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Bayesian calibration of wake-oscillator model for fluid structure interaction

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Abstract: The accurate prediction of structural instability caused by vortex shedding behind bodies or by nonlinear unsteady aerodynamic is fundamental to avoid the degradation of structural performance or even failure of the system. Numerous approaches can represent analytical models to modeling both the structure and fluid. The CFD (Computational Fluid Dynamics) approaches consists of solving the Navier-Stokes equations directly, mostly limited by heavily computational costs that, many times, are tough to satisfy in the practical engineering. To increase the expectations of solving practical problems, the use of phenomenological surrogate models, are an alternative approach for the underlying physics, where phenomenological equations emulate the fluid dynamic forces acting on the structure, have become an essential tool to simplify the analysis and can be a very useful tool in broad industrial applications. However, constructing accurate surrogate models introduce additional challenges that will be addressed in this work. Most of these models present a series of empirical parameters that need to be calibrated from experimental data. To build an accurate phenomenological model we need putting this parameter variability in the general context of Uncertainty Quantification (UQ). We present a phenomenological model for fluid-structure interaction to be calibrated. In the first stage of this processes, we do global sensitivity analysis for the empirical parameters of the model, where uncertainty source is introduced earlier using the Sparse Grid Stochastic Collocation method. After this, a backward parameter estimation analysis is done using a Bayesian technique to calibrate these empirical parameters, through exploring posterior density functions. Synthetic data were generated as reference simulating experimental data to show the calibration technique used. This kind of analysis can help to understand the effects of varying empirical parameters in the response variables. The influence of these parameters and other coefficients that affect the dynamical response is analyzed and also discussed.

Keywords: Unsteady flows, Reduced models, Calibration

INTRODUCTION

The complexity involved in engineering systems has been, frequently, tackled with the use of sophisticated computational models. That, from the decision makers' standpoint, requires the use of robust and reliable numerical simulators. Often, the reliability of those simulations is disrupted by the inexorable presence of uncertainty in the model data. The vortex-induced vibrations of floating structures play an essential role in the design of offshore engineering. The accurate prediction of structural instability is critical due that the vortex shedding behind bluff bodies may lead to degradation of structural performance or even structural failure. Numerous approaches can represent analytical VIV models to modeling both the structure and fluid. The CFD (Computational Fluid Dynamics) approaches consists of solving the Navier-Stokes equations directly, mostly limited by heavily computational costs. In this sense, surrogate models where phenomenological equations emulate the fluid dynamic forces acting over the structure, see [1], can be an interesting approach to help to obtain predictions with lower computational cost than CFD and become a useful tool with broad industrial applications. In this work, we present a phenomenological vortex induced vibration model proposed in [2] that captures essential features of the VIV dynamics 2DOF, and we try to improve its characteristics through calibration techniques. The reliability of simulations many time is disrupted by the inexorable presence of uncertainty in the model data, such as inexact knowledge of system forcing, initial and boundary conditions, physical properties of the medium, as well as parameters in constitutive equations. These situations underscore the need for efficient uncertainty quantification methods for the establishment of confidence intervals in computed predictions, the assessment of the suitability of model formulations, and the support of decision-making analysis. Putting the parameter variability of the model in the general framework of Uncertainty Quantification (UQ) we emulate experimental data for calibrating the model of interest using Statistical Bayesian methods as an efficient tool and demonstrating their capabilities for this purpose. So, we first have done a forward sensitivity analysis to identify relevant empirical input parameters using the Adaptive Sparse Grid Stochastic Collocation method (ASGC), [3], [4]. After this, a backward parameter estimation analysis was done using a Bayesian technique to estimate the values of these empirical parameters. This kind of analysis can help to understand the behavior of the structure to critical situations and the effects of varying empirical parameters in the response variables. The influence of these parameters and other coefficients that affect the dynamical response is analyzed. This paper is organized as follows. In Section we present the governing equations of the phenomenological model adopted for the analysis. Section 1 we present the numerical results for the forward uncertainty quantification analysis. Section 2 the calibration of the present model. The paper ends with a summary of our main findings in Section 3.

MATHEMATICAL MODEL

A phenomenological model for an elastically supported 2DOF cylinder with wake oscillators models that replace the vortex shedding mechanisms of the flow is presented below like showed in [5], this model is similar to models presented in [6], [1], [7], and was adopted to do this work due that its captures important features of the VIV dynamics with good agreement. As we can see in the scheme of the Figure 1, the cylinder is free to vibrate in both directions, i.e. in cross-flow and in-line directions where only as a reference of the physics we present the solution obtained with *Comsol Multiphysics 4.4* [8] for the lock-in velocity. Therefore, to modeling both the structure and fluid, we approach an analytical VIV model

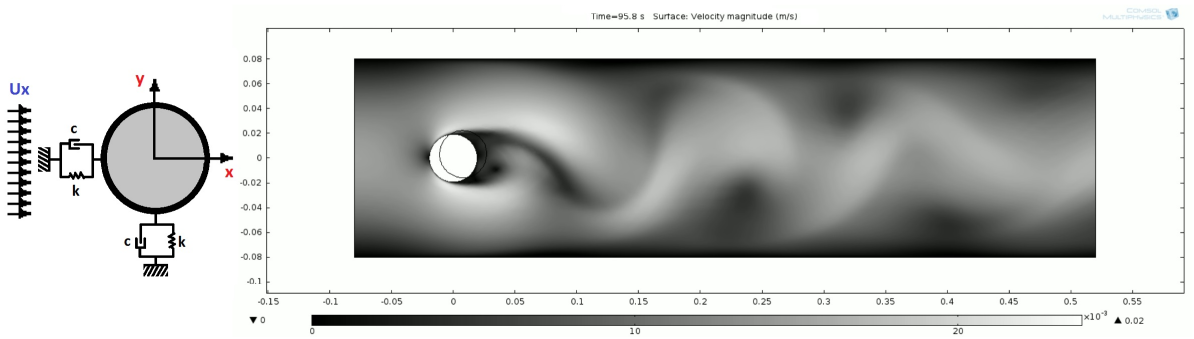

Figure 1: Numerical setup and FEM/Comsol solution for the lock-in velocity

incorporating a Van der Pol type equation as the governing equation for the fluid force that acts on the structure. Therefore, based on this approximation of non-conservative oscillator equations with nonlinear damping and empirical parameters, the effects of the fluid dynamics around the body are incorporated to predict the behavior of the system. The structure oscillator equations and the coupling wake for this cylinder can be represented for the equations of motion regarding the displacements in cross-flow y and in-line x directions as,

$$m\ddot{x} + r_s\dot{x} + hx = \frac{1}{2}\rho_f C_L D|U_R|\dot{y} + \frac{1}{2}\rho_f C_D D|U_R|(U-\dot{x}) \tag{1}$$

$$m\ddot{y} + r_s\dot{y} + hy = \frac{1}{2}\rho_f C_L D|U_R|(U-\dot{x}) + \frac{1}{2}\rho_f C_D D|U_R|\dot{y} \tag{2}$$

$$\ddot{w} + 2\varepsilon_x\Omega_F(w^2-1)\dot{w} + 4\Omega_F^2 w = (A_x/D)\ddot{x} \tag{3}$$

$$\ddot{q} + \varepsilon_y\Omega_F(q^2-1)\dot{q} + \Omega_F^2 q = (A_y/D)\ddot{y} \tag{4}$$

where the hydrodynamics forces, represented in the Eqs. (2,3) are the result of the vortexs-induced forces *Drag* and *Lift*. The mass m considered is the structural per unit of length $m_s = \frac{1}{4}\pi\rho_f D^2$ plus the added mass m_a and where the following system parameters are introduced for the equations,

$$q = 2\frac{C_L}{C_{L0}} \qquad w = 2\frac{C_D^{fl}}{C_{D0}^{fl}} \qquad |U_R| = \sqrt{(U-\dot{x})^2 + \dot{y}^2} \qquad \Omega_F^2 = 2\pi S_t\frac{U_{red}}{D}$$

and

$$C_L = \frac{C_{L0}q}{2} \qquad C_D = C_{D0}(1+Kq^2) + \frac{C_D^{fl}}{2} \qquad m = m_s + m_a$$

being Ω_F^2, frequency of vortex shedding, St, Strouhal number, C_L, C_D, C_{L0}, C_{D0}, lift and drag coefficients for fluctuating and stationary components, m_s, m_a, structural and added mass, D diameter of cylinder, r_s, structural damping, h, stiffness coefficient, ρ_f: fluid density, Ax, Ay, ε_x, ε_y, K, empirical wake oscillators parameters. The dimensions of the numerical model are, $D = 0.0381m$, the aspect ratio of this system is therefore L/D = 0.0381, the structural mass is $m = 6.72$ kg; the added mass is $m = 1.14$ kg; the structural damping coefficient $r_s = 0.0913$ and the stiffness constant is $h = 1.076$ kN/m. The vortex shedding lift coefficient $C_{L0} = 0.3$ is taken as in [9], the reference drag is taken as $C_{D0} = 0.12$ and $C_D^{fl} = 0.2$ drag fluctuating stationary, which was obtained from the experiments reported in [10]. The Strouhal number considered is $St = 0.19$ and the empirical parameters $A_x = 8$ and $A_y = 18$, $\varepsilon_x = 0.3$, $\varepsilon_y = 0.03$ respectively correspond to cross-flow and in-line amplification factors with $K = 1$ parameter for tuning the cross-flow and in-line motions like in [7]. Figures 2 presents the amplitude of vibration as a function of reduced velocity for the in-line and cross-flow vs. experimental results from [10]. Figure 3 shown the cylinder trajectories computed for with a increasing reduced velocity.

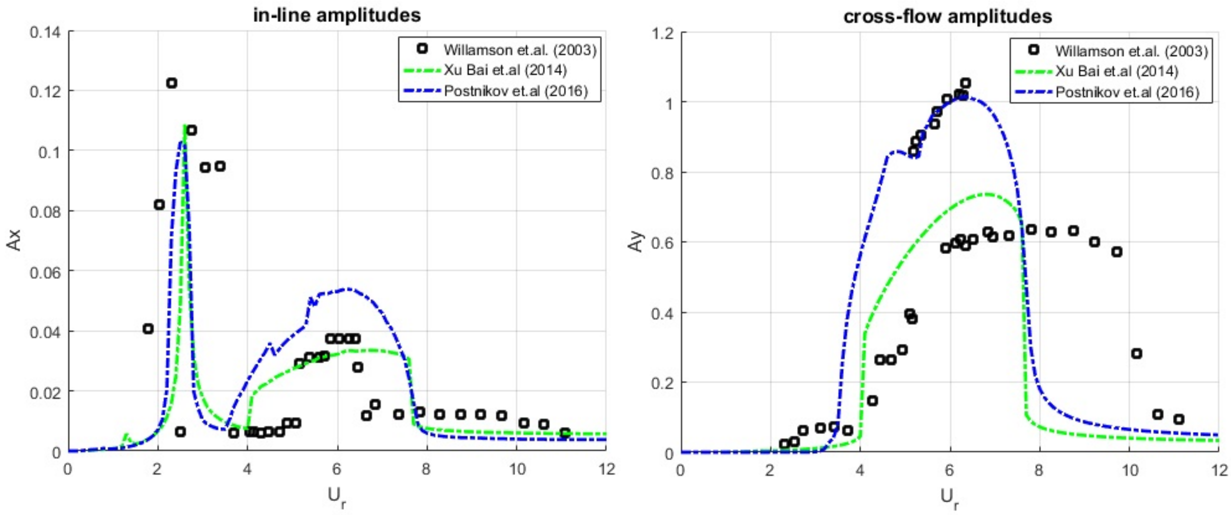

Figure 2: Comparison and experimental amplitudes for cross-flow and in-line versus reduced velocity. Experimental from: N.Javtis C.H.K, Williamson, [10]

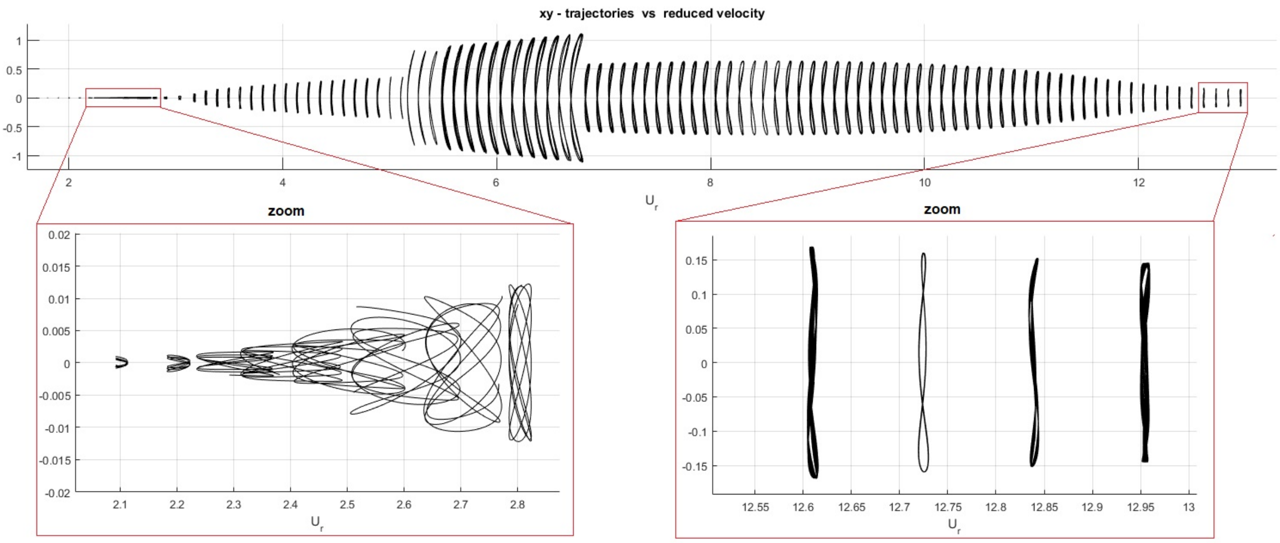

Figure 3: Trajectories of cylinder vs increasing reduced velocity

Parameters of phenomenological models for constructing accurate surrogate models introduce additional challenges that will be addressed in this work. Most of these models present a series of empirical parameters that need to be calibrated from experimental data. Due to this reason, the calibration is an activity that leads to parameters subject to uncertainties that can be modeled using experimental data as references. In the following sensitivity analysis is done to determine the impact of variability in the input parameters on the response of the model.

1 SENSITIVITY ANALYSIS

The sensitivity analysis in this section attempts to establish how the response of the model depends on the attributed values for some input parameters. With this aim, following we briefly introduce the non-intrusive (ASGC) method, described in detail in [4], to estimate the sensitivity for some of these parameters. We have used this method to identify those that have the greatest impact on the output, i.e., amplitudes in-line and cross-flow, for the analyzed U_r range. The idea of this method is building an interpolation function of the stochastic space built through either full-tensor product of 1D interpolation functions. From this function it is possible calculate easily the useful statistics of the solution $\mathbb{E}[u(x)]$ and $\mathbb{V}ar[u(x)]$. In the non-smoothness condition case in the stochastic space it the adaptive strategy improves the interpolation function in the stochastic space, see [11]. Thus, assuming the following probabilistic models for the parameters,

$$C_{D0} = \overline{C_{D0}}(1+\delta_1\xi_1) \qquad C_{L0} = \overline{C_{L0}}(1+\delta_2\xi_2)$$

with $\delta_i = 0.05$ variation over the mean and ξ_i independent and identically distributed uniform random variables where overbears denote the mean (expected) values, δ_i the percentile variation over the mean and ξ_i independent and identically distributed uniform random variables taking values into $[-1, 1]$. We analyze these coupling parameters C_{D0} and C_{L0} varying the reduced velocity U_r in the range $[0, 12]$. The adaptive sparse grid interpolation and integration schemes were implemented using a script of the *Tasmanian* approximation toolkit, a collection of robust libraries for high dimensional integration and interpolation as well as parameter calibration, see [12] for details.

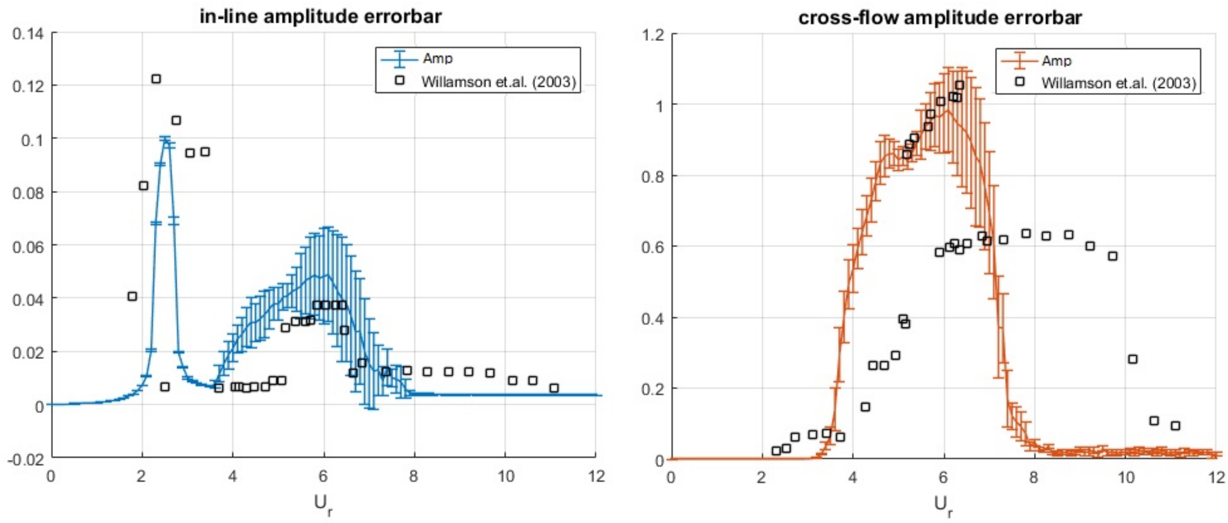

Figure 4: In-line and cross-flow prediction bars for amplitude in steady state for C_{D0} and C_{L0} input random parameters

Figure 4, shows the maximum mean amplitude in the steady-state regime is plotted as a function of the reduced velocity for in-line and cross-flow with prediction bars for amplitude in steady state for C_{D0} and C_{L0} input random parameters. It was possible to see in these graphs the impact of the variability in the parameters $\overline{C_{D0}}$ and $\overline{C_{L0}}$ mainly in the U_r in the range $[6, 8]$, for this reason we will try, from the calibration to estimate the values of these parameters taking as reference the original values due were identified as those that have the highest sensitivity in this range of reduced velocity.

2 BAYESIAN PARAMETER CALIBRATION

The Bayesian inference its a valuable tool for estimation of parametric and structural uncertainties of physical systems constrained by differential equations. Sampling techniques, such as Markov chain Monte Carlo (MCMC), have frequently employed in Bayesian inference [13, 14] [15, 16]. However, MCMC methods are, in general, computationally expensive [15, 16], because a large number of forwarding model simulations is needed to estimate the PDF and sample from it. In this sense, the use of phenomenological models of complex systems can help solve MCMC simulations for applications that would require the solution of models prohibitively large. Thus, a strategy to improve the efficiency of MCMC simulations is using substitution models, such as the one presented in this paper, which has already been developed used in a wide variety of problems. The model substitution by a surrogate one allows approximating the response of an original function, which is usually computationally expensive, by a cheaper substitute to execute. Thus, when the measurement error is ε_i we employ the statistical model,

$$\gamma_i = f(Q) + \varepsilon_i, \quad i = 1, ..., n,$$

where $f(Q)$ denotes the parameter-dependent model response and γ_i, ε_i, and Q are random variables representing measurements error, and parameters. The likelihood function $\pi(\upsilon|q)$ incorporates information provided by the samples and is the mechanism through which data informs the posterior density as detailed in [17] Hence, the Bayesian framework can be stated as follows: given measurements υ_{obs}, we find the posterior density $\pi(q|\upsilon)$. The likelihood function for this analises will be assumed with $\varepsilon_i \sim N(0, \sigma^2)$ where $\sigma^2)$ is fixed.

$$\pi(\upsilon|q) = \frac{1}{(2\pi\sigma^2)^2} e^{-SS_q}/2\sigma^2$$

where

$$SS_q = \sum_{i=1}^{n} (\varepsilon_i - f_i(q))^2$$

the sum of squares error. Through Bayesian inference techniques, given measurements v_{obs} we can find the posterior density $\pi(q/v_{obs})$, where $\pi(q/v_{obs})$ provides the complete distribution of parameters Q based on the observations v that can be estimated. For the coupling parameters C_{D0} and C_{L0} to be estimated, we generating synthetic data noise, with $\sigma = 0.05$, for the x and y response in the range of U_r emulating experimental data. Figure 5 shows the In-line and cross-flow model response with this added synthetic noise for $U_r = 6.4$ as example. Figure 6, present samples of the full probability distribution of the parameters C_{D0} and C_{L0} obtained by samplings from the estimated posterior distribution. Figure 7, shown the right the scatter plot of the parameters, displaying no correlation between them and in the left portrays the relative error over $n = 10000$ chain iterations. Figure 8 show the coupling parameter C_{D0} and C_{L0} evolution values for increasing Ur before and after calibration processes with significant variations over the original values. Applying these results to the original model it is possible to appreciate a significant improvement in the response of the amplitudes as in Figure 9 shows. The amplitudes after calibration (red line) with the original results (blue line) model and with the experimental data observing a marked improvement in the response of the phenomenological model after calibration process. Finally, the Figure 10 also confirms these results showing the effect of the frequencies, with a definite improvement concerning with the original parameters response model.

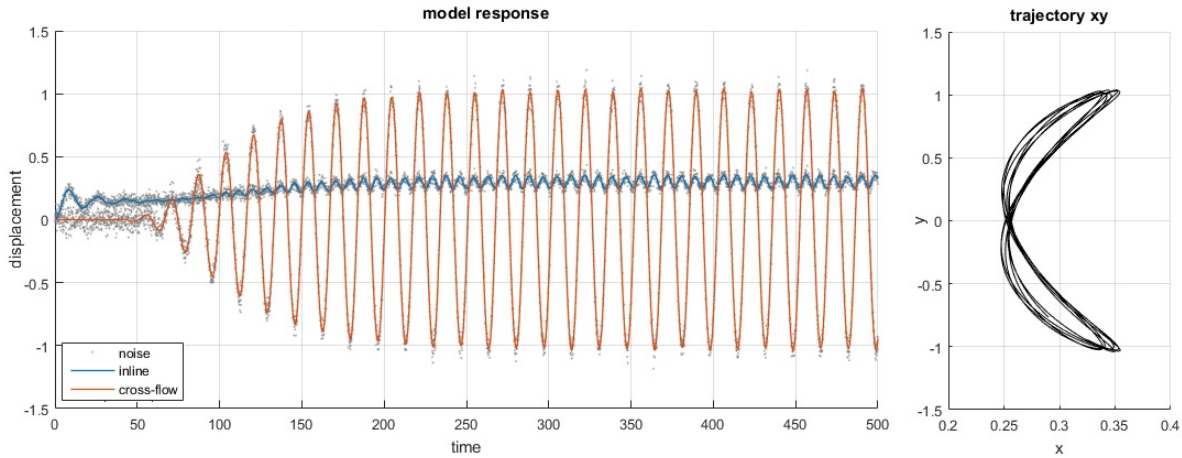

Figure 5: Left:Inline and cross-flow model response with added synthetic data noise. Right: trajectory for reduced velocity $U_r = 6.4$

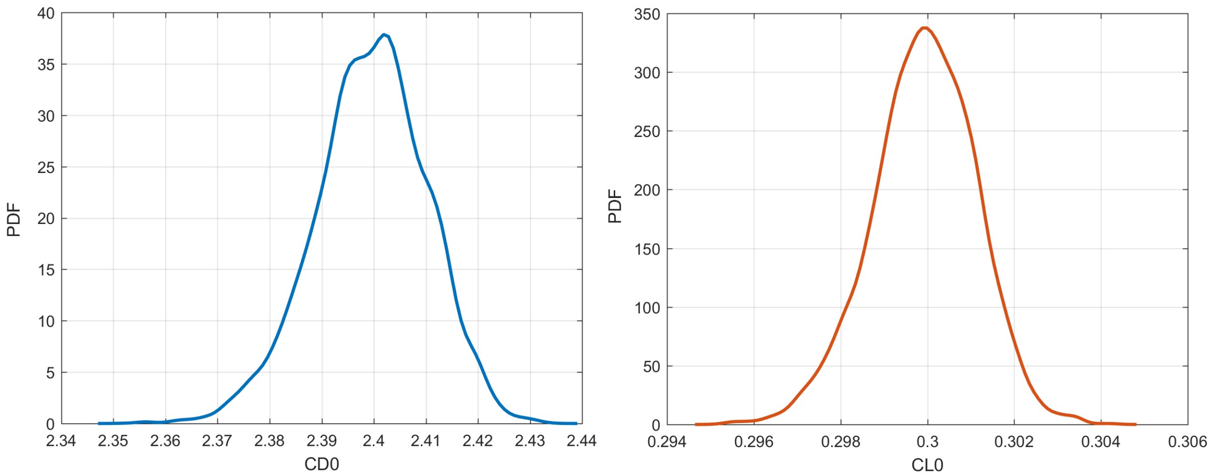

Figure 6: Estimated probability densities functions for C_{D0} (Left) and C_{L0} (Right) for $U_r = 6.4$.

3 FINAL REMARKS

This work is concerned with constructing accurate phenomenological surrogate models. For this, a phenomenological model was adopted showed to be practical to emulate the vortex induced vibration and to provide some insight into the phenomenon. Due to its simplicity, it allows you to identify important aspects of observing during experiments. However, a careful calibration required for some empirical parameters of the wake oscillator equations. Here, we first explored the capability of a forward analysis to identify highest sensitivity parameters to apply later a calibration technique to improve the response of the model allowing to deals with the vibrational response of submerged structures excited by

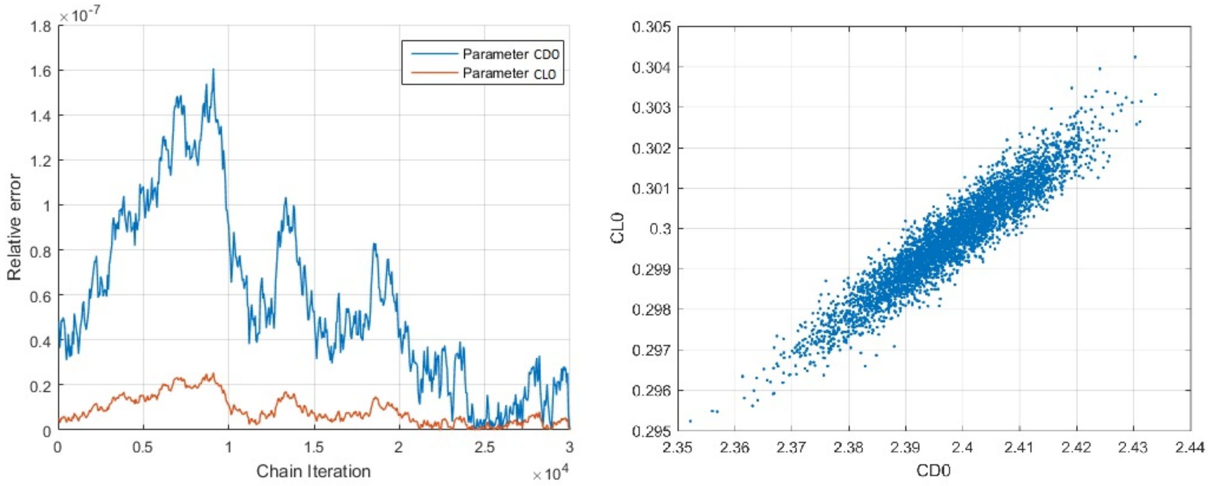

Figure 7: Left:Relative error of C_{D0} and C_{L0} coupling parameters. Right: Scatter of parameters for $U_r = 6.4$.

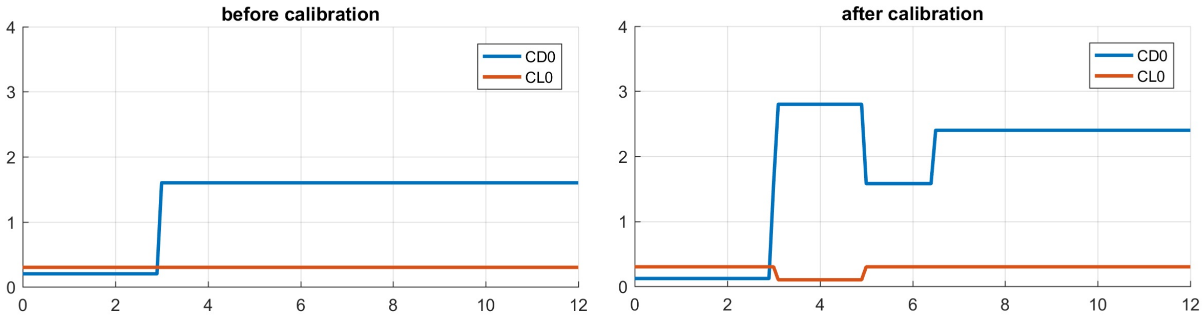

Figure 8: C_{D0} and C_{L0} functions, before calibration (Left) and after calibration (Right)

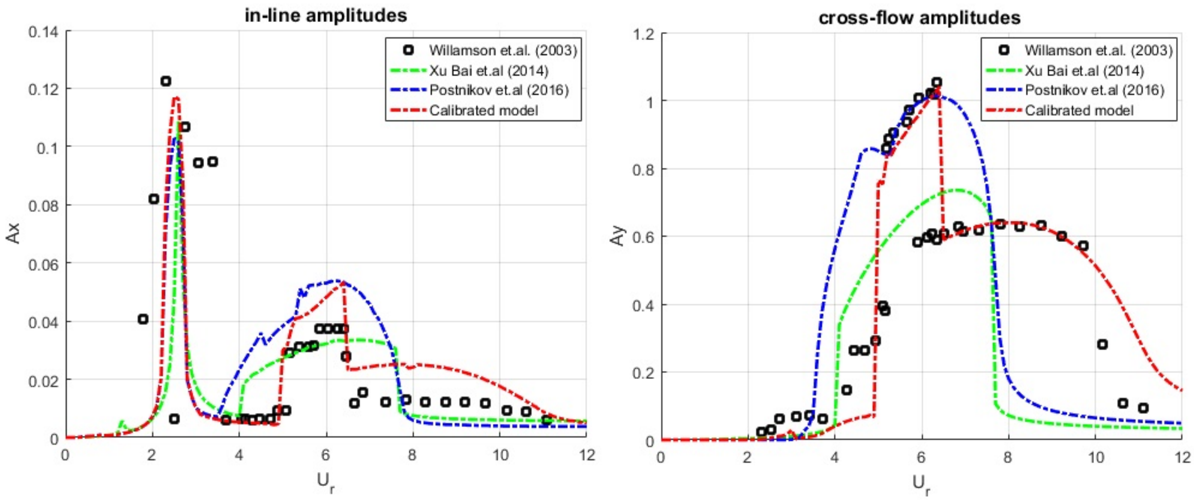

Figure 9: In-line and Cross-flow amplitudes after calibration process vs reduced velocity. Experimental from: N.Javtis C.H.K, Williamson

vortex detachments in the surrounding flow. An Adaptive Sparse Grid Collocation Method was used to estimate the statistical moments to evaluate the sensitivity of the stochastic input parameters. Two parameters, identified as the most relevant to be calibrated due to the impact on the system response were chosen to be calibrated. Therefore, we proceeded to calibrate them with a Bayesian technique using synthetic experimental data. This process performed across the range of reduced velocities, and as a result, new values for C_{DO} and C_{LO} obtained that had a positive impact on the response of the original model. The model improved over the original in the ability to predict the behavior of the reduced speed range with good agreement with the experimental data. The results obtained are preliminary and need to be analyzed in

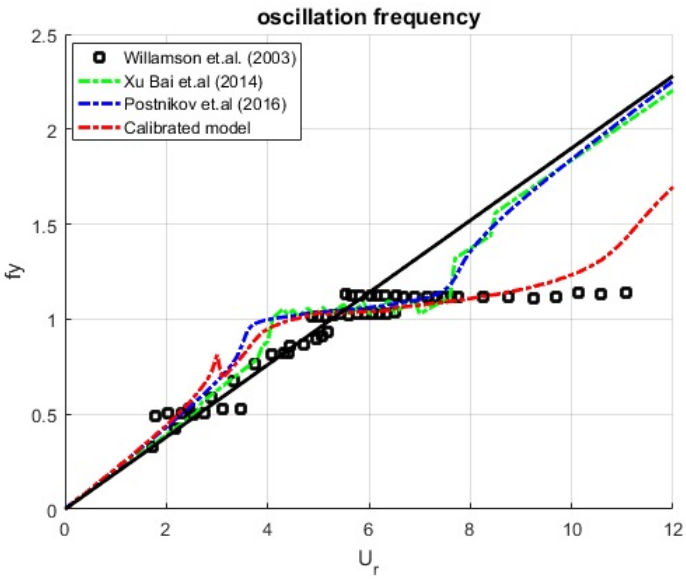

Figure 10: Cross-flow frequencies after calibration process vs reduced velocity. Experimental from: N.Javtis C.H.K, Williamson

greater depth. This numerical experiment illustrates the ability of Bayesian machinery to perform a successful calibration using synthetic data. Accordingly, experimental data can be used to obtain better information that will be important in responding to some of the issues raised successfully in calibration. Future work will include the use of this technique with actual experimental results to analyze the feasibility and performance of the proposed approach.

ACKNOWLEDGMENTS

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