DINAME 2019 - Proceedings of the XV International Symposium on Dynamic Problems of Mechanics,
M.A. Savi, T.G. Ritto and W.M. Bessa (Editors), ABCM, Buzios, RJ, Brazil, March 10th to 15th, 2019

Numerical Investigation of a Jeffcott Rotor Undergoing Impact Against an SMA Coated Stator

Alberto Paiva [1], **Rodrigo Veronese Moreira** [1] **and Alex Neves Brandão Mendes** [1]

[1] Universidade Federal Fluminense – Volta Redonda School of Engineering – Department of Mechanical Engineering
420 Trabalhadores Av. – Volta Redonda – RJ – Brazil – 27.255-125
albertopaiva@id.uff.br, rveronese@id.uff.br and alexbrandao@id.uff.br

Abstract: This work involves physical modeling and numerical simulation of a Jeffcott-based rotor-stator system with four degrees of freedom (two planar coordinates for each component), such that the stator inner surface is subjected to contact and is coated with a shape memory alloy layer, whose constitutive behavior is described by a first-order phase transition polynomial constitutive model. Moreover, the dynamical response strongly depends on this shape memory coating layer temperature, which affects its stiffness and directly influences the impact forces. In this work, this element temperature is subjected to two competing phenomena: heating deriving from friction during impact and cooling as a result of convection caused by the interaction with the surrounding environment. The results show the model sensitivity to the convective coefficient, indicating that a more complex behavior is associated to lower values of this parameter. Besides the results presented herein, a thorough investigation of the nonlinear behavior pattern is conducted by means of basins of attraction (as a matter of fact, parameters spaces), varying three main system parameters – namely: rotating speed, convective coefficient and friction coefficient.

Keywords: Jeffcott rotor, shape memory alloys, Falk model, nonlinear dynamics, chaos.

INTRODUCTION

A very common phenomenon in rotating dynamics is the intermittent contact between rotor and stator, which induces undesirable behaviors that may both compromise endurance and prevent the system from working properly. In the last decades, the development of intelligent materials has brought up new possibilities to avoid such inconvenience, taking advantage of particular properties of this kind of materials, for instance: intrinsic energy dissipation and adaptability.

The first attempts towards rotordynamics modeling took place in the late nineteenth century with Rankine and, afterwards, De Laval. In the beginning of the twentieth century, Jeffcott proposed a simple (but useful) model for a planar motion of a symmetric rotor, with two decoupled *Degrees of Freedom* (**DOF**). Since then, other complicating effects have been incorporated into these precursor models; nevertheless, a great hiatus took place, until scientific contributions start to arise in the literature.

Concerning more recent studies, Choy & Padovan (1987) modeled a rotor-stator Jeffcott system and conducted numerical simulations, in order to understand nonlinear effects originated from rubbing effect, due to Coulomb friction. Muszynska & Goldman (1995) made a nonlinear analysis of the aforementioned system based on bifurcation diagrams varying rotation speed and damping coefficient. Their results are in good agreement with experimental observations that attest that higher values of the damping coefficient inhibit chaotic behavior. Edwards et al. (1999) and Al-Bedoor (2000) included the shaft torsional effects as a third DOF. Popprath & Ecker (2007) studied the rotor-stator interaction, taking into account the stator inertial effects, including impact and friction between rotor and stator. Vlajica et al. (2017), besides considering the torsional effect, introduced rotor-stator contact (impact and friction) but the stator is massless. Some other authors scrutinized the rotor-stator contact dynamics with innovative approaches, experimental verifications and even providing approximate analytical solutions (Pavlovskaia et al. 2004; Lahriri & Santos 2013; Chavez et al. 2015; Fonseca et al. 2016). Another critical phenomenon that takes place in rotordynamics concerns the gyroscopic effects, due to both unbalance and asymmetry. There are several articles dedicated to this subject as, for instance: Riemann et al. (2010) and Ozşahin (2014), among others.

Recently, with the expanding **SMA** exploration (Paiva and Savi, 2006; Lagoudas, 2008, Cisse et al. 2016), new possibilities have arisen for nonlinear dynamic systems (Bernardini and Rega, 2010; Savi, 2015), including nonsmooth systems (Santos and Savi, 2009; Sitnikova et al. 2010) and rotor dynamics applications (Silva et al. 2013).

The dynamical model presented in this work deals with a 4-degree of freedom (**DOF**) rotor system consisting of a planar Jeffcott rotor and a stator, assuming 2 DOF for each. The stator inner surface is coated with an **SMA** layer and is subjected to intermittent contact and friction with the rotor. The **SMA** behavior is based on the polynomial model proposed by Falk (1980). The contact forces between rotor and stator are affected by the **SMA** layer mechanical behavior, which depends on its temperature. On the other hand, the **SMA** layer temperature is affected by the contact forces themselves (inducing heating due to friction) and by the interaction with the surrounding environment (inducing cooling due to convection). Since the **SMA** temperature is calculated considering these two competing phenomena, it incorporates one more **DOF** in the mathematical formulation.

The results involve a parameters space analysis (rotation speed *versus* convective coefficient), concerning the contact nature between rotor and stator (no contact, intermittent contact and permanent contact); the dynamical pattern (periodicities, chaotic, hyperchaotic) and the stabilized temperature of the **SMA** element in the steady-state regime.

MATHEMATICAL MODELING

This section is devoted to the formulation of the dynamical equations of motion for the rotor-stator physical model presented in Fig. 1. The system consists of a four-degree of freedom system, being two of them associated with a Jeffcott planar rotor and the other two related to a surrounding stator, with planar motion as well. The rotor has mass m_r and rotating speed Ω, whose center of mass (point $\boldsymbol{G}$) has an eccentricity e from its geometric center (point $\boldsymbol{O}$), due to unbalance. The stator with mass m_s encloses the rotor with a constant radial gap γ, measured in a rest position of both rotor and stator, considering the respective weight forces. Both rotor and stator have linear supports with respective equivalent stiffness and damping k_r; c_r and k_s; c_s. The inner surface of the stator is coated with an **SMA** wear layer, which may undergo contact and friction with the rotor.

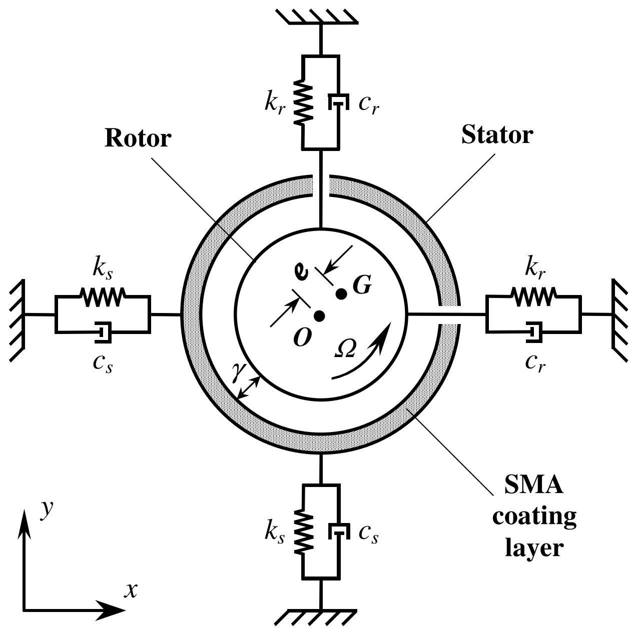

Figure 1 – Rotor-stator physical model

Equation (1) presents the equations of motion, where acceleration due to gravity is given by g. Rotor displacements in x and y directions are given by x_r and y_r (leftward in Eq. 1); analogously, x_s and y_s, stand for the stator (rightward in Eq. 1).

$$m_r \ddot{x}_r + c_r \dot{x}_r + k_r x_r = m_r e \Omega^2 \cos(\Omega t + \varphi) + F_{Cx} \quad \text{and} \quad m_s \ddot{x}_s + c_s \dot{x}_s + k_s x_s = -F_{Cx}$$
$$m_r \ddot{y}_r + c_r \dot{y}_r + k_r y_r = m_r e \Omega^2 \sin(\Omega t + \varphi) + F_{Cy} - m_r g \qquad m_s \ddot{y}_s + c_s \dot{y}_s + k_s y_s = -F_{Cy} - m_s g \tag{1}$$

The contact forces are given in x and y directions by F_{Cx} and F_{Cy} (leftward equations in Eq. 2), respectively. Regarding contact, it occurs every time the modulus of the relative displacement between rotor and stator $|\underline{r}_{sr}|$ exceeds the gap γ, i.e.: $|\underline{r}_{sr}| \geq \gamma$. Contact forces concerning the **SMA** layer are presented by Eq. (2) for normal and tangential directions and are represented by F_{Cn} and F_{Ct} (rightward equations in Eq. 2), respectively; while μ is the friction coefficient. The **SMA** behavior is based on the polynomial model proposed by Falk (1980), which is described as a function of the indentation δ, defined as $\delta = |\underline{r}_{sr}| - \gamma$ for contact conditions. The constants A, B and C are related to the original Falk parameters and stator geometry. T_l is the **SMA** layer temperature, while T_A and T_M represent the triggering transformation temperatures for austenitic and martensitic phases, respectively.

$$F_{Cx} = F_{Cn} \frac{1}{|\underline{r}_{sr}|}\left[-(x_r - x_s) + \mu(y_r - y_s)\right] \quad \text{and} \quad F_{Cn} = A(T_l - T_M)\delta - B\delta^3 + \frac{C}{(T_A - T_M)}\delta^5$$
$$F_{Cy} = F_{Cn} \frac{1}{|\underline{r}_{sr}|}\left[-(y_r - y_s) - \mu(x_r - x_s)\right] \qquad F_{Ct} = \mu F_{Cn} \tag{2}$$

The **SMA** layer temperature evolution is considered in Eq. (3), which establishes its temperature rate of change as a function of contact forces and convective phenomena. The ambient temperature is given by T_{amb}, while the convection coefficient is represented by h. Rotor radius is represented by r_r and λ is the fraction of heat stemming from friction that contributes to the **SMA** internal energy increase. The remaining constants, related to the **SMA** layer are: mass m_l, specific heat c_l and convection area A_l. The whole set of equations is numerically solved using the 4$^{\text{th}}$-order *Runge-Kutta* method.

$$\dot{T}_l = \frac{1}{m_l c_l}\left[\lambda F_{Ct} r_r \Omega - h A_l (T_l - T_{amb})\right] \tag{3}$$

NUMERICAL RESULTS AND CONCLUDING REMARKS

The results presented herein focus on the convective coefficient influence over the system dynamics. Figure 2 presents the bifurcation diagrams for the stator displacement in the *x*-direction and the system converged Lyapunov exponents, as a function of the rotating speed for three different convective coefficients: $h = 50$ W/m^{2}K, $h = 175$ W/m^{2}K and $h = 300$ W/m^{2}K. It is worthwhile to notice that these results are achieved after both dynamical behavior stabilization and **SMA** temperature stabilization.

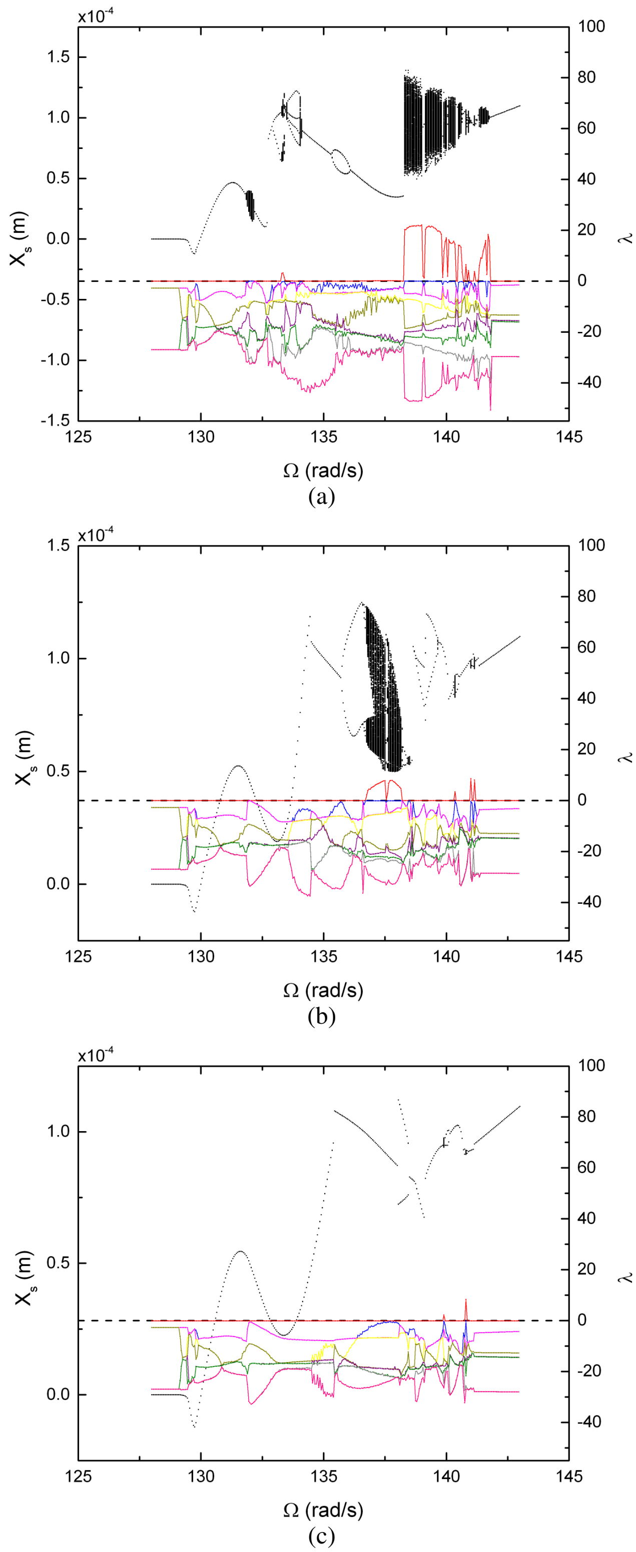

Figure 2 – Bifurcation diagrams and converged Lyapunov exponents for different convective coefficients. (a) 50 W/m^{2}K; (b) 175 W/m^{2}K and (c) 300 W/m^{2}K.

From previous investigations, out of scope of this work, it has been found that the stator dynamical behavior is, in general, more complex than the rotor response. Moreover, x and y-direction displacements present the same qualitative response. Therefore, the stator displacement in the x-direction is adopted for this analysis. In the bifurcation diagrams, the rotating speed is varied from $\Omega = 128$ rad/s to $\Omega = 143$ rad/s, which corresponds to a range where the contact behavior is intermittent.

The low-convective coefficient ($h = 50$ W/m^{2}K) diagram, in Fig. 2(a), shows some regions with high density of points, which correspond to one positive Lyapunov exponent, for instance, around $\Omega = 132$ rad/s and from $\Omega = 138$ rad/s to $\Omega = 142$ rad/s. Therefore, the behavior of such regions can be classified as chaotic. Outside these regions, the behavior is mostly periodic. Considering an intermediate value of the convective coefficient ($h = 175$ W/m^{2}K), shown in Fig. 2(b), the second chaotic region moves leftward from the previous range ($\Omega = 138$ rad/s to $\Omega = 142$ rad/s) to a region from $\Omega = 137$ rad/s to $\Omega = 138$ rad/s. Besides from this change, the overall periodic behavior increases when compared to the low-convective coefficient condition. At last, Fig. 2(c) shows the results for a high-convective coefficient ($h = 300$ W/m^{2}K) condition. Now, the behavior becomes mostly periodic, except for two single chaotic points near $\Omega = 140$ rad/s and $\Omega = 141$ rad/s, respectively.

A comparison between the bifurcation diagrams leads to the conclusion that a much more complex behavior is associated to lower values of the convective coefficient. This behavior can be explained from the fact that higher **SMA** layer temperatures are achieved in such cases, which, based on its thermomechanical behavior, leads to higher contact stiffness. Higher contact stiffness tends to increase the discontinuity in the system, which corresponds to the main nonlinearity in the equations of motion.

REFERENCES

Al-Bedoor, B.O.; 2000; “Transient Torsional and Lateral Vibrations of Unbalanced Rotors with Rotor-to-Stator Rubbing”; Journal of Sound and Vibration; Vol. 229; N. 3; pp. 627–645.

Bernardini, D. & Rega, G.; 2010; “The Influence of Model Parameters and of the Thermomechanical Coupling on the Behavior of Shape Memory Devices”; International Journal of Nonlinear Mechanics; Vol. 45; N. 10, pp. 933–946.

Chávez, J.P., Hamaneh, V.V. & Wiercigroch, M.; 2015; “Modelling and Experimental Verification of an Asymmetric Jeffcott Rotor with Radial Clearance”; Journal of Sound and Vibration; Vol. 334; pp. 86–97.

Choy, F.K. & Padovan, J.; 1987; “Non-Linear Transient Analysis of Rotor-casing Rub Events”; Journal of Sound and Vibration; Vol. 113; N. 3; pp. 529–545.

Cisse, C., Zaki, W. & Zineb, T.B.; 2016 “A Review of Constitutive Models and Modeling Techniques for Shape Memory Alloys”; International Journal of Plasticity; Vol. 76; pp. 244–284.

Edwards, S., Lees, A.W. & Friswell, M.I.; 1999; “The influence of Torsion on Rotor/Stator Contact in Rotating Machinery”; Journal of Sound and Vibration; Vol. 225; N. 4; pp. 767–778.

Falk, F.; 1980; “Model Free-energy, Mechanics and Thermodynamics of Shape Memory Alloys”; ACTA Metallurgica; Vol.28; pp. 1773–1780.

Fonseca, C.A.L.L., Aguiar, R.R. & Weber, H.I.; 2016; “On the Non-Linear Behaviour and Orbit Patterns of Rotor/Stator Contact with a Non-conventional Containment Bearing”; International Journal of Mechanical Sciences; Vol. 105; pp. 117–125.

Lagoudas, D.C.; 2008; “Shape Memory Alloys Modeling and Engineering Applications”; Ed. Springer; 435 pp.

Lahriri, S. & Santos, I.F.; 2013; “Theoretical Modelling, Analysis and Validation of the Shaft Motion and Dynamic Forces During Rotor-Stator Contact”; Journal of Sound and Vibration; Vol. 332; pp. 6359–6376.

Muszynska, A. & Goldman, P.; 1995; “Chaotic Responses of Unbalanced Rotor/Bearing/Stator Systems with Looseness or Rubs”; Chaos, Solitons and Fractals; Vol. 5; N. 9; pp. 1683–1704.

Özşahin, O., Özgüven, H.N. & Budak, E.; 2014; “Analytical Modeling of Asymmetric Multi-Segment Rotor-Bearing with Timoshenko Beam Model Including Gyroscopic Moments”; Computers & Structures; Vol. 144; pp. 119–126.

Paiva, A. & Savi, M.A.; 2006 “An Overview of Constitutive Models for Shape Memory Alloys”; Mathematical Problems in Engineering; Article Id. 56876, Vol. 2006; 30 pp.

Pavlovskaia, E.E., Karpenko, E.V. & Wiercigroch, M.; 2004; “Non-Linear Dynamic Interactions of a Jeffcott Rotor with Preloaded Snubber Ring”; Journal of Sound and Vibration; Vol. 276; pp. 361–379.

Popprath, S. & Ecker, H.; 2007; “Nonlinear Dynamics of a Rotor Contacting an Elastically Suspended Stator”; Journal of Sound and Vibration; Vol. 308; Issues 3–5; pp. 767–784.

Riemann, B., Castro, H.F., Cavalca, K.L., Nordmann, R. & Rinderknecht, S.; 2010; “Development of an Efficient Simulation Model for Rotors in Hydrodynamic Bearings Including Fluid-Induced Instability and Gyroscopic Effects”; Proceedings of the 8th IFToMM - International Conference on Rotordynamics; Vol. 1; 6 pp.

Santos, B.C. & Savi, M.A.; 2009 “Nonlinear Dynamics of a Nonsmooth Shape Memory Alloy Oscillator”; Chaos, Solitons and Fractals; Vol. 40; N. 1; pp. 197–209.

Savi, M.A.; 2015 “Nonlinear Dynamics and Chaos in Shape Memory Alloy Systems”; International Journal of Non-Linear Mechanics; Vol.70; pp. 2–19.

Silva, L.C., Savi, M.A. & Paiva, A.; 2013 “Nonlinear Dynamics of a Rotordynamic Nonsmooth Shape Memory Alloy System”; Journal of Sound and Vibration; Vol. 332; pp. 608–621.

Sitnikova, E., Pavlovskaia, E., Wiercigroch, M. & Savi, M.A.; 2010 “Vibration Reduction of the Impact System by an SMA Restraint – Numerical Analysis”; International Journal of Nonlinear Mechanics; Vol. 45; N. 9; pp. 837–849.

Vlajica, N., Champneys, A.R. & Balachandran, B.; 2017; “Nonlinear Dynamics of a Jeffcott Rotor with Torsional Deformations and Rotor-Stator Contact”; International Journal of Non-Linear Mechanics; Vol. 92; pp. 102–110.

RESPONSIBILITY NOTICE