DINAME 2019 - Proceedings of the XVIII International Symposium on Dynamic Problems of Mechanics,
M.A. Savi, T.G. Ritto and W.M. Bessa (Editors), ABCM, Buzios, RJ, Brazil, March 10th to 15th, 2019

Modeling and Characterization of a Flexible Rotor Supported by Active Magnetic Bearings Using Model Reduction Techniques

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Abstract: Active magnetic bearings (AMBs) have attracted the attention of researchers to find new applications and make them viable for industrial applications. Nowadays, AMBs technology has achieved a considerable level of maturity and is one of the most promising solutions for several applications involving rotating machinery. In such devices, the rotating shaft levitates suspended by magnetic forces, preventing from any wear due to mechanical contact between the bearings and the rotor journals. AMBs are lubricant free, reaching high speeds without any relevant heating. In this context, the present work is devoted to the design of a reduced model of an experimental test rig composed by a flexible shaft supported by two radial AMBs. For this purpose, the present study was divided into two parts. The first part is dedicated to the development of individual numerical models to represent the dynamic behavior of each subsystem that composes the rotor system, as based on the design characteristics of each subsystem. Thereafter, the models of the subsystem are assembled together to obtain a representative model of the entire system. Then, model reduction techniques are applied to make the system simpler and still representative. Next step is the correlation of the numerical models with the experimental results for validation purposes. Based on the data obtained, the stability analysis of the system is performed by considering existing standards for magnetic bearings. The obtained results demonstrate the representativeness of the reduced model.

Keywords: Active Magnetic Bearing, Reduced Model, Model validation, Control

INTRODUCTION

The development of AMBs along the years allowed their practical application in the industry (Schweitzer et al., 2009). AMB technology has become a promising solution for applications in rotating machines. In this case, electromagnetic forces suspend the rotating shaft, which prevents it from wear due to mechanical contact between the windings and the rotor retainers. This scheme permits the shaft to work at high speeds without any relevant heating, thus making lubrication unnecessary. As an electromechanical system is formed, the AMB can be classified as a mechatronic component (Schweitzer et al., 2009). In this way, through dedicated software, it is possible to actuate on this system composed of mechanical and electronic parts. In addition, inherent capabilities associated with smart materials ensure them a leading position for the development of innovative design of smart rotating machines, opening possibilities for new technology of both active vibration control (AVC) and structural health monitoring (SHM). AVC is considered as the most sophisticated vibration attenuation strategy nowadays. This technique is based on the application of dynamic forces to control undesired vibration amplitudes. In a simplified way, an AVC system is composed of sensors, actuators, and a control unit. The sensors are responsible for providing information about the controlled variables. The control unit possesses the function of processing the information from the sensors and determining the command signals, as calculated by using control algorithms. Finally, the actuators convert the command signals, provided by the control unit, into effective actions upon the system. In (Koroishi, 2009), for instance, a hybrid bearing based on electromagnetic actuators was employed to attenuate the vibration amplitudes of a rotating system by using different control strategies. Numerical simulations are widely used to simulate the dynamic behavior of complex physical phenomena, leading quite often to heavy computational effort, which, in many cases, justifies the formulation of reduced models, i.e., lower order models, which are efficient and accurate enough, while requiring affordable computation time to integrate the equations that represent the dynamic behavior of the system (Benner et al., 2015). Besides, lower order models are preferred in the present context since they allow for simple and efficient controller design. Nonetheless, there are numerical conditions that limit the usefulness of model reduction techniques. This often occurs when some vibration modes are nearly uncontrollable or unobservable (Safonov et al., 1989) or are close to the right half-plane of the s plane. In these cases, reduction techniques associated with pole-zero simplification, as proposed in the present work, may be used as an alternative to minimize numerical problems. In this context, the present work describes the first part of a research project devoted to the investigation of a reduced model applied to a rotating machine supported by AMBs. The first problem that should be addressed is the development of a representative computational model, as based on the manufacturer specifications. Thus, various control strategies can be applied to the system. Therefore, the process of modeling the system components is presented next, as well as the assembling of the entire model. Finally, the updating of both numerical transfer functions and stiffness parameters as based on experimental results is performed. Finally, the reduced model techniques are applied to the full system.

MODELING AND CHARACTERIZATION

The first step was dedicated to developing individual computational models for each main component of the system (shown in Fig. 1), which was performed based on the design specification provided by the manufacturer. The following step is associated with the assembling of these component models into a global one.

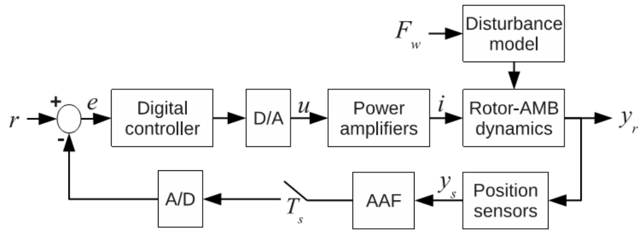

Figure 1: Main components of an AMB system.

This way, the paper will be divided into five sections presenting the sub-systems that will compose the global model. The sub systems will be organized as follows: Active Magnetic Bearings (AMBs), Rotor Model, Amplifier, Controller and Reduction Model. In the last topic, the technique of model reduction is applied to the global system developed previously as well as the obtained results are presented.

REDUCTION MODEL TECHNIQUES

Reduction model techniques are important in several applications involving finite-dimensional systems whose models are obtained through experimental procedures or by using numerical methods (Ohta et al., 1999). A lower-order model is preferred since the system analysis and control can be more easily implemented. Additionally, lower degree controllers are usually preferred due to hardware limitations (Anderson, 1989). Balanced truncation model reduction techniques have been developed and proved to be useful in many cases. They are based on the calculation of the Hankel singular values of a linear system (Glover, 1984). Taking into account the standard linear time-invariant dynamic system:

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{1}$$

$$y(t) = Cx(t) + Du(t) \tag{2}$$

the transfer function of the system is:

$$G(s) = D + C(sI - A)^{-1}B \tag{3}$$

Given that the eigenvalues of A, $\lambda_i(A)$ $\forall i$ are assumed to be strictly localized in the left half-plane, then the controllability gramian and the observability gramian can be defined, respectively, as:

$$P \triangleq \int_0^\infty e^{At} BB^* e^{A^*t} dt \tag{4}$$

$$Q \triangleq \int_0^\infty e^{A^*t} C^* C e^{At} dt \tag{5}$$

being $[^*]$ the representation of the complex conjugate transpose of a matrix. Taking into account that $\Re(\lambda_i(A)) < 0$ $\forall i$, then the Hankel singular values are defined by Eq. (6):

$$\sigma_i(G(s)) \triangleq [\lambda_i(PQ)]^{\frac{1}{2}}, \sigma_i(G(s) \geq \sigma_{i+1}(G(s)) \quad \forall i \tag{6}$$

The balanced truncation model reduction proposed by (Pernebo et al., 1989), considered as being the most used model reduction technique, is based on the fact that there exists an invertible state-space transformation T_{BAL} ε $\Re^{mxm}$ such that the transformed and reduced system given by (Safonov et al., 1989):

$$\left(\begin{array}{c|c} A_{BAL} & B_{BAL} \\ C_{BAL} & D_{BAL} \end{array}\right) = \left(\begin{array}{c|c} T_{BAL}^{-1} A T_{BAL} & T_{BAL}^{-1} B \\ C T_{BAL} & D \end{array}\right) \tag{7}$$

has controllability and observability grammians, respectively, in the form:

$$P_{BAL} = T_{BAL}^{-1} P (T_{BAL}^{-1}) = diag(\Sigma_1, \Sigma_2, 0, 0) \quad \varepsilon \quad \Re^{mxm} \tag{8}$$

$$Q_{BAL} = T_{BAL}^{T} Q (T_{BAL}) = diag(\Sigma_1, 0, \Sigma_3, 0) \quad \varepsilon \quad \Re^{mxm} \tag{9}$$

where Σ_1, Σ_2, and Σ_3 are positive definite diagonal matrices and $\Sigma_1 = diag(\sigma_1, \ldots, \sigma_m)$. In the reduction process used to obtain a k^{th} order model $\hat{G}(s)$ from a m^{th} order $G(s)$, $k \leq m$, (Glover, 1984) proved that the reduced model guarantees an error bound according to Eq. (10):

$$\bar{\sigma}(G(jw) - \hat{G}(jw)) \leq 2 \sum_{i=k+i}^{m} \sigma_i, \quad \forall w \tag{10}$$

where $\bar{\sigma}$ indicates the largest singular value of A. Numerical difficulties, however, limits the usefulness of model reduction techniques. This occurs because the calculations are complicated and sensitive to numerical errors. The solution may be badly conditioned when the matrix PQ has a high condition number, i.e. when some modes of the system are nearly uncontrollable or unobservable (Safonov, 1989) or are localized close to the right half-plane of the s plane. In order to reduce the bad numerical conditioning of the process, this work proposes the use of pole-zero simplification procedure either before or after the use of a model reduction method. In high-order systems with the presence of poles and zeros near or located in the right half-plane, this methodology guarantees the efficiency of the techniques based on the calculation of Hankel singular values. A model of lower order is generated, which is mathematically compatible with the original one. The pole-zero simplification is based on the cancelation of poles and zeros pairs that are located in the s plane inside a predetermined tolerance region tol. Thus, given the transfer function, Eq. (11):

$$G(s) = \frac{(s-z_1)(s-z_2)\ldots(s-z_j)\ldots(s-z_{nz})}{(s-p_1)(s-p_2)\ldots(s-p_j)\ldots(s-p_{nz})} \tag{11}$$

each pole p_l is compared with all zeros, and if the condition below is satisfied:

$$|p_i - z_j| \leq tol \tag{12}$$

then, the pole and zero being analyzed are excluded from the transfer function. Figure 2 shows the poles and zeros of the sub-systems in an open-loop model composed by the rotor, shaft reference, disturbances, sensor noise, and a control command. Figures 3 compares the full and reduced models.

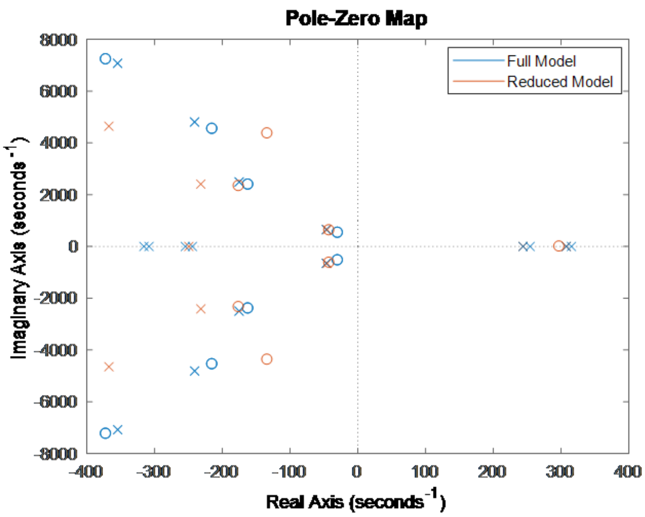

Figure 2: Pole and Zero Map comparing the models for the actuator V_{13} in open loop.

In this case, the rotor plant was modeled as a *SISO* (Single Input, Single Output) 58^{th} order system, in which rigid body modes are responsible for the instability of the transfer function. As a final step, an association between the balanced truncation model reduction proposed by Moore and the pole-zero simplification scheme was performed. The focus was devoted to the application sequence of the techniques as a function of the complexity and stability of the original transfer function. Clearly, the quality and the order of the resulting reduced models were also verified.

Figure 9 shows the comparison between the full and reduced models (proposed model and MATLAB® Reduce toolbox) for the open-loop configuration. In the reduction criterion, a tolerance of 10^{-5} and a frequency band between 0 and 1200 $[Hz]$ was used. Through the reduction, the actuators V_{13} and W_{13}, of the coupled bearing, presented 6^{th} order transfer functions. The free bearings, given by V_{24} and W_{24}, obtained 6^{th} order transfer functions.

In the context of the comparison performed, it is possible to observe that the proposed model shows a more faithful behavior regarding the complete AMB model as compared with the MATLAB® reduction toolbox for the same order.

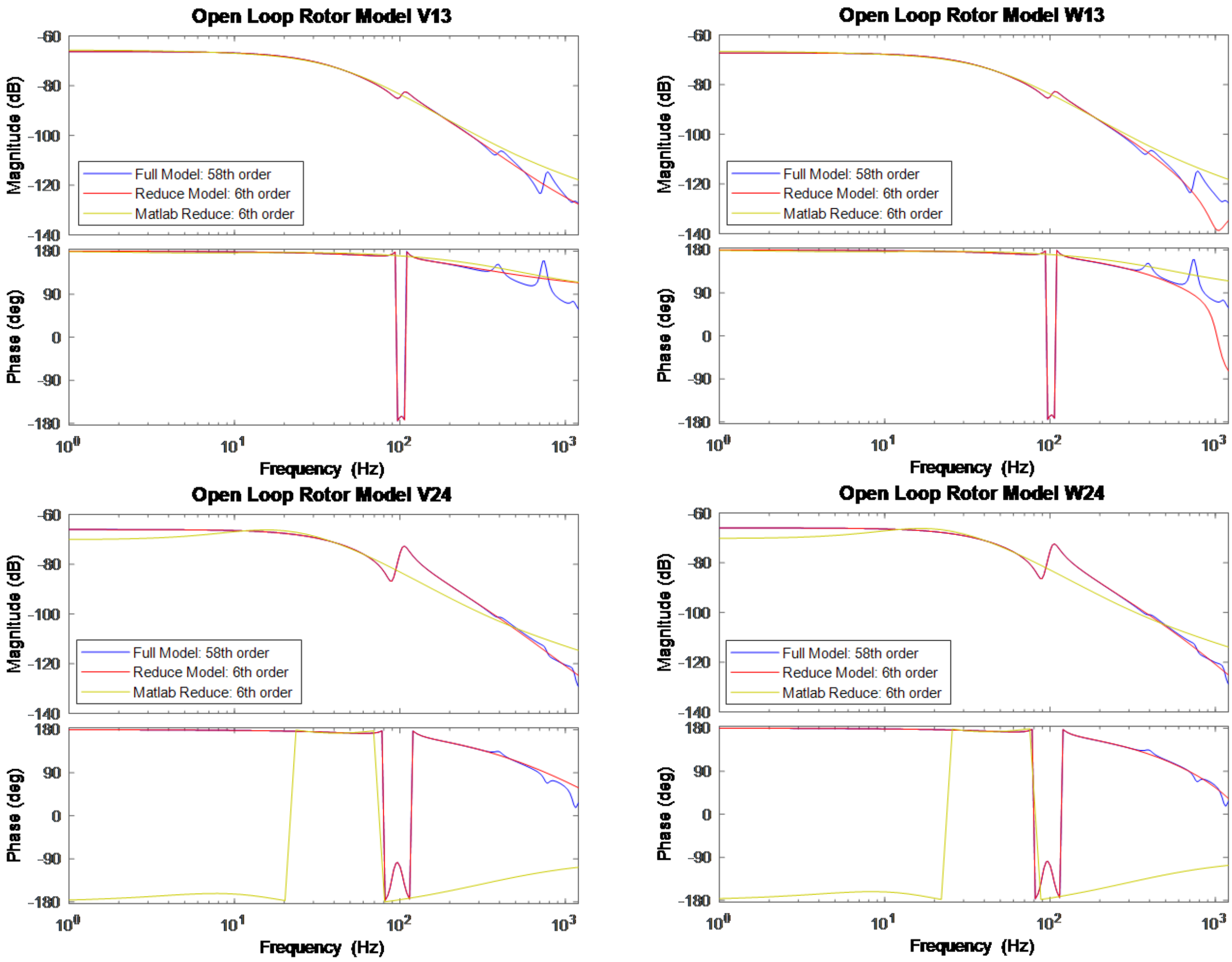

Figure 3: Comparison between the full and reduced models for Coupled and Free bearing transfer functions.

ACKNOWLEDGMENTS

The authors are thankful for the financial support provided to the present research effort by CNPq (574001/2008-5 and 304546/2018-8), FAPEMIG (TEC-APQ-3076-09, TEC-APQ-02284-15, and TEC-APQ-00464-16), and CAPES.

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