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Isolae present in electromechanical systems with nonlinearities: a tradeoff between risk and usefulness

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Abstract: This paper presents some highlights on the complex dynamics of electromechanical (EM) systems with nonlinear characteristics. In particular, it focuses on isolated branches in the frequency response, also known as "isolae". EM systems are used in a wide range of applications including mechanical isolation, actuation and energy harvesting. Nonlinearities that naturally occur in EM systems can lead to unexpected and consequently dangerous responses if not adequately taken into account. Knowing the full dynamics of these systems reduces the risk of failure in operation, but also allow for a smarter design, where the nonlinearities are not feared but used to improve the performance of the EM system. For example, in energy harvesting, nonlinearities can be used to broaden the frequency band of operation. In this study the following nonlinear parameters are considered: stiffness, inductance and capacitance. The study was conducted using direct integration approaches and numerical continuation together with analytical methods such as harmonic balance: the frequency responses were computed and the nonlinear parameters that can result in isolae were identified. The results show that EM systems with nonlinear parameters present a variety of bifurcations and that some of these can be advantageous for vibration isolation and energy harvesting.

Keywords: Electromechanical, Nonlinear Dynamics, Energy Harvesting, Isolae, Isolation

INTRODUCTION

Electromechanical (EM) systems with nonlinear parameters can exhibit complex responses featuring bifurcation, changes of period (e.g. period doubling), chaotic behaviour and change in stability. In addition Isolae, or isolated solutions can appear in the frequency response of these systems. These periodic responses belong to solution branches that are detached from the primary solution branch, making their prediction more complicated and therefore more troublesome. For example, numerical continuation methods are quite ineffective in discovering the existence of isolated branches as they require a solution belonging to the isolated branch to initiate the procedure.

A valid solution to this problem is to use brute force numerical integration (for example using Runge-Kutta methods) with several initial conditions. However, the computational cost is quite high and in case of a system with many degrees of freedom, exploring the entire hyperspace for initial condition might not be feasible. Usually, numerical continuation and direct integration are used in a complementary fashion, where the numerical integration is used to explore a partition of the state space and to compute the solutions in that partition and the numerical continuation is used to computed the branches to which those solutions belong.

Isolae are detached solutions, therefore that the above procedure is not always reliable as it depends on the mesh size used to explore the space of the initial conditions – in case of solutions with extremely small basin of attraction, very small meshes might be required to select initial conditions that lead to those solution. Finer meshes give better accuracy but with a tremendous increase in the computational cost. To solve this issue an analytical method to compute a first solution from the isolae was given in Hill, Neild and Cammarano (2016), for weakly nonlinear structures. This methodology can be applied by using, for instance, the harmonic balance method.

The computation of isolae is important because the responses belonging to these branches can have large amplitude and therefore they can be harmful. In some applications, though, large amplitude are desirable – in those cases isolae offer great potential to improve the performance of the system. For example in nonlinear suspensions undetected isolae may cause large oscillations and lead to catastrophic scenarios. However, in energy harvester system, isolae could be useful to harvest more energy over a broader range of frequency.

This paper investigates isolae in electromechanical systems. with cubic stiffness, based on Preumont (2006) and Kovacic and Brennan (2011). For this nonlinear component the dynamic response of the system and its possible applications will be discussed.

However, in the last section some initial considerations on the effects caused by nonlinear inductors and capacitors are given. In particular a moving core inductance based on Ho, Nguyen and Woo (2011) and Mendrela (1999) and capacitance of a MIM capacitor based on Kusters and Petersons (1963) and Wenger, et al. (2009) are considered.

NONLINEAR STIFFNESS

The electromechanical system is depicted in Fig. 1, where m is the mass, c is the damping coefficient, k is the linear stiffness coefficient, k_n is the cubic nonlinear stiffness coefficient of the mechanical subsystem; L is the inductance, C_0 is the capacitance and R is the resistance of the electrical subsystem. The coupling between the mechanical and electrical subsystems is made by a moving coil transducer which has a constant $B = 2\pi nrM$, where n is the number of turns in the coil, r is the radius of the turns and M is the magnetic field. e represents an AC voltage source, f the magnetic force, x is the displacement of the mass and x_0 is the displacement of the base.

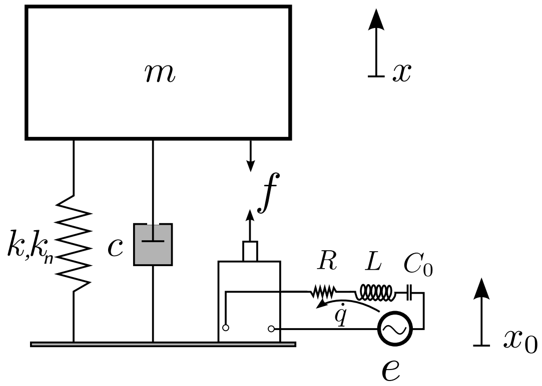

Figure 1: Electromechanical system.

The normalized equations of the system are given by

$$\begin{aligned} &x'' + 2\xi_m(x' - x_0') + (x - x_0) + \gamma(x - x_0)^3 - \tau_m q' = 0 \\ &q'' + 2\xi_e\lambda q' + \lambda^2 q + \tau_r\tau_m(x' - x_0') = 0 \end{aligned} \tag{1}$$

with the following normalized parameters:

$$\tau = \omega_m t \qquad \omega_m = \sqrt{\frac{k}{m}} \qquad \omega_e = \sqrt{\frac{1}{LC_0}} \qquad \xi_m = \frac{c}{2m\omega_m} \qquad \xi_e = \frac{R}{2L\omega_e}$$

$$\Omega_m = \frac{\omega}{\omega_m} \qquad \lambda = \frac{\omega_e}{\omega_m} \qquad \tau_m = \frac{B}{m\omega_m} \qquad \tau_e = \frac{B}{L\omega_m} \qquad \tau_r = \frac{\tau_e}{\tau_m}$$

Figure 2 shows the bifurcation diagram obtained by using Auto 07p for numerical continuation and Runge-Kutta for numerical integration (RK45) for system in Eq. (1). The parameters considered are $\tau_m = 1$, $\tau_r = 0.1$, $\xi_m = 0.1$, $\xi_e = 0.1$, $\lambda = 3$ and $\gamma = 15$.

One would expect based on the single-degree-of-freedom (SDOF) system that there would be two stable solutions and one unstable solution at a frequency band, while only one stable solution at other frequency bands. However, as this is a two-degree-of-freedom system more periodic responses were obtained, as it is shown in Fig. 2. There are two backbone curves, one main branch of solutions which follows the above expectation and two isolae. As stated in Hill, Neild and Cammarano (2016), in order to an isola exist, there must be a backbone curve that crosses it, which suggests there is an isolated backbone curve which was not computed.

Figure 3 shows the basins of attraction of this system for $\Omega_m = 2.5$ in (a) and $\Omega_m = 4$ in (b). The blue initial conditions lead to the bigger isola depicted in Fig. 2, the red ones lead to the smaller isola and the yellow ones lead to the main branch.

By observing Fig. 3 (b), it is clear that the basin of attraction of the smaller isola is inside the basin of attraction of the main branch. If one wants the EM system to behave as an isolator, this is a risky nonlinearity. However, if one wants the system to behave as an energy harvester, this smaller isola could be interesting to improve the harvesting band. The bigger isola is, possibly, a dangerous response for both applications due to its very large amplitude.

ONGOING WORK

The study with the nonlinear stiffness is being expanded using the harmonic balance method to identify the presence of isolae and the same methodology is being applied to EM systems with nonlinear inductance and capacitance, one at a time.

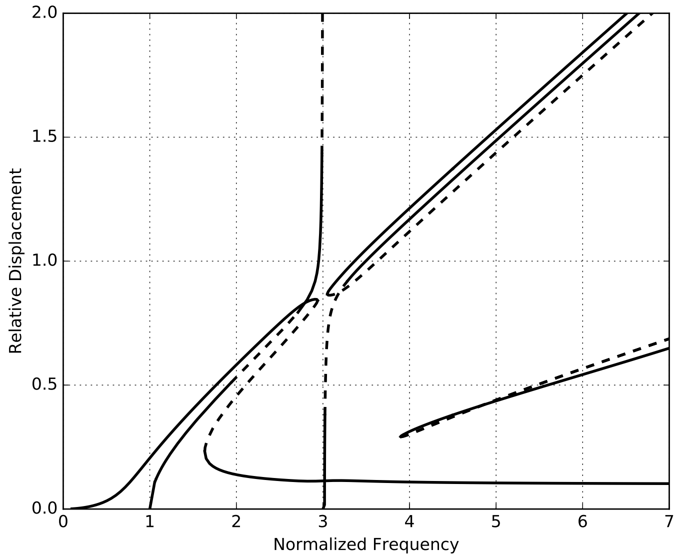

Figure 2: Bifurcation diagram of system 1.

Nonlinear Inductance

Based on Ho, Nguyen and Woo (2011) and Mendrela (1999), the inductance can be nonlinear due to a moving core in a coil according to its relative displacement z. The equation which models this behavior is:

$$L(z) = L_0 + \frac{A}{\sigma\sqrt{\pi/2}} e^{-2(z/\sigma)^2} \tag{2}$$

where L_0 is the inductance without the core, A is a function coefficient, σ is a standard deviation and e is the Euler's number. Considering this, the model becomes:

$$\begin{gathered} m\ddot{x} + c(\dot{x} - \dot{x}_0) + k(x - x_0) - B\dot{q} = 0 \\ \frac{d(L\dot{q})}{dt} + R\dot{q} + \frac{q}{C_0} + B(\dot{x} - \dot{x}_0) = 0 \end{gathered} \tag{3}$$

Nonlinear Capacitance

Based on Kusters and Petersons (1963) and Wenger, et al. (2009), the capacitance can have quadratic voltage coefficient $C_v(v) = C_0(1 + \alpha v^2)$, where v is the voltage and $\alpha = 2n_2 n_0/(k_0 d^2)$ is the quadratic coefficient, with n_2 being the nonlinear refractive index, n_0 the linear refractive index, k_0 the linear dielectric constant and d the thickness of the dielectric layer.

The voltage in a capacitor is defined by:

$$dv = \frac{1}{C_v} \dot{q}(t) dt \tag{4}$$

Hence:

$$q = C_0 v + \frac{\alpha C_0 v^3}{3} \tag{5}$$

As one must obtain voltage (v) in function of the charge (q) in order to model the circuit, solving Eq. (5) results in only one real solution. This solution can be approximated by a Taylor expansion:

$$v(q) = \frac{q}{C_0} - \frac{\alpha q^3}{3C_0^3} + \frac{\alpha^2 q^5}{3C_0^5} \tag{6}$$

Considering the 5^{th} order Taylor expansion, the EM model becomes:

$$\begin{gathered} m\ddot{x} + c(\dot{x} - \dot{x}_0) + k(x - x_0) - B\dot{q} = 0 \\ L\ddot{q} + R\dot{q} + \frac{q}{C_0} - \frac{\alpha q^3}{3C_0^3} + \frac{\alpha^2 q^5}{3C_0^5} + B(\dot{x} - \dot{x}_0) = 0 \end{gathered} \tag{7}$$

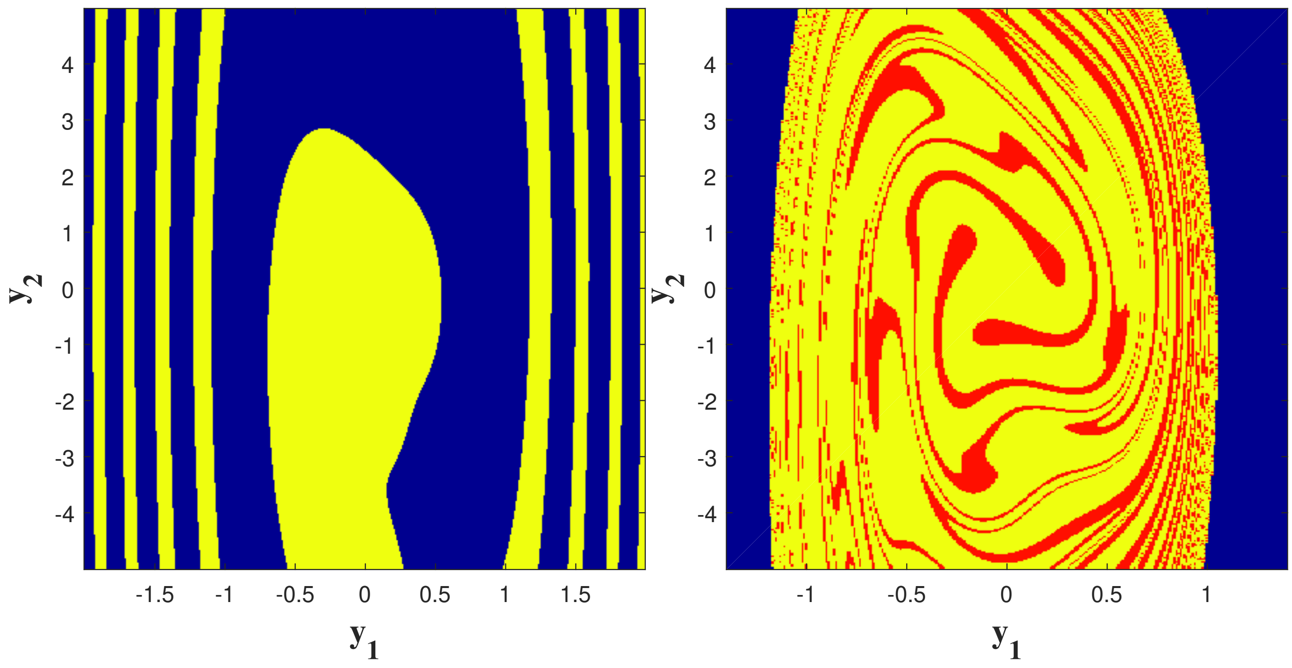

Figure 3: Basins of attraction. $\Omega_m = 2.5$ (a) and $\Omega_m = 4$ (b).

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