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Analysis in Acoustic Duct with a Periodic Side-Branch Tubes Array

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Abstract: The proposed paper presents the behavior of a duct with periodic side-branch tubes commonly used as a device for noise control. They increase the resonance bandwidth to improve the transmission loss in the duct-tube assembly. One way to obtain a greater attenuation range is to place the tubes periodically along the duct. It generates forbid frequency bands commonly known as bandgaps, where the propagating harmonic waves are attenuated. These band gaps are generated based on the spatial frequency of areas of incompatible impedance that produce Bragg scattering effect and local resonance effect on the tubes. This work investigates the band gaps created in the duct-tube system using three methods: transfer matrix method (TMM), finite element method (FEM) and wave finite element (WFE) method. A system duct-tube is simulated with the intention to obtain the dispersion diagrams and forced response. The results are compared between the methods (TMM and WFE) to evaluate their accuracy and efficiency. Simulations using different unit-cell geometries and the number and length of tubes in the unit-cell are under way and will be presented in the full paper.

Keywords: metamaterial, bandgaps, duct, noise control

INTRODUCTION

The use of acoustical silencers is widely applied for noise control in combustion engines and ventilation systems. Side-branch tubes have been used extensively in acoustic engineering as passive devices to reduce noise in narrow frequency bands. In the last decades, some approaches have been proposed to improve the performance of side-branch tubes. They are concentrated on tuning the resonance frequency to enlarge the resonance bandwidth and improve the transmission loss. An approach to broader attenuation range in a duct consists in to distribute side-branch tubes periodically along the duct. Figure 1 shows a schematic view of such system. This generates some frequency bands, known as prohibited bands or bandgaps, were harmonic waves do not propagate. Commonly studied in the area of phononic crystals and metamaterials, bandgaps are generated based on the spatial frequency of mismatched impedance areas which produce Bragg scattering effect and/or local resonance effect.

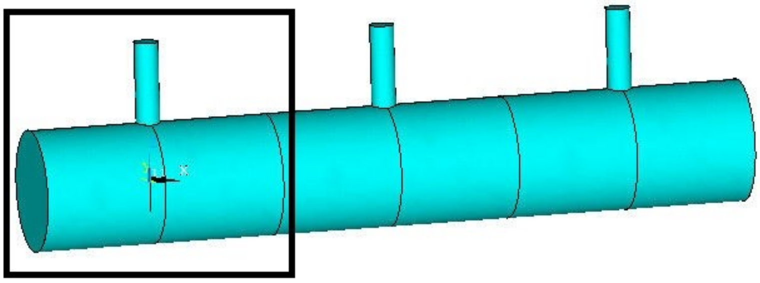

Figure 1 – Periodic duct with side-branch tubes.

This paper investigates the bandgaps created in side-branch tubes in periodic system using acoustic transfer matrix method (TMM) and wave finite element (WFE) method. The TMM is based on the plane wave acoustic formulation for acoustic system to obtain a transfer matrix (Munjal, 1982; Wang *et al*, 2016). In the following, Floquet-Bloch theorem is applied on the transfer matrix to obtain an eigenvalue problem, whose solutions are the wavenumbers (dispersion diagrams) and the corresponding wave modes (wave amplitudes) caused by Bragg scattering and local resonance.

Recently developed to evaluate the behavior of periodic structures from a unit-cell modeled by finite element (FE) method, the WFE is applied here to calculate bandgaps in acoustic system (Ichchou and Mencik, 2005). In this method the dynamic stiffness matrix of a unit-cell of the duct-HR acoustic system is modeled by FE and transformed in the transfer matrix form. Using Floquet-Bloch's theorem results in an eigenvalue/vector problem, whose solution gives the wavenumbers and the corresponding wave mode. WFE has been used for free and forced wave propagation in vibration analysis with applications to one and two-dimensional structural models (Duhamel *et al*, 2006; Mace and Maconi,2008).

Here, WFE will be applied to model acoustic metamaterial waveguide with side-branch tubes. Numerical predictions of the dispersion diagrams of a periodic side-branch tube system calculated by TMM model and WFE models are compared. An important result of this paper is to demonstrate the accuracy, flexibility and efficiency of WFE for modeling acoustic metamaterial system.

THEORETICAL FOUNDATION

The TMM has been widely used as a tool for calculate complex systems due to its computational efficiency and flexibility. Some researchers, Munjal (1987) and Singh *et al* (2008), have used the TMM for analyzing acoustic duct systems. Although many research papers and acoustic text books (Beranek and Vér, 1992; Kinsler, 1992), provide a brief outline of the TMM, they do not include some important details of a particular system.

This paper investigates the bandgaps created in acoustic periodic system using acoustic transfer matrix method (TMM) and wave finite element (WFE) method. The TMM is based on the plane wave acoustic formulation for a sidebranch tube unit-cell to obtain a transfer matrix (Munjal, 1987). In the following, Floquet-Bloch theorem is applied on the transfer matrix to obtain an eigenvalue problem, whose solutions are the wavenumbers (dispersion diagrams) and the corresponding wave modes (wave amplitudes) caused by Bragg scattering and local resonance (Campos *et al*, 2017; Wang *et al*, 2008).

$$\mathbf{Tq} = e^{\mu}\mathbf{q} \tag{1}$$

where $\mathbf{T}$ is the transfer matrix, $\mathbf{q}$ is the state vector, e is the exponential function and $\mu = -ikd$ is the attenuation constant with $k = k(1+i\eta)$ as the complex wavenumber, d the unit-cell length and i the imaginary unit. For the spatial periodic distribution the band gaps are generated by Bragg scattering, which appears around frequencies governed by $L = n(\lambda/2)(n = 1,2,...)$. Before apply periodicity condition eq. 1 can be rewritten in a more convenient form as:

$$\mathbf{q}_r = \mathbf{T}\mathbf{q}_L \tag{2}$$

where $\mathbf{q}_r$ and $\mathbf{q}_L$ now are the state vector at the right and left position of the unit-cell, respectively. Let's consider now consecutive unit-cells, m and $m+1$, in the structure shown in fig. 1. The volume velocity and acoustic pressure compatibility and continuity condition produces $\mathbf{q}_L{}^{(m+1)} = \mathbf{T}\mathbf{q}_r{}^{(m)}$.

The Floquet-Bloch theorem for wave propagation in an infinite periodic system applied to consecutive unit-cells, generates:

$$\mathbf{T}\mathbf{q}_L = e^{\mu}\mathbf{q}_L \tag{3}$$

which is the Bloch wave eigenvalue/vector problem, where e^{μ} is the Bloch wave number and $\mathbf{q}_L$ are the corresponding wave vectors.

Recently developed to evaluate the behavior of periodic structures from a unit-cell modeled by finite element (FE) method, the WFE is applied here to calculate bandgaps in an acoustic system (Ichchou and Mencik, 2005). In this method the dynamic stiffness matrix of a unit-cell of the side-branch tube acoustic system is modeled by FE and transformed in the transfer matrix form. Using Floquet-Blochś theorem results in an eigenvalue/vector problem, whose solution gives the wavenumbers and the corresponding wave modes. WFE has been used for free and forced wave propagation in vibration analysis with applications to one and two-dimensional structural models (Cook *et al*, 2002; Duhamel *et al*, 2006; Mace and Maconi, 2008).

The equation shown below gives the estimate of the ratio of acoustic pressure p_L at the duct exit to the input volume velocity v_0. Z_L is the radiation impedance of an unflanged open end of the duct. If v_0 is set to unity $(1m^3/s)$, then p_L represent the estimates of acoustic pressure at the duct exit described by $p_L = \frac{Z_L}{A_L + B_L Z_L}$ (Singh *et al* 2008).

Wave Finite Element Method

In this section the finite element method starts from the assembly of the mass $\mathbf{M}_a$, and stiffness $\mathbf{K}_a$ acoustical matrices extracted from ANSYS software. Then, we assemble the dynamic stiffness matrix described by:

$$\mathbf{D} = \mathbf{K}_a - \omega^2\mathbf{M}_a \tag{4}$$

The dynamic acoustic stiffness matrix can be rewritten by partitioning the degrees of internal freedom, right and left. So we have:

$$\begin{bmatrix} \mathbf{D_{ii}} & \mathbf{D_{il}} & \mathbf{D_{ir}} \\ \mathbf{D_{li}} & \mathbf{D_{ll}} & \mathbf{D_{lr}} \\ \mathbf{D_{ri}} & \mathbf{D_{rl}} & \mathbf{D_{rr}} \end{bmatrix} \begin{Bmatrix} \mathbf{p_i} \\ \mathbf{p_l} \\ \mathbf{p_r} \end{Bmatrix} = \begin{Bmatrix} \mathbf{0_i} \\ \mathbf{f_l} \\ \mathbf{f_r} \end{Bmatrix} \tag{5}$$

In this method, the dynamic stiffness matrix of a unit cell of the entire structure is modeled by finite elements is used to apply the periodicity condition in a harmonic perturbation propagating through the duct-tube system. By rearranging and partitioning the eq. 5 as the transfer matrix the formulation yields:

$$\left\{ \begin{array}{c} \mathbf{p_r} \\ -\mathbf{f_r} \end{array} \right\} = \left[\begin{array}{cc} -\mathbf{D_{lr}}^{-1}\mathbf{D_{ll}} & -\mathbf{D_{lr}^{-1}} \\ \mathbf{D_{rl}} - \mathbf{D_{rr}}\mathbf{D_{lr}}^{-1}\mathbf{D_{ll}} & -\mathbf{D_{rr}}\mathbf{D_{lr}^{-1}} \end{array} \right] \left\{ \begin{array}{c} \mathbf{p_l} \\ \mathbf{f_l} \end{array} \right\} \quad \textbf{or} \quad \mathbf{q}_r = \mathbf{T}_{WFE}\mathbf{q}_l \tag{6}$$

where, $\mathbf{T}_{WFE}$ is transfer matrix obtainded by WFE method.

Applying to the Floquet-Bloch theorem solves the similar eigenvalue/eigenvector problem by the equation.

$$\mathbf{T}_{WFE}\mathbf{q}_L = e^{\mu}\mathbf{q}_L \tag{7}$$

RESULTS AND DISCUSSION

A periodic duct-tube acoustic system as shown in was simulated by TMM and WFE with the geometry and material properties given by Table 1.

Table 1 – Geometric parameters and material properties.

Geometry/Properties	Value
Periodic part length (m)	0.9
Unit-cell length (m)	0.3
Duct diameter (m)	0.15
Tube diameter (m)	0.03
Tube length (m)	0.1
Sound velocity (m/s)	343.24
Air density (kg/m^3)	1.2041

Dispersion diagram of the side-branch tube system is shown in fig.1. As expected the method TMM and WFE presents a good agreement for all frequency band. The bandgaps represents the non-propagative waves they will be located in the negative imaginary part of kL in the graphic. It can be seen two band gaps, they can be identified as a locally resonant bandgap which occur around the natural frequency of the side-branch tubes. It can be seen that both methods can identify the bandgap but presents differences in the bandgap position and width obtained with TMM and WFE.

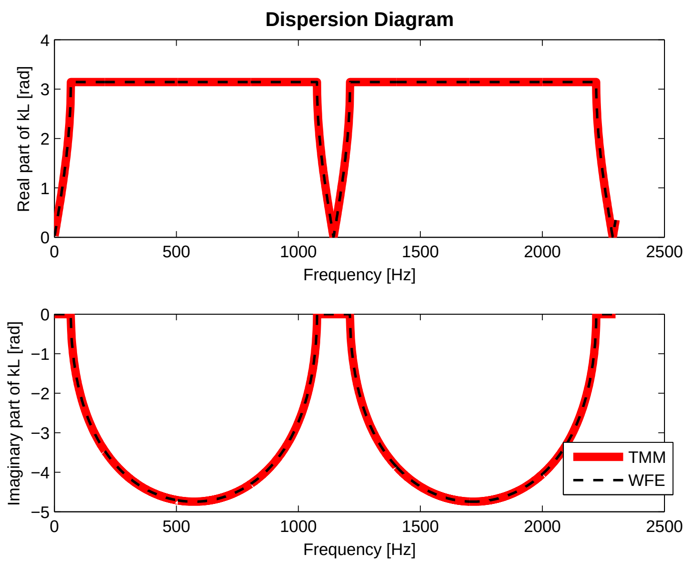

Figure 2 – Dispersion diagram of the side-branch tubes system (real and imaginary parts).

The forced response obtained for side-branch tubes is shown in fig. 3. It is possible to observe the present intervals between the frequency bands between 78 to 1080 Hz and 1230 to 3080 Hz approximately, which agrees with the result found in the dispersion diagram, where in that same frequency band is located the local resonance bandgap caused by the side-branch tubes.

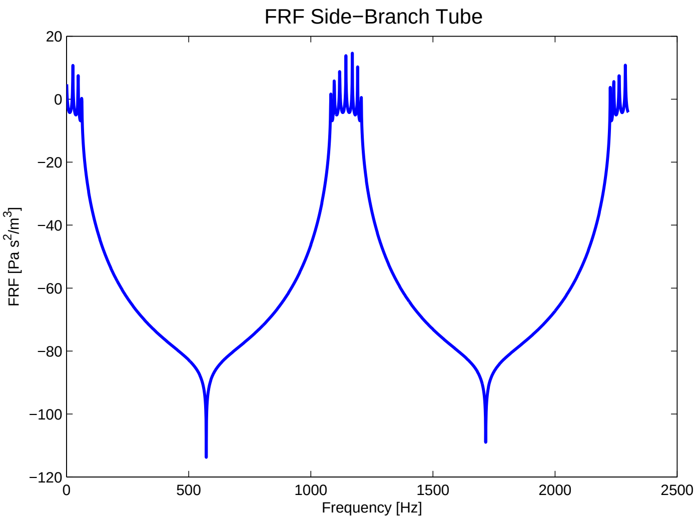

Figure 3 – Forced response to three unit-cells of the side-branch tubes system.

CONCLUSION

The methodology used in this work to determine attenuation in ducts with a side-branch tubes can be used to optimize acoustic system to improve noise control. The acoustic mass and stiffness matrices extracted from a commercial software (ANSYS) were performed and processed using Matlab to calculate dispersion diagrams of a periodic system with WFE method efficiently and precisely as compared with transfer matrix method (TMM).

Results show that both methods TMM and WFE are able to identify bandgaps generated by periodicity (Bragg effect) and locally resonators. Must be noticed that due the plane wave hypothesis assumed in all acoustic formulations evaluated, as the frequency band increase the dispersion diagram results can present variations.

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