Dynamic Response of a Beam Excited by a Moving Load at Different Velocities

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Abstract: The objective of this study is to investigate the response of an Euler-Bernoulli beam excited by a moving force with constant velocity. The dynamic response is obtained in terms of deflection. The effective solution for deflection of such a case can be obtained by applying the integration and the differentiation rules for the beam differential equation. A code to solve the problem is written and is verified with the results from reference obtained by others analytical and numerical methods. The dynamic responses are simulated for different velocities developed by the force and how they affect the beam response.

Keywords: Moving force, dynamic response, different velocities.

INTRODUCTION

The dynamic response of a beam subjected a under moving force action ia a important issue in structural engineering. Several works on vibration of simply-supported beams subjected a under moving forces are found in the literature. Frýba (1999) analyzed the influence of different kinds of moving loads on various structures, with diversify end conditions, damping, velocity and types of forces: punctual or distributed. Olsson (1990) and Kurt (2016) evaluated on the fundamental problem of moving load and presented a comparative study about the analytical and finite element method solutions. Law (1999) identified of moving force in both time and frequency domains.

Cancellations phenomenon and resonances in beams under moving load have been reported by numerous researchers. By example, Savin (2001) analyzed the dynamic amplifications for forced and free vibrations presented that for some optimal periodic length corresponding to the wavelength of the load, theoretically, some responses can be canceled. Uzzal *et al* besides evaluating the behavior of the structures subject to the moving load, also analyzed the effect that the movement of the force causes to propagate by the same until the foundation, specific case for larger structures like bridges, catwalks, among others.

Several methods can be used to solve the mobile load problem, the most constrained are: finite elements, developed by Nguyen and Duhamel (2008), spectral, Azizi *et al* (2012) analyzed continuous structures when subjected to mobile loads by SEM, differences finite, other semi-analytical methods are also used by Kurt (2016) and the as well as this article, which analyze analytically how the work of Foda and Abduljabbar (1998) from the formulation to the functions of Green. Currently, several commercial software have algorithms implemented in their bases to solve problems of this type, through transient analysis.

The standard methods of analyzing the beam dynamics are only if the coefficients in the well-know differential equation of the beam are constants. The theoretical problem is procedure for obtaining the analytical solution is complex. However, through mathematical software for numerical analysis, the solution reduces to the form quickly. Thus, we can observe that the mobile load analysis is a broad field of research and the possibilities of developing new techniques for parameter weighting is a reality.

THEORETICAL FOUNDATION

In this paper, it is consider the Euler-Bernoulli linear elastic beam theory subjected to simple supported boundary condition and excited by a moving load. Figure 2 shows a schematic view of the beam.

The general differential equation of motion for the simply-supported beam excited by a moving point load can be written as (Frýba, 1999):

$$EI\frac{\partial^4 u(x,t)}{\partial x^4} + \rho A\frac{\partial^2 u(x,t)}{\partial t^2} = \delta(x - ct)P(t), \tag{1}$$

where L is the length, A is the cross-section area, ρ is the mass density, E is the Young's modulus, I is the inertia moment, P is the point load, c is the load velocity, δ is the Dirac delta function, $u(x,t)$ is the deflection, x is the space
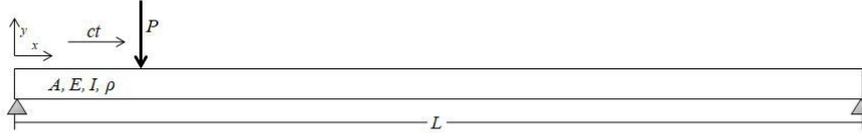
Figure 1 – Simply-supported Euler-Bernoulli beam model subject to a moving load.

position and t is the time.

The force begins its movement at $x = 0$ and $t = 0$. It runs through the entire length of the beam, ending at $x = L$ and $t = t_f$. Thus, these are the conditions for determining the transverse displacements presented by the elastic line of the beam. Therefore, the boundary conditions for simply-supported beam and initial conditions are given by:

$$\begin{cases} u(0,t) = u(L,t) = 0 \\ \frac{\partial^2 u(0,t)}{\partial x^2} = \frac{\partial^2 u(L,t)}{\partial x^2} = 0 \end{cases}, \quad \begin{cases} u(x,0) = 0 \\ \frac{\partial u(x,0)}{\partial t} = 0. \end{cases} \tag{2}$$

For the static case, the maximum deflection presented by the elastic curve of the beam can be found by using the method of integration of Macaulay. For the simply-supported beam the maximum deflection will be at the middle length of the beam (Fig. 2), where a and b are arbitrary points.
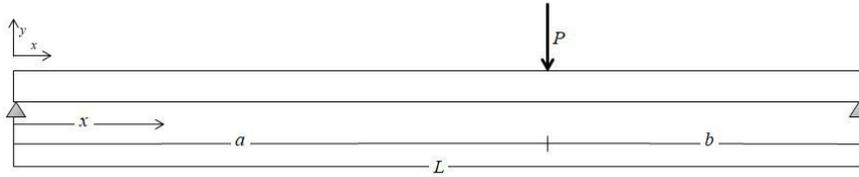

Figure 2 – Force applied at random point on beam.

By applying to the Eq.(1) it has,

$$EI\frac{d^2 u(x)}{dx^2} = \left(\frac{Pb}{L}\right)x - P(x - a). \tag{3}$$

By integrating twice Eq.(3) the equation of the elastic curve is:

$$u_o\left(\frac{L}{2}\right) = \frac{1}{48}\frac{PL^3}{EI}. \tag{4}$$

From the Eq.(1), it is possible to derive de natural frequencies as:

$$\omega_n = n^2\sqrt{\frac{\pi^4 EI}{\rho L^4}}, \tag{5}$$

where n is the $n - th$ frequency.

For a moving point load with constant intensity, $P(t)$, and velocity, c, the load is a function of time and space is given by:

$$P(x,t) = \begin{cases} \delta(x - ct)P(t), & \text{if } 0 \leq t \leq t_f \\ 0, & \text{if } t_f < t \end{cases}, \tag{6}$$

The solution of Eq.(1) can be obtained by decomposing into to a series of uncoupled ordinary differential equations. Frýba (1999) proposes and developed this solution in series for a displacement presented by the elastic line expressed as:

$$u(x,t) = u_o \sum_{n=0}^{\infty} \sin\left(\frac{n\pi x}{L}\right)\frac{1}{n^2(n^2 - \alpha^2)}[\sin(n\omega t) - \sin(\omega_n t)], \tag{7}$$

where $\alpha = c/c_{cr}$ is the velocities ratio and the critical velocity, $c_{cr} = (\pi/L)\sqrt{EI/\rho}$.

NUMERICAL RESULTS AND DISCUSSION

In order to validate the implemented computational code, the results were compared with that obtained by Paganelli (2014). Then, all simulated examples use the same beam geometry and material properties (6061 aluminum) as used in Paganelli's work (Tab.1). The simple-supported beam is excited by a point force with amplitude $P = 7.0$ kN and velocity $c = 2.108$ m/s.

Table 1 – Beam geometry and material properties.

Geometry and Properties	Value
Length (L) [m]	1.0715
Cross-section area (A) [m^2]	6.6834×10^{-3}
Mass density (ρ) [kg/m^3]	2763.6
Young's modulus (E) [GPa]	72.4

Figure 3 shows a comparison between results obtained with the implemented code (Fig. 3(a)) and Paganelli's work (Fig. 3(b)). Should be noticed that Paganelli presents experimental and simulated (Timoshenko and Euler-Bernoulli beam theory) results calculated in $x = \frac{7L}{16}$. It can be seen that code results presents similar behavior as the simulate results obtained by Paganelli. This is a relevant factor for the validation of the implemented code.
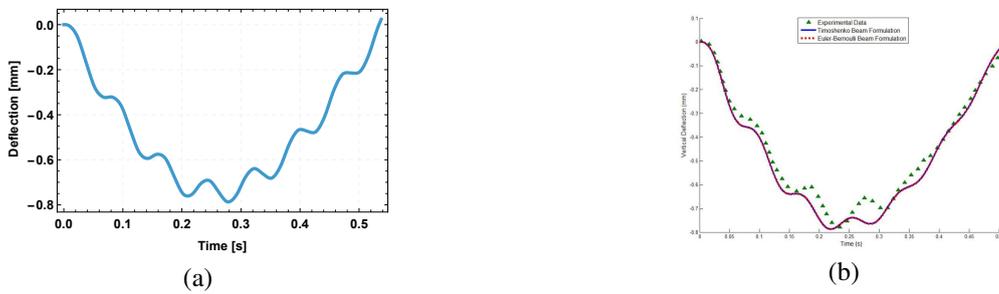

Figure 3 – Beam deflection results from:(a) implemented code (Euler-Bernoulli beam theory); (b) Paganelli's work (Paganelli, 2014).

A trend that is presented in the results obtained is that there is a speed ratio, where the critical velocity is a dependent variable of the boundary condition, so it is observed that the maximum displacement suffers an increase with the increase of the ratio, with proportionality, since it decreases with the reduction of the rate of velocities.
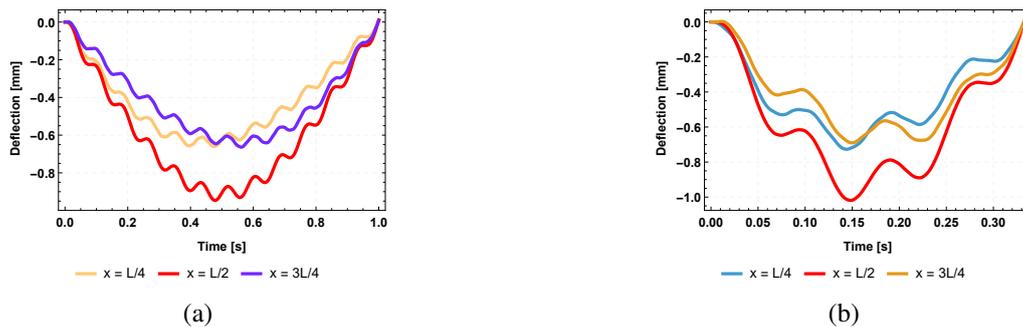

Figure 4 – Beam deflection results from different velocities: (a) $c = 1$ **m/s; (b)** $c = 3$ **m/s; (c)** $c = 5$ **m/s; (d)** $c = 7.5$ **m/s.**

In the dynamic response, three points were chosen to verify the behavior: $x = \frac{L}{4}$, $\frac{L}{2}$ and $\frac{3L}{4}$. Due to the condition, it is noted that there is symmetry between the points from the midpoint. It is possible to observe the effect that the velocity causes in the deflection presented by the beam. With the increase of velocity, there is a reduction of the amplitude of the oscillations and the periodicity of the same. Figure 4 shows the dynamic response of the beam for 4 velocity types: $c = 1$, 3, 5 and 7.5 m/s, once 2 m/s was implemented together with the comparison of Paganelli's work.

Figure 5 shows the three-dimensional diagram behavior of the beam together with the time and length of the beam and effect of moving loads with different speed conditions on dynamic responses. Notice that maximum deflection is increasing with velocity of load. Although the load intensity is kept constant and the same in all cases, the deflection changes for each velocity, increasing along with the speed.
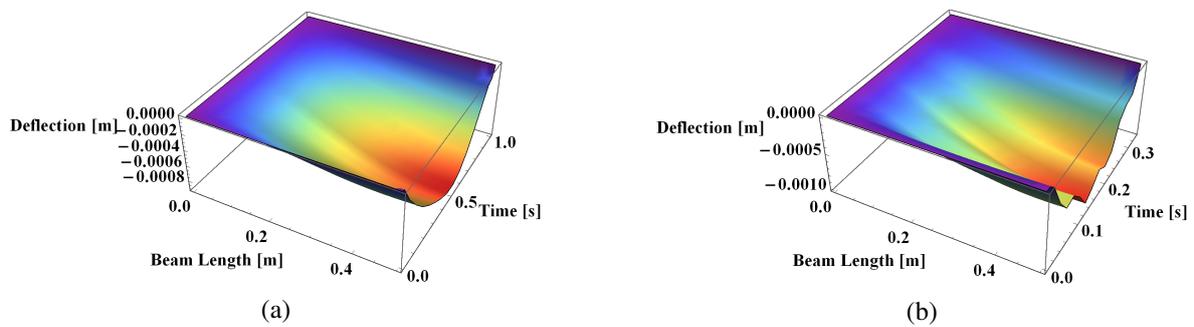
(a)

(b)

Figure 5 – Beam deflection results from different velocities :(a) $c = 1$ **m/s; (b)** $c = 3$ **m/s; (c)** $c = 5$ **m/s; (d)** $c = 7.5$ **m/s.**

CONCLUSIONS

The objective of this study is to investigate the dynamic response of an Euler-Bernoulli beam under a moving load with constant velocity and magnitude. The maximum displacement load does not always occur when the load is in the average range position of the beam. This behavior is a consequence of both the reflected beam boundary and the displacement load velocity.The presented solutions can be applied in the analysis of dynamics and reliability of beams subjected to a moving loads. The load is a point force and beam limits are limited as simply-supported. This study gives basic understanding about dynamics response of the beams subjected to a moving loads. However, for good understanding, further investigations including the combined effect of anisotropy structures and transverse shear should be needed. Beams with periodic structures, different boundary conditions, materials, and other excitation forces will be investigate at respect effects on the displacement in future works.

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