Rotor Crack Identification Based on Mode Shape and Derivative Order Using Combination Resonances

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Abstract: Several works on crack detection techniques dedicated to rotating machines have been proposed along the years. Previously, the second order derivative of the shaft mode shapes was used for crack identification purposes revealing satisfactory results. In this case, the second derivative of both the healthy and the faulty rotor mode shapes are compared. Any deviation on the resulting curves is interpreted as the crack location. However, the vibration responses of the rotor must be measured for many positions along the shaft. Additionally, good results are obtained only if the rotor operates close to one of its critical speed. Recently, a crack identification methodology based on a nonlinear approach was proposed. This technique uses external applied diagnostic forces at specific frequencies attaining combination resonances in the spectral responses of the flexible shaft with crack. In this context, the present work proposes to use both methodologies to identify the presence and the location of cracks in a rotating machine. The advantage of this technique is that the shaft can operate in its normal condition, which is usually far away from any critical speed.

Keywords: rotating machines, crack detection, modal shapes, combination resonances

INTRODUCTION

Rotating machines represent an important class of systems widely used in industry. The safety operation condition of these types of equipment has been desirable especially when it implies a reduction of costs or eventual economic losses due to unplanned shutdowns. Thus, most of the investigations on rotordynamics were motivated by malfunctions. The appearance of failures is typical in several components, including shafts, which are subject to combined bending, axial, and torsional loads. In order to avoid sudden failures, Structural Health Monitoring (SHM) techniques have been used and demonstrated to be a promising tool. According to Cavalini Jr et al. (2016), vibration-based methods allied to representative numerical models lead to satisfactory results even when the damage appears at inaccessible locations of the machine. For instance, incipient cracks that appear under the surface of the shaft.

Iwatsubo et al. (1992) proposed an SHM technique based on periodical external excitation for crack detection in rotating machines. The authors observed the appearance of combination harmonics due to the nonlinear behavior of the cracked system. A similar method was discussed by Ishida and Inoue (2006). However, the combination resonance approach is not efficient to indicate the location of the crack along the shaft. Thus, the methodology proposed by Iwatsubo et al. (1992) should be combined with other techniques.

In this context, the present work proposes to associate the combination resonances approach with the second order derivative of the shaft mode shapes constructed from the obtained combination resonances. The second order derivative of the beam mode shapes was used for crack identification purposes revealing satisfactory results (Pandey et al. (1991) and Ismail et al. (2006)). In this case, the second derivative of both the healthy and the faulty rotor mode shapes are compared. Any deviation on the resulting curves is interpreted as the crack location. However, the vibration responses of the rotor must be measured for many positions along the shaft. Additionally, good results are obtained only if the rotor operates close to one of its critical speed.

The advantage of the technique proposed in the present contribution is that the shaft can operate on its normal condition, which is usually distant from any critical speed. In this case, an external diagnostic excitation with a specific frequency is applied to the rotating machine. The frequency of the diagnostic force is determined by using the method of multiple scales. The rotating machine used in this work is composed of a flexible horizontal shaft, two rigid discs, and two self-aligning ball bearings. The vibration responses of the rotating machine were determined by using a finite element model (FEM).

ROTOR MODELING

The FEM model used to represent the rotor was formulated based on the Timoshenko beam theory with 35 finite elements, as shown in Fig. 1. It is composed by a flexible steel shaft with 850 mm length and 19 mm diameter (E = 182 GPa, ρ = 7930 kg/m^3, and ν = 0.29), two rigid discs D_1 (node #18; 2.637 kg) and D_2 (node #25; 2.649 kg), both of steel with 150 mm diameter and 20 mm thickness (ρ = 7850 kg/m^3), and two self alignment ball bearings (B_1 and B_2, located at the nodes #6 and #33, respectively). For this configuration, the first and second forward whirl critical speeds of the rotating machine were 53.06 Hz and 207.81 Hz. The first and second backward whirl critical speeds were 51.85 Hz and 190.15 Hz.

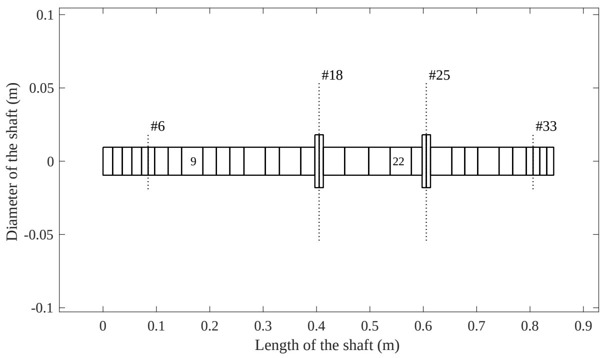

Figure 1 – Shaft-bearing-disc finite element model.

The dynamic behavior of cracked flexible shaft supported by the self-alignment ball bearings is mathematically represented as presented by Eq. (1).

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{D}\dot{\mathbf{q}} + \mathbf{K}(\Omega t)\,\mathbf{q} = \mathbf{W} \tag{1}$$

where $\mathbf{M}$ is the mass matrix, $\mathbf{D}$ is the damping matrix, $\mathbf{K}(\Omega t)$ is the stiffness matrix with variable values due to the crack existence, Ω is the rotation speed of the shaft, Ωt is the angular position of the shaft, t is the time, $\mathbf{W}$ stands for the weight of the rotating parts, and $\mathbf{q}$ is the generalized displacement vector. Regarding the crack simulation, the finite element with crack was obtained by using first the linear fracture mechanics theory to determine the additional flexibility due to the crack. This formulation is explained next, assuming a beam element containing a transverse crack with depth α.

Crack breathing model

The model proposed by Mayes and Davies (1984) is currently used to represent the breathing behavior of cracks with weight dominance. A simple mathematical function describes the mechanism for opening and closing the crack. However, this model is not able to correlate the additional flexibility of a shaft due to the crack depth. Thus, in this work, the additional flexibility is calculated by using the linear fracture mechanics theory.

Linear fracture mechanics

The Castiglione theorem is used to determine the displacement of the shaft with a crack q_i in the direction of the applied load P_i, as presented by Eq. (2) (Darpe et al., 2004).

$$q_i = \frac{dU}{dP_i} = \frac{\partial U^0}{\partial P_i} + \frac{\partial U^c}{\partial P_i} \tag{2}$$

where U^0 is the elastic strain energy of the shaft element with crack and U^c is the additional strain energy due to the crack presence, as described by Eq. (3), in which the concepts of fracture mechanics are used.

$$U^c = \int_A J(A)\,dA = \int_A \frac{1-\upsilon}{E}\left[\left(\sum_{i=1}^{6} K_{Ii}\right)^2 + \left(\sum_{i=1}^{6} K_{IIi}\right)^2 + (1+\upsilon)\left(\sum_{i=1}^{6} K_{IIIi}\right)^2\right] \tag{3}$$

where E is the Young's modulus, ν is the Poisson's ratio, K_{Ii}, K_{IIi}, and K_{IIIi} are the stress intensity factors (SIF) associated with the crack load modes I, II, and III, respectively. In this case, only the crack load mode I is considered since the principal load is applied normal to the crack plane (Anderson, 2005). The additional flexibility c_{ij} is obtained as presented in Eq. (4).

$$c_{ij} = \frac{\partial^2 U^c}{\partial P_i \partial P_j} \tag{4}$$

where the resulting integrals were calculated by using the procedure described in Papadopoulos (2004). Equation (5) shows that the additional flexibility matrix due to the crack is included on the flexibility matrix of the healthy shaft (c_0) to obtain the resulting flexibility of the shaft element with crack.

$$\mathbf{c_{CE}} = \mathbf{c_0} + \mathbf{c} \tag{5}$$

Mayes' breathing model

The Mayes' model suggests that a cosine function governs the opening and closing behavior of the crack. The crack is fully open when the η coordinate coincides with the negative Z coordinate (180° to the Z axis) and fully closed when the η coordinate coincides with the positive Z coordinate (0° or 360° to the Z axis), as shown in Figure 3. The stiffness of the shaft with a crack in rotating coordinates ($\mathbf{k}_{RMayes}$ in a matrix representation) according to the Mayes' model is given by Eq. (6).

$$\mathbf{k}_{RMayes} = \begin{bmatrix} k_{M\xi} + k_{D\xi}C_1 & 0 \\ 0 & k_{M\eta} + k_{D\eta}C_1 \end{bmatrix} \tag{6}$$

where $k_{M\xi} = \left(k_0 + k_\xi\right)/2$, $k_{D\xi} = \left(k_0 - k_\xi\right)/2$, $k_{M\eta} = \left(k_0 + k_\eta\right)/2$, $k_{D\eta} = \left(k_0 + k_\eta\right)/2$, and C_1 = cosθ (C_1 = cos $i\theta$; i = 1,2,3, ...). The stiffness k_ξ and k_η are obtained from the inverse of $\mathbf{c}_{CE}$ ($\mathbf{k}_{CE} = \mathbf{c}_{CE}^{-1}$; $k_\xi = k_{CE}(1,1)$ and $k_\eta = k_{CE}(2,2)$. Similarly, k_0 is obtained from the inverse of the flexibility matrix $\mathbf{c}_0$ ($\mathbf{k}_0 = \mathbf{c}_0^{-1}$; $k_0 = k_0(1,1) = k_0(2,2)$). In fixed coordinates, the stiffness of the shaft with crack ($\mathbf{k}_{FMayes}$ in a matrix representation) is determined by applying the transformation given by Eq. (7).

$$\mathbf{k}_{FMayes} = \begin{bmatrix} C_1 & S_1 \\ -S_1 & C_1 \end{bmatrix}^T \mathbf{k}_{RMayes} \begin{bmatrix} C_1 & S_1 \\ -S_1 & C_1 \end{bmatrix} \tag{7}$$

Combination resonances and the second order derivative of the mode shape

The method begins by defining a set of diagnostic forces with different frequencies Ω_{diag} (i.e., the frequency of the external force applied to the structure) that will induce the rotor to combination resonances. The frequency of each force is determined by using the method of multiple scales. According to Cavalini Jr et al. (2016), the conditions required for a combination resonance follows by Eq. (8):

$$\Omega_{diag} = |n\Omega - \omega_\Omega| \tag{8}$$

where ω_Ω is a critical speed (forward whirl) of the rotor, Ω is the rotation speed, and $n = \pm1, \pm2, \pm3, ...$

In this work, ω_Ω = 53.06 Hz (first forward critical whirl speed of the rotating machine) and Ω = 1200 RPM. Cavalini Jr et al. (2016) showed that the existence of peaks associated with the combination resonances is obtained by using $n = 2$. The diagnostic force was applied along the X direction at the node #33 of the rotor finite element model with an amplitude of 25 N. Figure 2 shows the DFTs obtained by using the diagnostic frequency Ω_{diag} = 13.44 Hz for the pristine condition of the shaft and 10%, 30%, and 50% depth breathing cracks located at the element #22. Note that peaks at combination resonances appear due to the crack existence. As expected, the harmonics of the rotation speeds can be observed in the spectral responses of the shaft with crack.

The crack identification methodology proposed in the present contribution was tested by considering cracks located at the element #9 and element #22, separately. The mode shape of the rotor was reconstructed by using the vibration amplitude of the shaft obtained at the combination resonance of 72.99 Hz. For this aim, DFT signals were measured in different positions of the shaft along the X direction. Then, the second order derivative of the reconstructed mode shape was calculated based on the derivatives of normalized modal displacements. The difference between the second order derivative of the mode shape for the pristine and faulty conditions of the shaft was used as an indication of the crack position. Figure 3 shows the obtained results, in which the location of both cracks (#9 and element #22) was successfully identified for various crack depths.

FINAL REMARKS

This work was dedicated to associate the combination resonance approach with the second order derivative of the mode shapes of a rotor system for crack identification purposes. According to the results, it was possible to observe that the conveyed approach can be used to identify the location of cracks along rotating shafts operating under normal conditions. Further research effort will be dedicated to comparing the numerical results with experimental data.

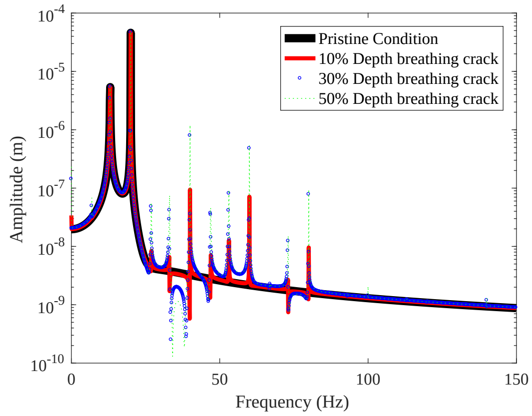

Figure 2 – DFTs obtained by using the diagnostic frequency Ω_{diag} = 13.44 Hz.

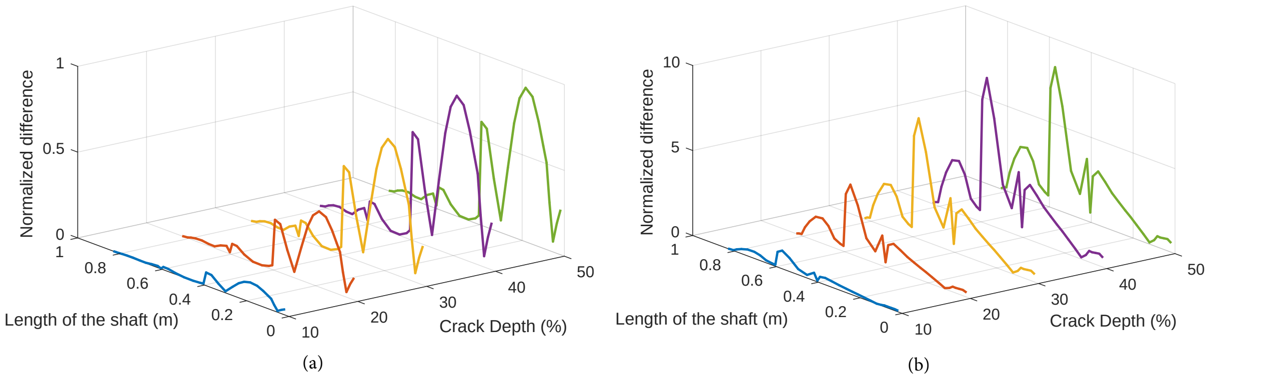

Figure 3 – Difference between the second order derivative of the mode shape for the pristine and faulty conditions of the shaft, crack located (a) in node element 9 and (b) in element 22.

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