DINAME 2019 - Proceedings of the XV International Symposium on Dynamic Problems of Mechanics,
M.A. Savi, T.G. Ritto and W.M. Bessa (Editors), ABCM, Buzios, RJ, Brazil, March 10th to 15th, 2019

Nonlinear vibrations analysis of functionally graded material cylindrical shells resting on a partial elastic foundation

Patrícia Corrêa Rodrigues [1], Frederico Martins Alves da Silva [1]

[1]School of Civil and Environmental Engineering - Federal University of Goiás. Av. Universitária, nº 1488 - St. Universitário. CEP 74605-200, Goiânia-Brazil

Abstract: In this work the resonance curves of a functionally graded cylindrical shells containing an axial flow fluid and resting on a partial elastic foundation are analyzed. The cylindrical shell's equilibrium equation is obtained considering the nonlinear Donnell's shallow shell theory. The axial flow fluid is described as inviscid, irrotational and incompressible being represented by a velocity potential. Pasternak model is used to evaluate the reaction of the elastic medium in a discontinuity region of the domain that it is described by Heaviside functions. The transversal displacements field is obtained by pertubation procedure and it is applyed to discretize the equilibrium equations by the standard Galerkin method. Backbones and resonances curves are used to identify the nonlinear forced behavior of a FGM shell conveying fluid and surrounded by a discontinuous elastic foundation. The results show that in some analyzed cases the backbone curves have a softening type behaviour. Also, it is found that the elastic foundation discontinuity modifies the nonlinear behavior, increasing the nonlinearities and changing the stable and unstable paths, as well as modifying the maximum vibration amplitudes of stable solution.

Keywords: Cylindrical shell, FGM, Partial elastic foundation, Nonlinear Vibrations.

INTRODUCTION

Functionally Graded Material (FGM) are composite materials formed of two or more constituent phases characterized by a continuous transition of material properties. Typically, FGM cylindrical shells are composed of metallic and ceramic materials varying through the thickness of the shell. Ceramic guarantees heat and corrosion resistance and metal provides mechanical strength. In the last two decades, several research works have been made to analyze the dynamic behavior of FGM structures. Birman and Byrd (2007) summarized the main scientific and technical developments in the area of FGM material between 2000 and 2006, including stress, stability and dynamic analyses and heat transfer issues. Recently, Thai and Kim (2015) presented a complete review of various theories for modeling and analysis of FGM shells. Montes (2015) analyzed free and forced nonlinear vibrations of FGM shells with a sandwich function graduation. The author compared Donnell and Sanders theories and investigated the influence of internal fluid and thermal effects in the linear and nonlinear shell behavior.

Kim (2015a) investigated a clamped-clamped FGM cylindrical shell partially resting on Pasternak elastic foundation with an oblique angle. Several linear results were obtained, varying the power fraction law, geometry, elastic foundation stiffness and oblique angle that is in contact with the elastic foundation. Continuing the researches, Kim (2015b) analyzed a FGM cylindrical shell partially buried in elastic foundation. The author studied the influence of the circumferential angle in contact with the elastic foundation, as well as the elastic foundation discontinuity along the shell length. Both works Kim (2015a) and Kim (2015b) have focused only on linear results.

The aim of the present work is to investigate the influence of the elastic foundation discontinuities in the cylindrical shell's nonlinear response. Nonlinear Donnell shallow shell theory is used to obtain equilibrium equation that is discretized by a standard Galerkin method. To obtain the nonlinear frequency-amplitude ratio (backbone curves) a Galerkin-Urabe method is applied to discretize the vibration amplitude in time, obtaining a system of algebraic equations, which is solved by using Newton-Raphson method. Also, resonance curves are used to identify the nonlinear forced behavior of a FGM shell conveying fluid and surrounded by a discontinuous elastic foundation, using continuation techniques in order to find the resonance curves. Fourth-order Runge-Kutta method is employed to find the phase-portrait and Poincaré sections to better understanding of nonlinear dynamical response of FG cylindrical shell.

THEORETICAL FORMULATION

Consider a simply-supported circular cylindrical shell of radius R, length L and thickness h. The shell material is assumed to be elastic, isotropic and homogeneous with Young's modulus E, Poisson ratio ν, density ρ and thermal coefficient α. The axial, circumferential and radial coordinates are denoted by x, y and z, respectively, and the corresponding mid-surface displacements are denoted by u, v and w, as shown in Fig. 1. The shell is resting on an elastic foundation with reaction P_k. Also, it is subjected to an axial load P_x, a lateral pressure p and a hydrodynamic pressure of an internal fluid P_H.

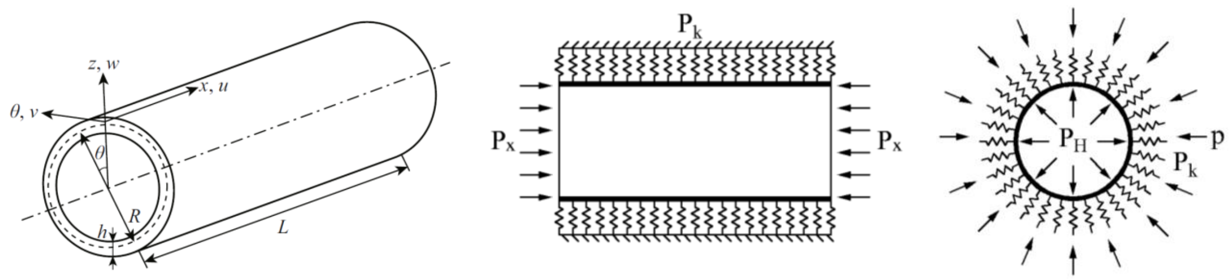


Figure 1 – Geometry of a circular cylindrical shell and external loads

Nonlinear Donnell shallow shell theory is used to obtain equilibrium equation, given by Eq. (1).

$$\rho_t\,\ddot{w}+2\eta_1\,\rho_t\,h\,\omega\,\dot{w}+\eta_2\,\frac{E_M\,h^3}{12\left(1-\nu_M{}^2\right)}\nabla^4\dot{w}+\frac{1}{R}N_\theta+\left(N_x\,\beta_x+N_{x\theta}\,\beta_\theta\right)_{,x}+\frac{1}{R}\left(N_\theta\,\beta_\theta+N_{x\theta}\,\beta_x\right)_{,\theta}$$

$$-\frac{1}{R}\left(M_{x\theta,\theta}+R\,M_{x,x}\right)_{,x}-\frac{1}{R^2}\left(M_{\theta,\theta}+R\,M_{x\theta,x}\right)_{,\theta}+P_H-p+P_k=0 \tag{1}$$

Considering the generalized Hooke’s law for stress components, the stress and moment resultant are obtained as:

$$\begin{Bmatrix} N_x \\ N_\theta \\ N_{x\theta} \\ M_x \\ M_\theta \\ M_{x\theta} \end{Bmatrix}=\begin{bmatrix} A_{11} & A_{12} & 0 & B_{11} & B_{12} & 0 \\ A_{12} & A_{22} & 0 & B_{12} & B_{22} & 0 \\ 0 & 0 & A_{66} & 0 & 0 & B_{66} \\ B_{11} & B_{12} & 0 & C_{11} & C_{12} & 0 \\ B_{12} & B_{22} & 0 & C_{12} & C_{22} & 0 \\ 0 & 0 & B_{66} & 0 & 0 & C_{66} \end{bmatrix}\begin{Bmatrix} u_{,x}+w_{,x}{}^2/2 \\ \left(v_{,\theta}+w\right)/R+w_{,\theta}{}^2/2\,R^2 \\ u_{,\theta}/R+v_{,x}+\left(w_{,x}\,w_{,\theta}\right)/R \\ -w_{,xx} \\ -w_{,\theta\theta}/R^2 \\ -w_{,x\theta}/R \end{Bmatrix}. \tag{2}$$

where ρ_t is the average density, given by $\rho_t=\int_{-h/2}^{h/2}\rho\,dz$; η_1 and η_2 are, respectively, the linear viscous damping and the viscoelastic material damping coefficients; A_{ij}, B_{ij} and C_{ij} (i, j = 1, 2, 6) are the extensional, coupling and bending stiffness defined as:

$$\left[A_{11},A_{22},A_{12},A_{66}\right]=\int_{-h/2}^{h/2}\left[\frac{E}{1-\nu^2},\frac{E}{1-\nu^2},\frac{E\,\nu}{1-\nu^2},\frac{E}{2\left(1+\nu\right)}\right]dz \tag{3}$$

$$\left[B_{11},B_{22},B_{12},B_{66}\right]=\int_{-h/2}^{h/2}\left[\frac{E}{1-\nu^2},\frac{E}{1-\nu^2},\frac{E\,\nu}{1-\nu^2},\frac{E}{2\left(1+\nu\right)}\right]z\,dz \tag{4}$$

$$\left[C_{11},C_{22},C_{12},C_{66}\right]=\int_{-h/2}^{h/2}\left[\frac{E}{1-\nu^2},\frac{E}{1-\nu^2},\frac{E\,\nu}{1-\nu^2},\frac{E}{1+\nu}\right]z^2\,dz\,. \tag{5}$$

To satisfy equilibrium equations, Airy’s stress function, *f*, is used with the following definition:

$$f_{,xx}=N_\theta \qquad f_{,\theta\theta}=R^2\,N_x \qquad f_{,x\theta}=-R\,N_{x\theta}\,. \tag{6}$$

The compatibility equation for FGM cylindrical shells can be considered by:

$$\left(\frac{2\,A_{12}}{R^2\,c_1}+\frac{1}{R^2\,A_{66}}\right)f^P_{,xx\theta\theta}+\frac{A_{11}}{c_1}\,f^P_{,xxxx}+\frac{A_{22}}{R^4\,c_1}\,f^P_{,\theta\theta\theta\theta}=-\frac{1}{R}\,w_{,xx}\left(w_{,\theta\theta}-1\right)+\frac{1}{R^2}\,w^2_{,x\theta}+\frac{A_{12}\,B_{11}-A_{11}\,B_{12}}{c_1}\,w_{,xxxx}$$

$$+\frac{A_{11}\,A_{22}\,B_{66}-A_{11}\,A_{66}\,B_{22}-A_{12}^2\,B_{66}+2\,A_{12}\,A_{66}\,B_{12}-A_{22}\,A_{66}\,B_{11}}{R^2\,A_{66}\,c_1}\,w_{,xx\theta\theta}+\frac{A_{12}\,B_{22}-A_{22}\,B_{12}}{R^4\,c_1}\,w_{,\theta\theta\theta\theta} \tag{7}$$

where c_1 is a constant defined as $c_1 = A_{11} A_{22} - A_{12}$.

In the present work, the variation of constituent materials across the FGM shell thickness is given by a power law distribution, written in Eq. (8). This variation is known as sandwich function (Montes, 2015).

$$V_M(z) = \left[\left(1 - \frac{2z}{h}\right)\left(1 + \frac{2z}{h}\right)\right]^{2N+1} \qquad V_C(z) = 1 - V_A(z) \tag{8}$$

where V_M and V_C are, respectively, the volume fraction of metal and ceramic; z is the distance from the middle surface of the shell and N is the volume fraction index.

The effective material properties, as Young's modulus E, Poisson ratio ν, density ρ and thermal coefficient α, varies with the volume fraction as $P = (P_M - P_C) V_M(z) + P_C$, where P_M and P_C are the chosen material properties of the metal and ceramic, respectively.

Partial elastic foundation

It is considered that some parts of the shell is buried in an elastic medium that behaves as an elastic foundation, as shown in Fig. 2. In order to describe the elastic foundation discontinuity, it is used the Heaviside step function, as a function of a discrete variable x, expressed in Eq. (9).

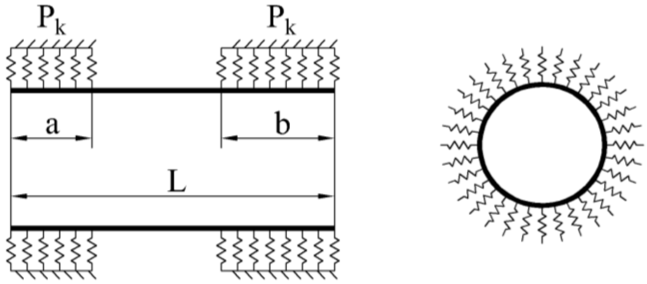


Figure 2 – Shell partially buried in elastic foundation with discontinuity along the length.

$$H(x-a) = \begin{cases} 1, & x > a \\ 0, & x < a \end{cases} \tag{9}$$

Pasternak model is used to evaluate the reaction of the medium P_k on the cylindrical shell, described as $P_k = (K_W w + K_P \nabla^2 w) H_x$, where K_W and K_P are the radial and transverse stiffness of the spring layer, respectively; ∇^2 is the Laplace operator and H_x is the Heaviside step function at x direction, expressed by $H_x = H(x) - H(x-a) + H(x-b) - H(x-L)$. For $H_x = 1$, elastic foundation becomes complete.

Internal fluid

The fluid-structure interaction is described by Païdoussis and Denise (1972) analysis. The fluid is considered as inviscid, irrotational and incompressible. Laplace equation is used to describe the velocity potential and the dynamic condition is given by Bernoulli equation. By employing the method of separation of variables to obtain the velocity potential and considering all boundary conditions. So, the hydrodynamic pressure is expressed as:

$$P_H = \frac{\rho_F}{q} \frac{I_n(q\,r)}{I_n'(q\,r)} \left(\ddot{w} + 2U\,\dot{w}_{,x} + U^2\,w_{,xx}\right) \tag{10}$$

where U is the mean axial flow velocity; ρ_F is the fluid density; I_n is the modified Bessel function of the first kind of order n; I_n' is the derivative of I_n with respect to the argument and $q = m\pi/L$.

Modal expansion for transversal displacements field

Part of the used modal expansion, expressed by Eq. (11), is obtained using perturbation techniques (Gonçalves, 1987; Silva, 2008; Montes, 2015). Considering a simply-supported shell, the modal expansion that can describe large amplitude vibration up twice shell's thickness is given by:

$$w_p = \overline{W}_{11}(t)h\cos(n\theta)\sin(m\pi x/L) + \overline{W}_{11}^c(t)h\sin(n\theta)\sin(m\pi x/L) + \overline{W}_{13}(t)h\cos(n\theta)\sin(3m\pi x/L) +$$
$$+ \overline{W}_{13}^c(t)h\sin(n\theta)\sin(3m\pi x/L) + \overline{W}_{02}(t)h\left[(3/4) - \cos(2m\pi x/L) + (1/4)\cos(4m\pi x/L)\right]. \tag{11}$$

In addition, asymmetric terms - Eq. (12) - is added in order to describe the behavior of the asymmetric partial elastic foundation. These terms were carefully added until to obtain the convergence of resonance curves of FG cylindrical shells, considering several types of foundation discontinuities (symmetric and asymmetric). The complete modal expansion used is given by $w = w_p + w_d$, generating a transversal displacement field with 13 degrees of freedom. It is important to notice that the number used in sum of Eq. (12) was checked to obtain the convergence of nonlinear resonance and backbone curves.

$$w_d = \sum_{j=2,4,6}^{8} \overline{W}_{1j}(t) h \cos(n\theta) \sin(j m \pi x/L) + \overline{W}_{1j}^{c}(t) h \sin(n\theta) \sin(j m \pi x/L) \tag{12}$$

NUMERICAL RESULTS AND DISCUSSIONS

Consider a cylindrical shell of radius R=2 m, length L=2 m and thickness h=0.01 m. The FGM shell is composed of nickel and silicon nitride, behaving as metal and ceramic material, respectively. The nickel properties are: $E_M = 2.051\times10^{11}$ N/m², $\alpha_M = 12.51\times10^{-6}$ 1/K, $\rho_M = 8900$ kg/m^3 and $\nu_M = 0.3$. Silicon nitride properties has the following properties: $E_C = 3.222\times10^{11}$ N/m², $\alpha_C = 7.47\times10^{-6}$ 1/K, $\rho_C = 2370$ kg/m^3 and $\nu_C = 0.24$. In all cases analyzed in this work, Fig. 3, the radial and transverse stiffness of the spring layer are, respectively, $K_W = 2.7\times10^7$ N/m^2 and $K_P = 2.7\times10^5$ N.m. Also, is considered volume fraction index N = 0.1, fluid velocity U = 20 m/s, magnitude of the lateral load $P_L = 5000$ N/m, $\eta_1 = 0.003$ and $\eta_2 = 0.0001$. For the present geometry the lowest natural frequency is obtained in Cases A, C and D for (m,n)=(1,7) and in Cases B and D for (m,n)=(1,8) where this wave-number changing is due to coupled problem: fluid-structure-elastic foundation, as shown in Fig. 4.

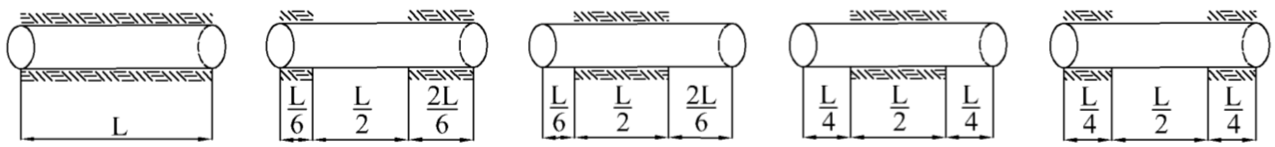


Figure 3 – Variation of elastic foundation. (a) Case A; (b) Case B; (c) Case C; (d) Case D; (e) Case E.

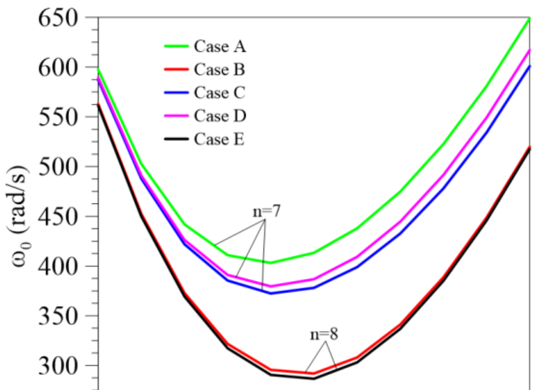


Figure 4 - Frequency spectrum versus the circumferential wave numbers

The shell is subjected by a lateral pressure defined as Eq. (13), where m and n are the number of longitudinal half-waves and the number of circumferential waves, respectively; P_L is the magnitude of the lateral load and ω_L is the excitation frequency.

$$p(t) = P_L \sin(m\pi x/L) \cos(n\theta) \cos(\omega_L t) \tag{13}$$

Modal expansion described in Eq. (11) and Eq. (12), with 13 degrees of freedom, is applied in the compatibility equation, expressed by Eq. (7), which is solved analytically. Then, substituting the considered modal solution and the obtained stress function into Eq. (1), a standard Galerkin method is employed to obtain a system with 13 ordinary differential equations, as a function of the number of degrees of freedom of w expansion. Resonance curves were evaluated applying continuation techniques in the discretized equilibrium equations using software AUTO. Backbone curves are obtained applying Urabe method, considering a temporal discretization given by Eq. (14) (Gonçalves, 1987; Montes, 2015), and the obtained algebraic system is solved by Newton-Raphson method.

$$\overline{W}_{11} = W_{11}\cos(\omega t) \qquad \overline{W}_{02} = W_{02}\cos(\omega t)^2 \qquad \overline{W}_{13} = W_{13}\cos(\omega t)^3 \qquad \overline{W}_{1j} = W_{1j}\cos(\omega t)\ \text{(j=2,4,6,8)}. \quad (14)$$

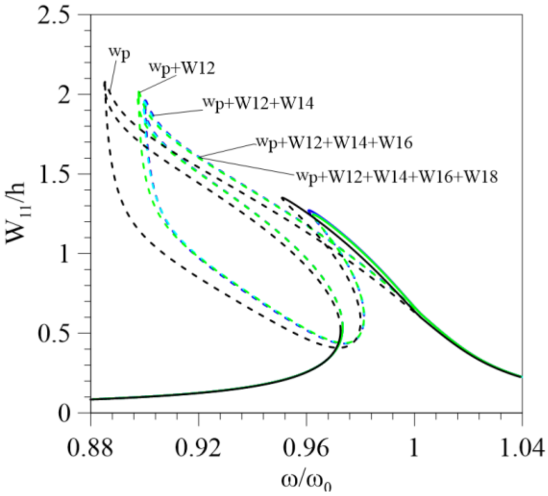
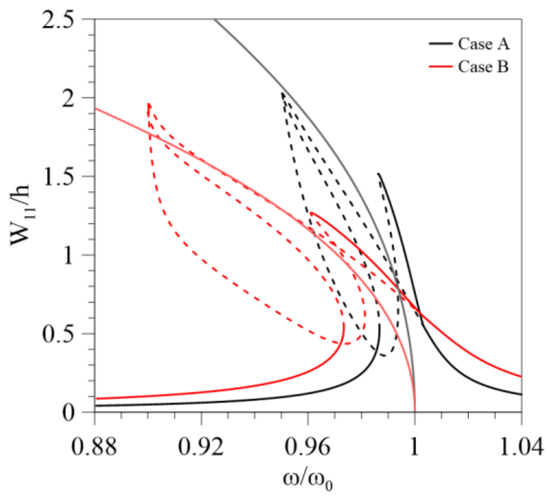
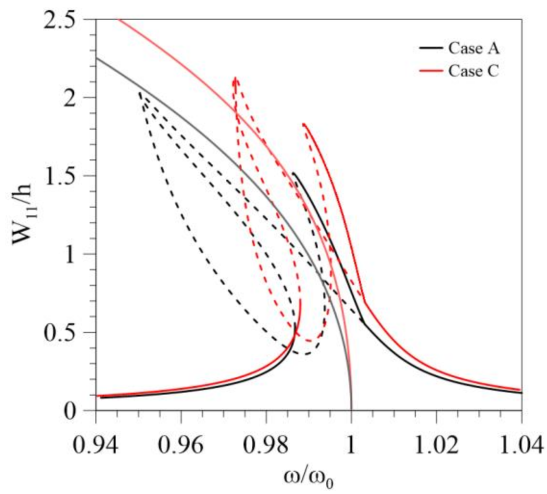


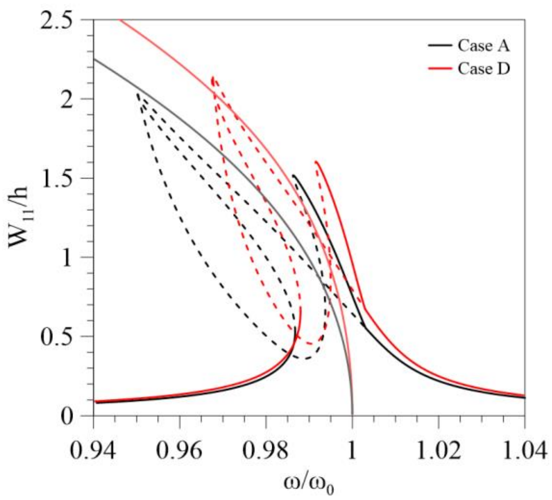
Figure 5 - Convergence of resonance curves with number of degree-of-freedom for Case B.



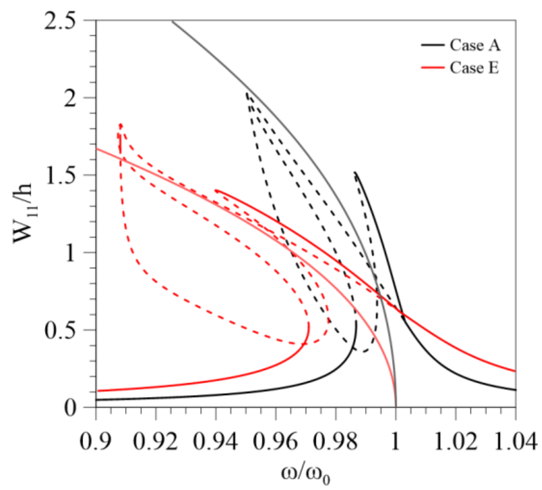
(a) Cases A and B



(b) Cases A and C



(c) Cases A and D



(d) Cases A and E

Figure 6 – Backbone and resonance curves for an FGM cylindrical shell. R=2m, L=2m, h=0.01m, m=1, N=0.1, U = 20 m/s, K_W=2.7×10^7 N/m^2, K_P=2.7×10^5 N.m, η_1=0.003, η_2 = 0.0001. P_L=5000 N/m.

Firstly, Fig. 5 shows the convergence of resonance curve with used number of degree-of-freedom for a case of asymmetric and discontinuous elastic foundation, which it is related to Case B (Fig. 3b). It is noted that convergence occurs adding asymmetric terms of Eq. (12) until j=8. When term W_{12} is added to w_p, the result approaches from the final response, which means that W_{12} is the main asymmetric term in equation. In order to verify the participation of the other terms, it is necessary to evaluate the maximum vibration amplitude of each one of the asymmetric terms, which it is shown in Fig. 7.

Fig. 6 presents nonlinear resonance curves and backbone curves for five chosen cases of elastic foundation. Case A represents a FGM cylindrical shell completely surrounded by an elastic foundation (Fig. 3a). Case B to Case E are related to a FGM cylindrical shell partially surrounded by an elastic foundation along the length. In Case B, elastic foundation is symmetrically concentrated at the ends of the shell (Fig. 3b), while in Case C the elastic foundation is asymmetrically concentrated at the ends of the shell length (Fig. 3c). Case D (Fig. 3d) and Case E (Fig. 3e) correspond to a shell resting on elastic foundation concentrated in the center of the shell symmetrically and asymmetrically, respectively. The resonance curves are represented with continuous and dashed lines, which refers to stable and unstable paths, respectively. The backbones curves are given by the continuous line, plotted in lighter color, and they indicate the nonlinear free vibration of FG cylindrical shells.

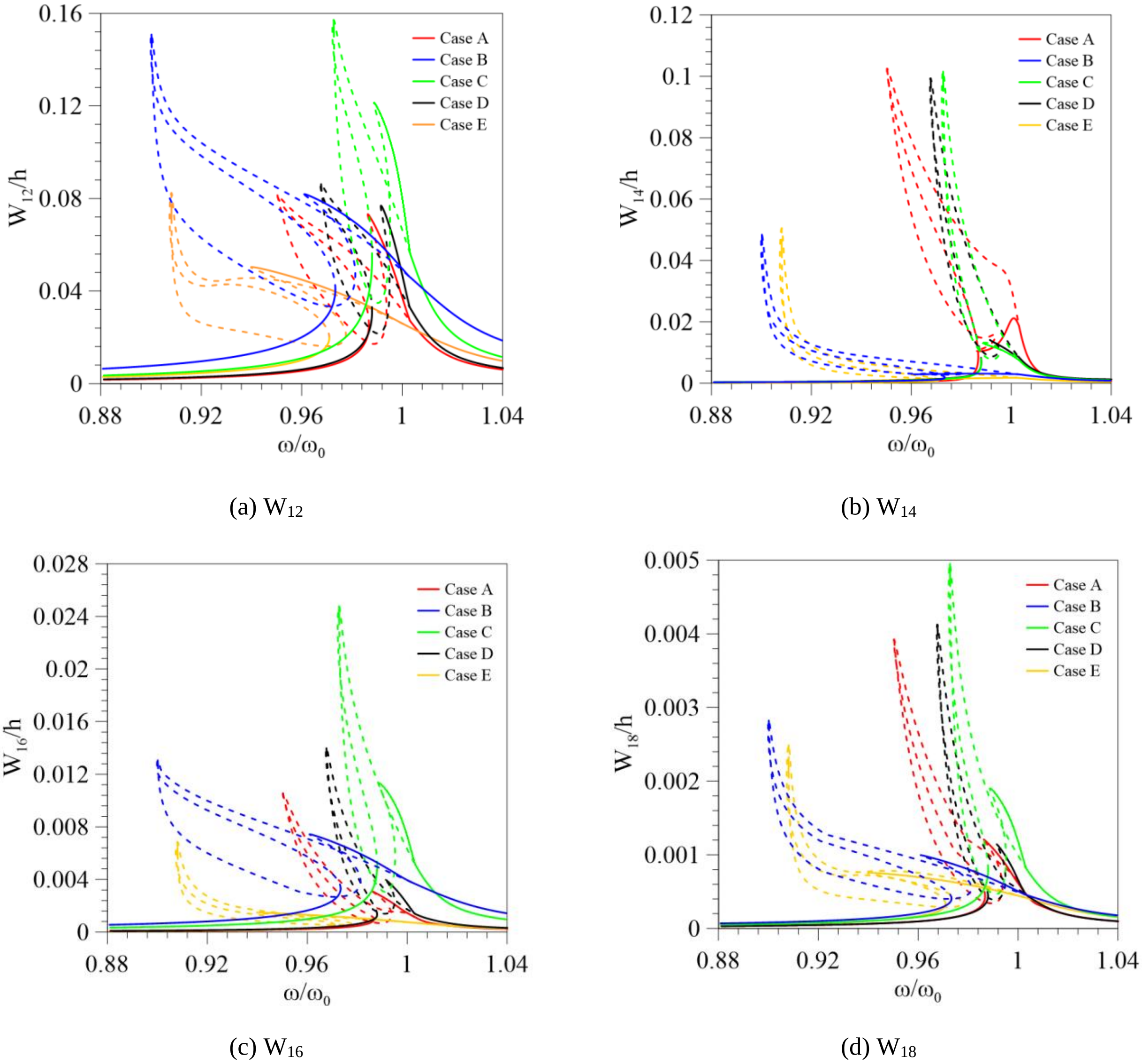
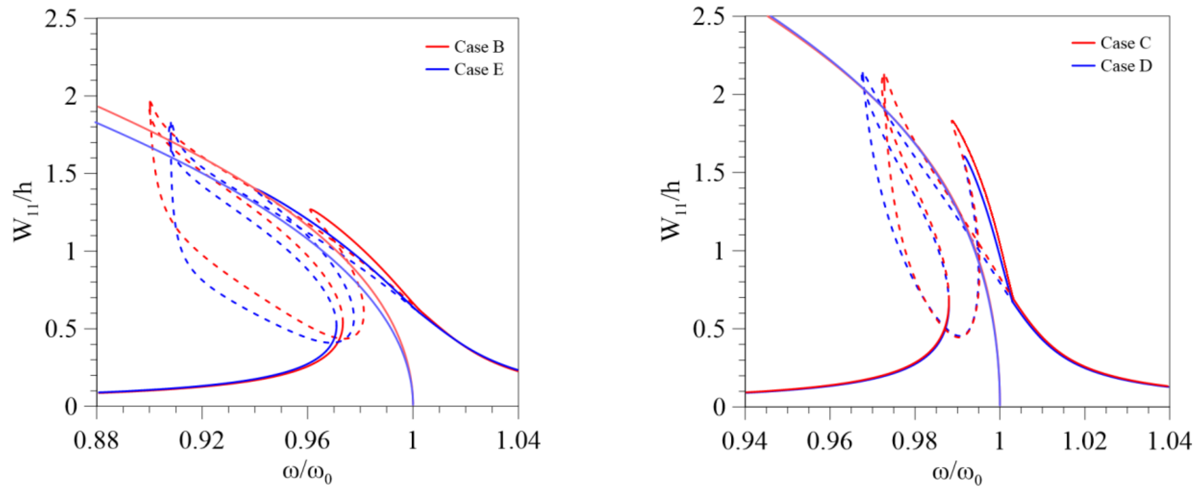


Figure 7 – Comparison of participation of the asymmetric terms in the forced nonlinear response for all analyzed case of elastic foundation.



(a) Cases B and E (b) Cases C and D

Figure 8 – Comparison between asymmetric and symmetric cases.

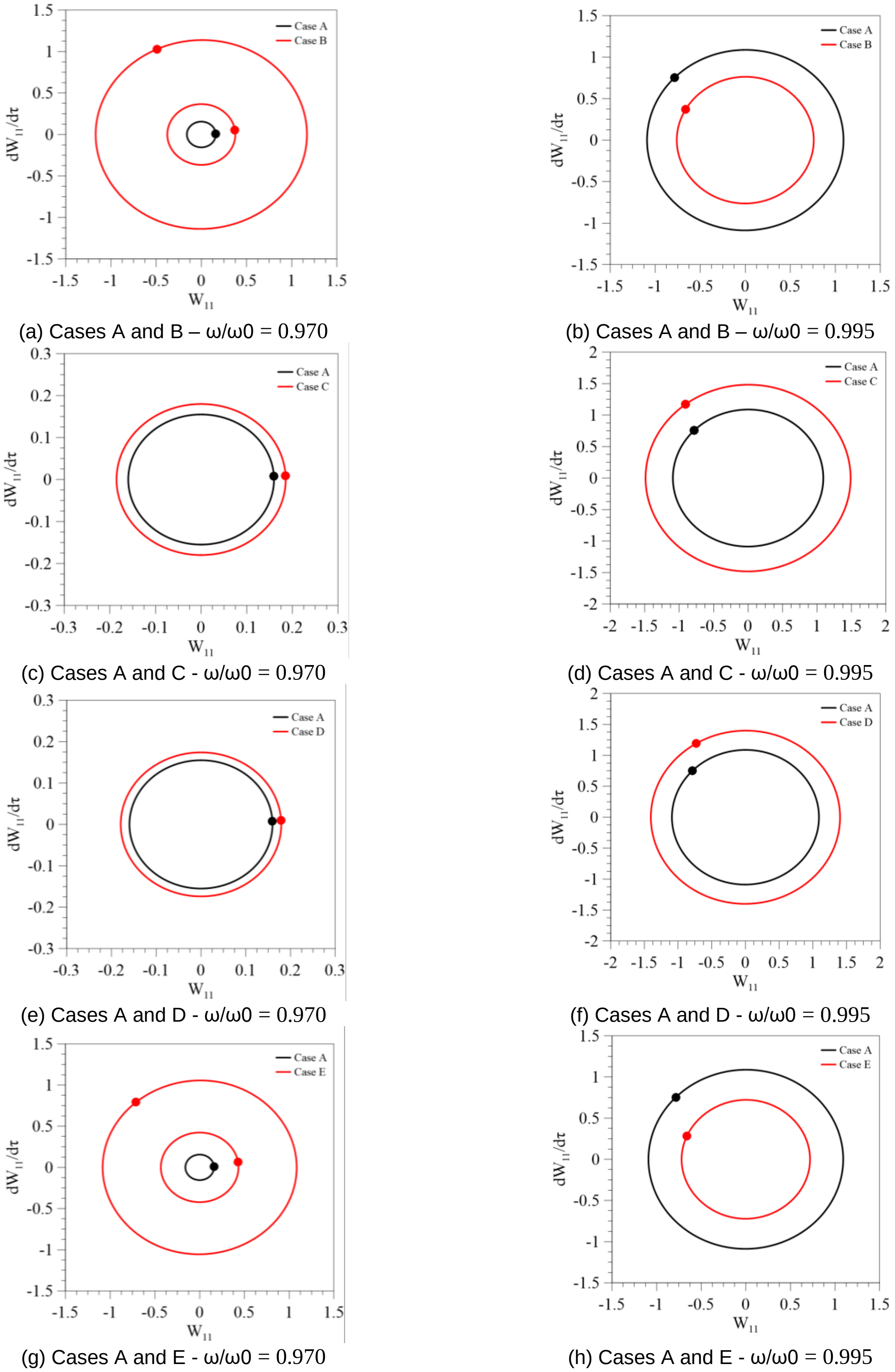
In Fig. 6, the backbone and resonance curve of Case A (shell completely surrounded by elastic foundation – Fig. 4a) exhibits a softening behavior with amplitude twice shell's thickness. However, it is observed that the elastic foundation discontinuity along the shell length (Figs. 6a-6d) modifies the forced vibration behavior. In all discontinuous cases (Case B to Case E), the resonance curves also present a nonlinear behavior with a softening type nonlinearity. However, for the analyzed conditions, it is shown that when the elastic foundation is concentrated at the center of the shell the global structural stiffness is increased changing the nonlinear behavior of resonance curves. Otherwise, when the elastic foundation is localized at the ends of the shell, it reduces the FG cylindrical stiffness of shell, increasing the softening effects on the resonance curves. In a practical point of view, if a structure is projected to keep itself completely buried, and posteriorly part of its elastic foundation is excavated, this situation can provoke a new nonlinear dynamical scenario, depending on how the elastic foundation is set up along the shell.

Fig. 7 shows the participation of each asymmetric term in the nonlinear response, considering all cases analyzed. Term W_{12} (Fig. 7a) reaches values in the order of 16% the shell thickness for cases with asymmetric discontinuities (Cases B and C), while for symmetric foundations as Cases A, D and E, this value is in the order of 8%. It occurs because W_{12} is the main asymmetric term, as previously discussed. For the other asymmetric terms, their participation in the shell behavior is reduced until reaches the order of 0.3% to 0.5% the shell thickness for terms W_{18}, which was considered small, compared to the thickness. It is important to note that the asymmetric terms also participates in the cases with symmetric elastic foundation (Cases A, D and E). In a linear analysis it does not happen because the displacements are symmetric. But when the forced nonlinear vibrations shell is evaluated, the asymmetric terms associate with the symmetric ones. However, their participation is small, considering that term W_{11} reaches about two times the shell thickness in Fig. 6.

Fig. 8 compares the resonance curves for symmetric and asymmetric cases. Fig. 8a corresponds to the cases that the discontinuity is located at the ends of the shell and Fig. 8b refers to the cases with discontinuity in the center of the shell. It is important to note that when Case B is compared to Case E, Fig. 8a, the symmetric discontinuity (Case E) reduces the shell stiffness, since in Case E there is an increase of nonlinearities. The same fact occurs with Cases C and D (Fig. 8b), where the symmetric case (Case D) shows a reduction of the stiffness. It is also observed that the discontinuities modify the stable and unstable paths, but for the analyzed conditions, there is not and adding or reduction of new equilibrium paths.

For each resonance curve shown in Fig. 6, some frequencies of excitation values were chosen in order to obtain projections of the phase-portrait and Poincaré sections. The projections of phase-portrait (W_{11} x $dW_{11}/d\tau$ plane) are plotted considering only the presented stable solutions in the resonance curves. The obtained results are plotted in Fig. 9 and they show the phase-portraits and Poincaré sections using $\omega/\omega 0=0.970$ and $\omega/\omega 0=0.995$ for Case A to E. The black curves represent the nonlinear dynamical results for Case A (FG cylindrical shell completely surrounded by an elastic foundation) while the red curves show the nonlinear dynamical response for a FG cylindrical shell with discontinuous cases of elastic foundation. Poincaré sections in all cases shown that the nonlinear response is periodic with period equal to one. Further, it is noted that the maximum amplitude of phase plane orbit are agree with the results found in the resonance curve (Fig. 6).

In Fig. 9, Case A presents a single solution for $\omega/\omega 0=0.970$. The same occurs with Cases C and D (Figs. 9c and 9e). However, the discontinuity in the ends of the shell (Figs. 9a and 9g), which corresponds to Cases B and E, generated another stable solutions with bigger displacements (bigger than the shell thickness) that were not in an initial case of design's FG cylindrical shell. It is shows that there is a coexistence of stable solutions for discontinuous cases, leading to a competition of solutions that can create unwelcome and not predicted vibration in the primary resonance region that could be very sensitivity to applied perturbation. For ω/ω0=0.995, the discontinuity of elastic foundation does not create another solutions. Despite that, in Cases C and D (Fig. 9d and 9f) there is an increase of the vibration amplitude to approximately 1.5 times the shells thickness. On the other hand, Cases A and E (Fig. 9b and 9h) reduced the vibration amplitude.



(a) Cases A and B – ω/ω0 = 0.970
(b) Cases A and B – ω/ω0 = 0.995
(c) Cases A and C - ω/ω0 = 0.970
(d) Cases A and C - ω/ω0 = 0.995
(e) Cases A and D - ω/ω0 = 0.970
(f) Cases A and D - ω/ω0 = 0.995
(g) Cases A and E - ω/ω0 = 0.970
(h) Cases A and E - ω/ω0 = 0.995

Figure 9 – Phase-portrait projections with its Poincarè sections.

Conclusions

In this work, the nonlinear behavior of FGM cylindrical shells resting on a partial elastic foundation is investigated. Applying nonlinear Donnell shallow shell theory, the resultant equilibrium equations are reduced to a system of nonlinear differential equations by the application of a standard Galerkin method. Resonance curves are obtained using continuation techniques. Backbone curves are solved applying Galerkin-Urabe and Newton-Raphson method. The results, for the present analysis, show that in all cases the backbone curves are of the softening type behavior. Also, it is found that the elastic foundation discontinuity modifies the nonlinear behavior, increasing the nonlinearities and changing the stable and unstable paths, as well as modifying the maximum vibration amplitudes. This change in structural behavior shows that the discontinuity of elastic foundation needs to be evaluated in structural engineering, since it can lead the structure to collapse when it is considered the dynamic analysis.

ACKNOWLEDGMENTS

This work was made possible by the financial support of the Coordination for the Improvement of Higher Level Personnel (CAPES), Brazilian Research Council (CNPq) and Goiás State Research Foundation (FAPEG).

REFERENCES

Birman, V., Byrd, L. W., 2007, “Modeling and Analysis of Functionally Graded Materials and Structures”. Applied Mechanics Reviews, Vol.. 60, No. 5, pp. 195-216.

Gonçalves, P. B., 1987, “Nonlinear dynamical interaction between fluid and thin shells” (in Portuguese). COPPE, Universidade Federal do Rio de Janeiro, Rio de Janeiro, 317 p.

Kim, Y. W., 2015a, “Free vibration analysis of FGM cylindrical shell partially resting on Pasternak elastic foundation with an oblique edge”. Composites Part B: Engineering, Vol. 70, pp. 263-276.

Kim, Y. W., 2015b, “Effect of partial elastic foundation on free vibration of fluid-filled functionally graded cylindrical shells”. Acta Mechanica Sinica, Vol. 31, pp. 920-930.

Montes, R. O. P., 2015, “Reduced order model to nonlinear vibration analysis of functionally graded cylindrical shell” (in Portuguese). Programa de Pós-Graduação em Geotecnia, Estruturas e Construção Civil da UFG, Universidade Federal de Goiás, Goiânia, Brazil, 148 p.

Païdoussis, M. P., Denise, J. P., 1972, “Flutter of thin cylindrical shells conveying fluid”. Journal of Sound and Vibration, Vol. 20, No. 1, pp. 9-26.

Silva, F. M. A., 2008, “Reduced order model to nonlinear oscillation and stability analysis of cylindrical shell” (in Portuguese). Pontifícia Universidade Católica do Rio de Janeiro, Rio de Janeiro, 182 p.

Thai, H-T.; Kim, S-E., 2015, “A review of theories for the modeling and analysis of functionally graded plates and shells”. Composite Structures, Vol. 128, pp. 70-86.

RESPONSIBILITY NOTICE