the constitutive equation (Reddy, 2003).

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{Bmatrix} \quad (1)$$

Equation 1 was written to a coordinate system coinciding with the principal material coordinate system. It can be represented by the reduced form

$$\{\sigma_i\} = [C_{ij}] \{\varepsilon_j\} \quad (2)$$

where $[C_{ij}]$ are relationship with stiffness matrix, it represents the material coefficients. Equation 2 can also be expressed in terms of strain-stress relations

$$\{\varepsilon_j\} = [S_{ij}] \{\sigma_i\} \quad (3)$$

where S_{ij} are the compliance coefficients ($[C] = [S]^{-1}$).

In terms of $[S]$

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{Bmatrix} = \begin{bmatrix} S_{11} & S_{12} & S_{13} & 0 & 0 & 0 \\ S_{12} & S_{22} & S_{23} & 0 & 0 & 0 \\ S_{13} & S_{23} & S_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & S_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & S_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & S_{66} \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{Bmatrix} \quad (4)$$

where

$$\begin{aligned} S_{11} &= \frac{1}{E_1}, & S_{12} &= -\frac{\nu_{21}}{E_2}, & S_{13} &= -\frac{\nu_{31}}{E_3} \\ S_{21} &= -\frac{\nu_{12}}{E_1}, & S_{22} &= \frac{1}{E_2}, & S_{23} &= -\frac{\nu_{32}}{E_3} \\ S_{31} &= -\frac{\nu_{31}}{E_1}, & S_{32} &= -\frac{\nu_{23}}{E_3}, & S_{33} &= \frac{1}{E_3} \\ S_{44} &= \frac{1}{G_{23}}, & S_{55} &= \frac{1}{G_{13}}, & S_{66} &= \frac{1}{G_{12}} \end{aligned} \quad (5)$$

The $E_{11}, E_{22}, E_{33}, G_{12}, G_{13}, G_{23}, \nu_{12}, \nu_{13}, \nu_{23}$ are independent material coefficients. In the reduced form it can be wrote to E_1, E_2, E_3 . And due to symmetric

$$\frac{\nu_{ij}}{E_i} = \frac{\nu_{ji}}{E_j} \quad (6)$$

The transformation to the material coordinate system (m) of the constitutive equation for a problem coordinate system (p) is necessary to apply a matrix of transformation Eq. 7. The material/problem stress relation is given by

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{yz} \\ \sigma_{xz} \\ \sigma_{xy} \end{Bmatrix} = \begin{bmatrix} c^2 & s^2 & 0 & 0 & 0 & -2sc \\ s^2 & c^2 & 0 & 0 & 0 & 2sc \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & c & s & 0 \\ 0 & 0 & 0 & -s & c & 0 \\ cs & -cs & 0 & 0 & 0 & c^2 - s^2 \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{Bmatrix} \quad (7)$$

where

$$c = \cos \theta \quad s = \sin \theta \quad (8)$$

In reduced form

$$\{\sigma\}_p = [T] \{\sigma\}_m \quad (9)$$

The relationship between σ_p and σ_m is given by

$$\{\sigma\}_p = [T] \{\sigma\}_m = [T] [C]_m \{\varepsilon\}_m = [T] [C]_m [T]^T \{\varepsilon\}_p = [C]_p \{\varepsilon\}_p \quad (10)$$

The matrix corresponding to a plane stress constitutive relations for an orthotropic material can be expressed by the plane stress-stiffness reduced form

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} \quad (11)$$

$$\begin{Bmatrix} \sigma_{yz} \\ \sigma_{xz} \end{Bmatrix} = \begin{bmatrix} Q_{44} & 0 \\ 0 & Q_{55} \end{bmatrix} \begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} \quad (12)$$

Where

$$\begin{aligned} Q_{11}^{k_l} &= \frac{E_1}{1 - \nu_{12}^{k_l} \nu_{21}^{k_l}}, & Q_{12}^{k_l} &= -\frac{E_2 \nu_{21}^{k_l}}{1 - \nu_{12}^{k_l} \nu_{21}^{k_l}}, & Q_{22}^{k_l} &= \frac{E_2 \nu_{21}^{k_l}}{1 - \nu_{12}^{k_l} \nu_{21}^{k_l}} \\ Q_{44}^{k_l} &= G_{23}^{k_l}, & Q_{55}^{k_l} &= G_{13}^{k_l}, & Q_{66}^{k_l} &= G_{12}^{k_l} \end{aligned} \quad (13)$$

k_l represents the laminae.

Applying the coordinate transformation matrix, the stress-strain relations are given by Reddy(2003).

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix} \quad (14)$$

$$\begin{Bmatrix} \sigma_{yz} \\ \sigma_{xz} \end{Bmatrix} = \begin{bmatrix} \bar{Q}_{44} & \bar{Q}_{45} \\ \bar{Q}_{54} & \bar{Q}_{55} \end{bmatrix} \begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} \quad (15)$$

where

$$\begin{aligned} \bar{Q}_{11} &= Q_{11} \cos^4 \theta + 2(Q_{12} + 2Q_{66}) \sin^2 \theta \cos^2 \theta + Q_{22} \sin^4 \theta \\ \bar{Q}_{12} &= (Q_{11} + Q_{22} - 4Q_{44}) \sin^2 \theta \cos^2 \theta + Q_{12}(\sin^4 \theta + \cos^4 \theta) \\ \bar{Q}_{16} &= (Q_{11} - Q_{12} - 2Q_{66}) \sin \theta \cos^3 \theta + (Q_{11} - Q_{22} + 2Q_{66}) \sin^3 \theta \cos \theta \\ \bar{Q}_{22} &= Q_{11} \sin^4 \theta + 2(Q_{12} + 2Q_{66}) \sin^2 \theta \cos^2 \theta + Q_{22} \cos^4 \theta \\ \bar{Q}_{26} &= (Q_{11} - Q_{12} - 2Q_{66}) \sin^3 \theta \cos \theta + (Q_{11} - Q_{22} - 2Q_{66}) \sin \theta \cos^3 \theta \\ \bar{Q}_{66} &= (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66}) \sin^2 \theta \cos^2 \theta + Q_{66}(\sin^4 \theta + \cos^4 \theta) \\ \bar{Q}_{44} &= Q_{44} \cos^2 \theta + Q_{55} \sin^2 \theta \\ \bar{Q}_{55} &= (Q_{55} - Q_{44}) \sin \theta \cos \theta \\ \bar{Q}_{45} &= Q_{55} \cos^2 \theta + Q_{44} \sin^2 \theta \end{aligned} \quad (16)$$

MATHEMATICAL EQUATIONS

From the Fig. 2 and using the Hamilton's principle, it is possible to obtain the governing differential equations of motion (Eq. 17) and associated boundary conditions for composite beams based on the first-order shear deformation theory (FSDT).

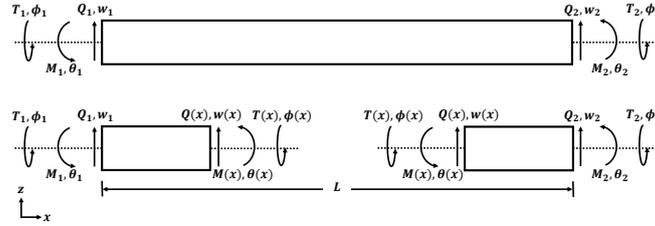


Figure 2 – Free body diagram.

According to Lee (2009), the equation of motion for a composite laminated beam is given as

$$\begin{aligned} \kappa GA(w_0'' - \theta') + P(w_0'' - y_{CA}\phi'') - \rho A(\ddot{w}_0 + y_{CM}\phi'') - c_1 \dot{w}_0 + f_w(x, t) &= 0 \\ GJ\phi'' + K_T\theta'' + P(y_{CA}w_0'' + r_g^2\phi'') - \rho J\phi'' - \rho A y_{CM}\dot{w}_0 - c_3\phi' + f_\phi(x, t) &= 0 \\ EI\theta'' + K_T\phi'' + \kappa GA(w_0' - \theta') - \rho I\dot{\theta} - c_2\dot{\theta} + f_\theta(x, t) &= 0 \end{aligned} \quad (17)$$

Where w_0 , ϕ and θ represent the displacements, ρ is the density material, A , I , and J are geometric properties, c_i is the viscous damping, y_{CM} and y_{CA} are the distance between the elastic axis and the mass axis, and elastic axis and the center of the area of cross-section, respectively. This equation represents the bending-shear-torsion coupled. Besides, the foregoing equation represent the bending-shear-torsion coupled. In the literature (Weissshaar et al., 1970), (Lee, 2009), the shear rigidity (κGA), the bending rigidity (EI), the bending-torsion coupling rigidity (KT) and torsional rigidity (GJ) are defined by

$$\begin{aligned} \kappa GA &= \kappa b A_{55} \\ EI &= b(D_{11} - D_{12}^2 D_{22}^{-1}) \\ K_T &= 2b(D_{16} - D_{12} D_{26} D_{22}^{-1}) \\ GJ &= 4b(D_{66} - D_{26}^2 D_{22}^{-1}) \end{aligned} \quad (18)$$

where A_{ij} is the extensional stiffness and D_{ij} is the bending stiffness which are function that depend of laminate ply orientation (Eq. 18), stacking sequence (Eq. 19) and composite material properties (Eq. 5) (Ashton et al., 1970), whereas κ is the shear correction factor that depend of the cross section of the composite beam and b is the width. It is very important to pay attention in the coordinate system to develop Eq. 4, since changing the axes of the coordinate system, would imply in different relations D_{ij} to define Eq. (4).

$$\begin{aligned}
A &= \sum_i^N \bar{Q}_{ij} (z_{i+1} - z_i) \\
B &= \sum_i^N \bar{Q}_{ij} (z_{i+1}^2 - z_i^2) \\
D &= \sum_i^N \bar{Q}_{ij} (z_{i+1}^3 - z_i^3)
\end{aligned} \tag{19}$$

Equation 19 is obtained by integrating Eq. 11. This integral provides the resultant forces and moments. Where $(z_{k+1} - z_1)$ is the laminate thickness (Fig. 1c) and \bar{Q}_{ij} is dependent of material coefficients and fiber orientation (Eq. 16).

Wavenumber

A way of analyzing how an elastic wave propagates in the structure is evaluating to the wavenumber. The wavenumber represents an analogous measure of the temporal frequency, however, it is the measure of the spatial frequency of the wave propagation.

It indicates if the wave is a dispersive or non-dispersive. In other words, non-dispersive curves indicate that all harmonic waves travel at the same velocity, i.e if the structure is excited in the direction that your wavenumber represents non-dispersive curves, the waves travel forward and backward with the same velocity. The dispersive curves indicate that there are the different velocities of wave propagation, i.e in the same direction, there are waves that propagate faster than other.

The wavenumber is dependent of the fiber direction. The relationship between ply orientation and wavenumber is given by solving the governing equations (Eq. 17) in the frequency domain. In this case, the displacements, forces and moments are represent in the spectral analysis forms (Doyle, 1997 and Lee, 2009).

$$\begin{aligned}
\{w(x,t), \theta(x,t), \phi(x,t)\} &= \sum_{n=0}^{N-1} \{W_n(x, \omega_n), \Theta_n(x, \omega_n), \Phi_n(x, \omega_n)\} e^{j\omega_n t} \\
\{Q(x,t), M(x,t), T(x,t)\} &= \sum_{n=0}^{N-1} \{Q(x, \omega_n), M(x, \omega_n), T(x, \omega_n)\} e^{j\omega_n t}
\end{aligned} \tag{20}$$

where

$$Q(x,t) = \kappa GA(W' - \Theta), M(x,t) = EI\Theta' + K_T\Phi', T(x,t) = GJ\Phi' + K_T\Theta' \tag{21}$$

The frequency domain governing equation is obtained by substituting Eq. 20 into Eq. 17, and considering that there is not viscous damping effect, it is given by simplified homogeneous governing equation

$$\begin{aligned}
\kappa GA(W' - \Theta) + P(W'' - y_{CA}\Phi'') + \omega^2 \rho A(W + y_{CM}\Phi') &= 0 \\
GJ\Phi'' + K_T\Theta'' + P(y_{CA}W'' + r_g^2\Phi'') + \omega^2 \rho J\Phi'' + \omega^2 \rho A y_{CM}W &= 0 \\
EI\Theta'' + K_T\Phi'' + \kappa GA(W' - \Theta) + \omega^2 \rho I\Theta &= 0
\end{aligned} \tag{22}$$

The general solutions can be assumed as

$$\{W(x) = ae^{-jkx}, \Theta(x) = \alpha ae^{-jkx}, \Phi(x) = a\beta e^{-jkx}\} \tag{23}$$

Therefore, the wavenumber (k) can be represented by substituting the Eq. 23 into Eq. 22.

$$\begin{aligned}
[-(\kappa GA + P)k^2 + \kappa GA j\alpha k + \beta(\rho A \omega^2 y_{CM} - Pk^2 y_{CA})]ae^{-jkx} &= 0 \\
[-\kappa GA k + \alpha(\rho I \omega^2 - EIk^2 - \kappa GA) - K_T \beta k^2]ae^{-jkx} &= 0 \\
[\rho A \omega^2 y_{CM} - Pk^2 y_{CA} - K_T \alpha k^2 + \beta(\rho J \omega^2 - Pk^2 r_g^2 - GJk^2)]ae^{-jkx} &= 0
\end{aligned} \tag{24}$$

Therefore, analyzing the equation above, it possible to note that the wavenumber has a strong dependency of the bending, torsion and shear rigidity, consequently, it is also influenced through of the fiber orientation. Equation 24 provides an eigenvalue problem. The solution can be found solving the equation below.

$$\begin{aligned}
(\rho A \omega^2 y_{CM} - Pk^2 y_{CA})(jK_T k^3 \kappa GA - (\rho I \omega^2 - \kappa GA - EIk^2)(\rho A \omega^2 y_{CM} - Pk^2 y_{CA})) \\
-jk\kappa GA(K_T k^2(\rho A \omega^2 y_{CM} - Pk^2 y_{CA}) - jk\kappa GA(\rho J \omega^2 - Pk^2 r_g^2 - GJk^2)) \\
+(\rho A \omega^2 - k^2 \kappa GA - Pk^2)((\rho I \omega^2 - \kappa GA - EIk^2)(\rho J \omega^2 - Pk^2 r_g^2 - GJk^2) - K_T^2 k^4) = 0
\end{aligned} \tag{25}$$

Analyzing the coefficients of the Eq. 25, it is noticeable that the solution will be dependent on the structural rigidity and beam inertial properties. Thus, to comprehend the relationship between fiber orientation and wavenumber, for a frequency range (ω), it was built the dispersion relation (Fig. 3).

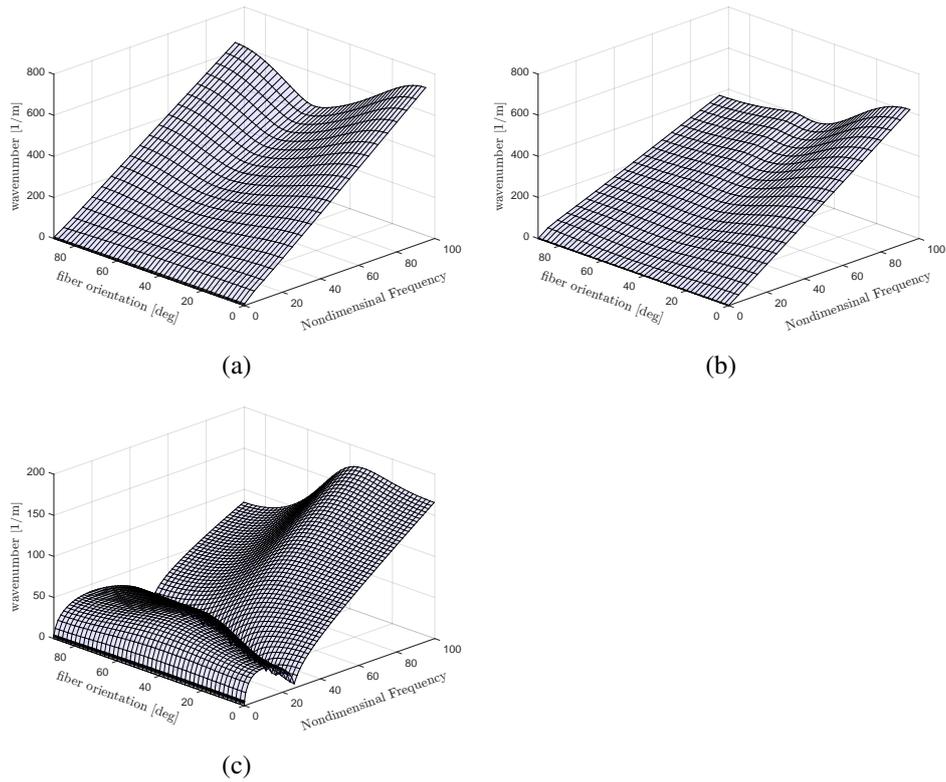


Figure 3 – The dispersion of wavenumber for different angles of fiber [0° at 90°] and frequency.

Figure 3 presents the relation between the frequency and wavenumber for different fibers orientations. It is possible to note that for Fig. 3a and Fig. 3c the characteristics of the curves are the same of the behavior of dispersive waves, whereas in the Fig. 3b, the dispersion presents a characteristic of non-dispersive waves.

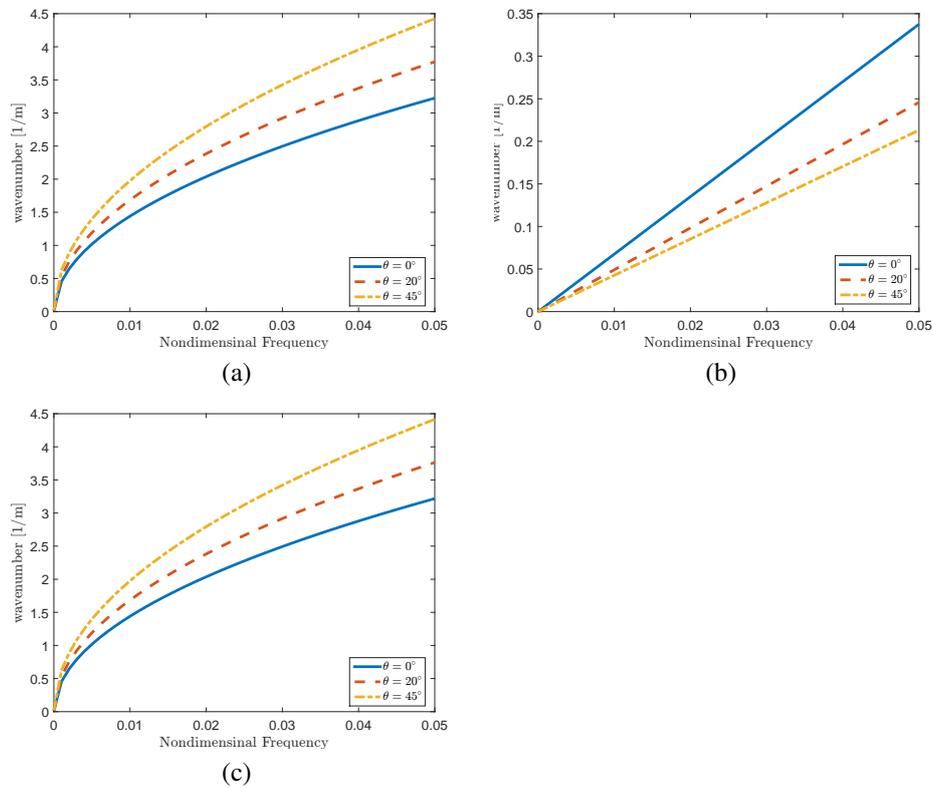


Figure 4 – Wavenumber for different angles of fiber [0° at 90°] and low frequency.

In Fig. 4a and Fig. 4c, the curves initially have the same behavior, however, there is a small difference between it. This difference is more expressive in higher frequencies how shown in the wavenumber relation(Fig. 3). The study of dispersion waves is important to understand the behavior of the structure subjected at some excitation. This allows designing a structure to attenuate some forms of vibration.

Numerical Example

A beam model is presented as an example to comprehend the dynamic behavior of a periodic structure. The beam is on free-free boundary condition, and it made of glass-epoxy material. The study considered the same beam properties presented by Teh and Huang (1980), Bannerjee and Foist (1996), and Lee and Jang (2010). Table 1 describes the material properties. The composite beam is made up of unidirectional fibers, it has single thick ply, and rectangular cross-section (Fig. 5). In Tab. 2 it is possible to obtain some configurations of periodic structures used to represent the composite beam.

Table 1 – Glass-epoxy composite material mechanical properties and geometric information.

Composite Properties	value	Geometry	value
E_{11} (GPa)	3.25	L (m)	0.1905
E_{33} (GPa)	9.71	b (m)	0.0127
G_{12}, G_{13} (GPa)	0.9025	h (m)	0.00318
G_{23} (GPa)	0.2356		
ν_{31}	0.2356		
ρ (kg/m ³)	1347		

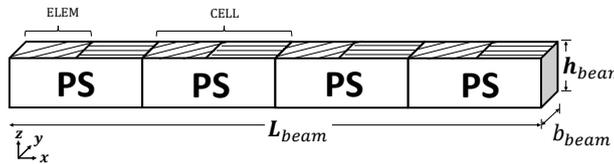


Figure 5 – Free-Free glass-epoxy composite beam. PS: Periodic structure.

Table 2 – Single thick Ply orientation.

Beam Configuration	Periodic Structures	Ply orientation(°)
$Elem_1$	-	0
$Elem_2$	-	45
$Cell_1$	-	$(45 - 0)_1$
$Cell_2$	5	$(45 - 0)_5$
$Cell_5$	10	$(45 - 0)_{10}$

RESULTS AND DISCUSSION

Following, it is presented the results of the frequency response for a beam under free-free boundary condition for different configurations of periodic coupling excited by an impulsive force.

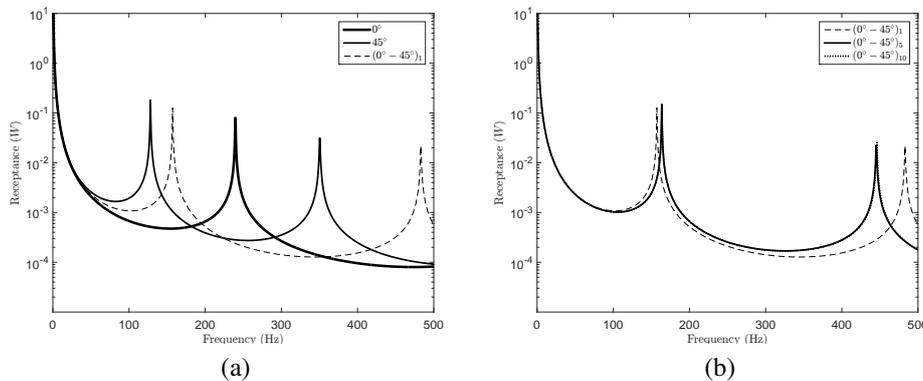


Figure 6 – Frequency response function of the effects of bending-shear-torsion coupling with $P = 0$ N. Free-free beam excited by impulsive force Q_2 . (a) and (b) Shear.

The results presented in Fig. 6, Fig. 7 and Fig. 8 shown the frequency response of a Free-Beam for each degree of freedom. In Fig. 6a and Fig. 7a coupling of single layer elements (45° and 0°) produces a variation in the frequency. These frequencies (first and second peak) are displaced. It is possible to note that the first peak occupied an intermediate position between the first peak of each of them, this also occurs to the second peak.

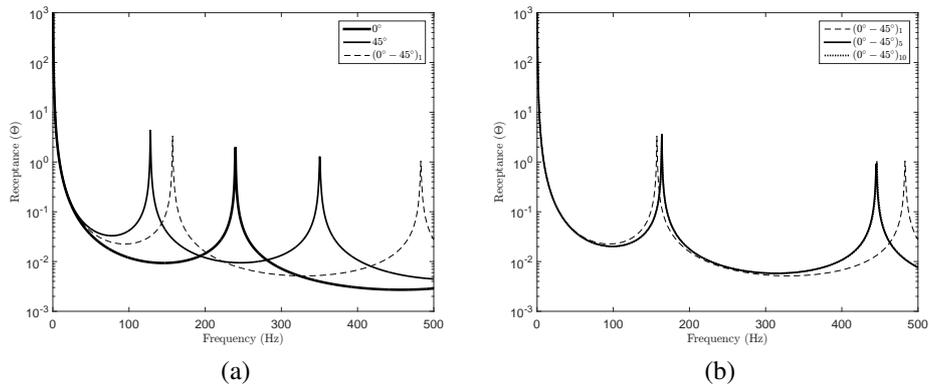


Figure 7 – Frequency response function of the effects of bending-shear-torsion coupling with $P = 0 N$. Free-free beam excited by impulsive force Q_2 . (a) and (b) Bending.

Analyzing Fig. 8a, the torsional displacement does not appear to layer element 0° , but the torsional coupling occur when the two layers (45° and 0°) are coupled. The first and second peak does not displace in the frequency to 5 and 10 periodic cells (Fig. 6b and Fig. 7b), although it presents changing in relation to a unique cell.

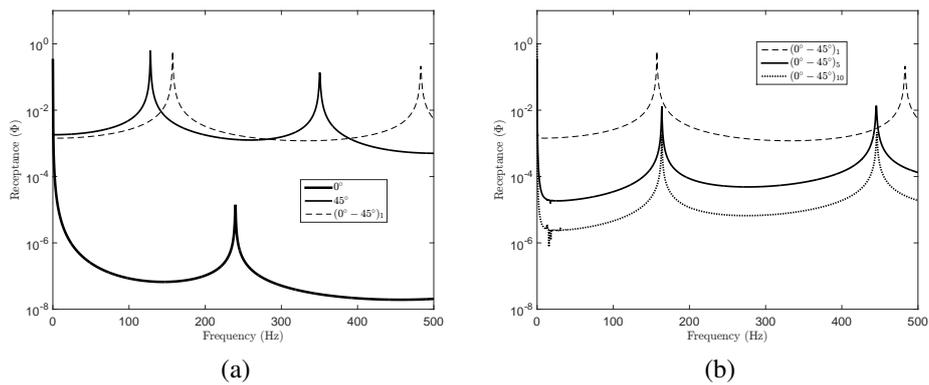


Figure 8 – Frequency response function of the effects of bending-shear-torsion coupling. Free beam excited by impulsive force Q_2 . (a) and (b) Torsion.

In Fig. 8b, the coupling does not produce a change of the frequency, however, it is remarkable that the amplitude is being each more time lower, this occurs due to the reduction of the element size, i.e this behavior probably occur due to the number of elements of the single layer of 0° producing a dis-coupling of the structure, therefore, in this case, the structure behave of the same manner that the single layer of 0° .

Figures 9 - 11 were analyzing for high frequency.

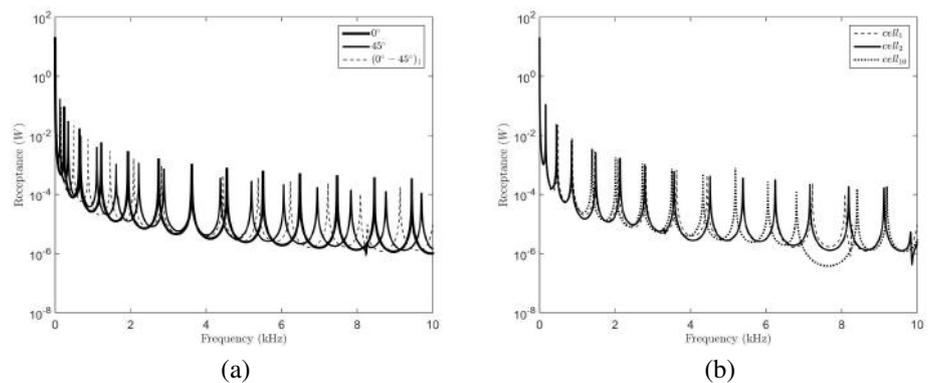


Figure 9 – Frequency response function of the effects of bending-shear-torsion coupling. (a) and (b) Shear.

In Fig 9a, there is a region between two resonances peaks, in which it is possible to observe an attenuation, probably it is caused by appearing to gap region, in this case, a band gap. In this band gaps do not occur a wave propagation, however, this effect is observed with adding for more periodic cell.

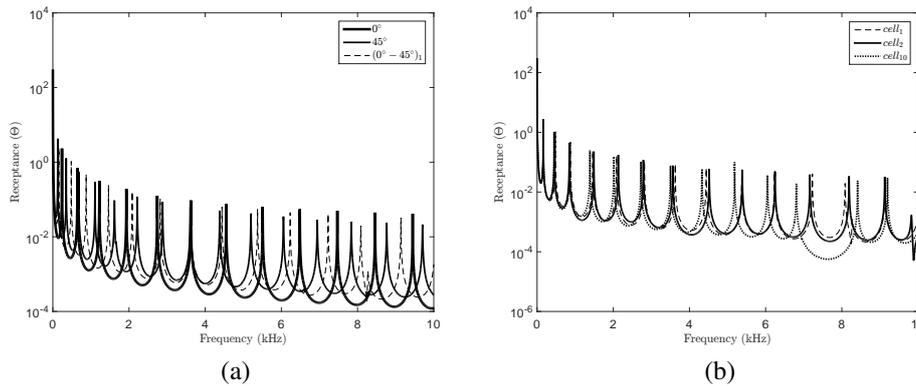


Figure 10 – Frequency response function of the effects of bending-shear-torsion coupling.(a) and (b) Bending.

Analyzing the frequency response to Θ and Φ displacements (Fig 10b and 11b), it is possible to observe the same effect that was found to W displacement. Band gap appears between the 6 and 9 kHz frequency range. Besides, the $cell_10$ results in Fig. 11b indicate that there is a dis-coupling for torsional displacement.

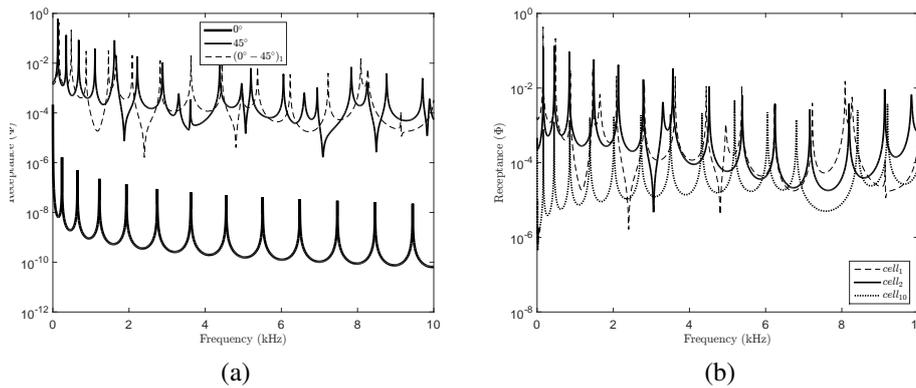


Figure 11 – Frequency response function of the effects of bending-shear-torsion coupling.(a) and (b) Torsion.

CONCLUSION

In this work, the dispersion relation for different fiber orientations was analyzed for a composite material beam with a bending-shear-torsion coupling.

The governing differential equations for composite laminated beams were presented based on the first-order shear deformation theory. Applying discrete Fourier transform (DFT) obtained a governing equation into the frequency domain, and it was used to represent the dependency of the wave number of the fiber orientation.

The effects of the periodicity were analyzed for a periodic cell with 0° and 45° fiber orientation, and its FRF response was compared to the frequency response of each one. The periodic cell provides a change in the natural frequencies, it was possible to observe for low frequency that the first peak moved to an intermediate position between the individual behavior of elements. However, the torsional displacement presented an uncoupling when the structure presents larger quantities of periodic cells. For high frequency, it can observed the presence of band gap to 6 at 9 frequency range, this indicates that for this frequency range there is not waves propagation.

Therefore, this paper has demonstrated the relationship between wavenumber, fiber orientation, and periodic structure. This study was important to understand the effects in the structural behavior, and your continuity will be able to provide the necessary knowledge to project mechanism to attenuate structural vibration.

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