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Investigation of a Finite Element Rotor Model with Angular Contact Ball Bearings under EHD Lubrication.

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Abstract: As the need for reducing costs and deadlines of machines project increases, more accurate models are necessary to better predict the behavior of mechanical systems. In this context, the present work aims to model a rotor supported in ball bearings with angular contact under Elastohydrodynamic (EHD) lubrication, obtaining stiffness and damping equivalent parameters for each bearing, in axial and radial directions. The dynamic equilibrium of the bearings 'spheres is carried out and used as entry value for the transient multi-level algorithm solution of EHD equations. Moreover, an optimization provides the equivalent coefficients for stiffness and dumping bearing forces that are introduced in the rotor as reaction forces. The complete rotor systems is developed with a finite element method, using Timoshenko bean theory, and solved in time domain, allowing the rotor and bearing responses analysis.

Keywords: Rotor, Angular Contact, Ball Bearing, EHD Lubrication.

INTRODUCTION

The present work purposes a finite element rotor model supported in ball bearings with angular contact and EHD lubrication regime. It intends to investigate the influence and the application of Nonato and Cavalca (2010) EHD contact behavior of a ball bearing in this rotor, comparing the results with Carrer, Bizarre and Cavalca (2018) lumped parameters model of a rotor, with a decentered disc with unbalanced radial mass and gyroscopic effect. To explore the angled contact of these bearings, external radial and thrust loads are applied to the disc. The insertion of the bearings in the system is done by applying its reaction forces with nonlinear parameters. These parameters are obtained by the optimization proposed by Bizarre (2018), integrating the multilevel solution of the Elastohydrodynamic lubrication model (EHD) presented by Nonato (2014), using as a first approach the dynamic equilibrium of the spheres (Harris, 1991) described in Radaelli (2013).

The system solution is completed in time domain due to the presence of non-linearities. The orbits and frequency responses for points of interest are shown and a Campbell diagram with rigid bearing is used to support the understanding of bearings' model effect in the whole system response.

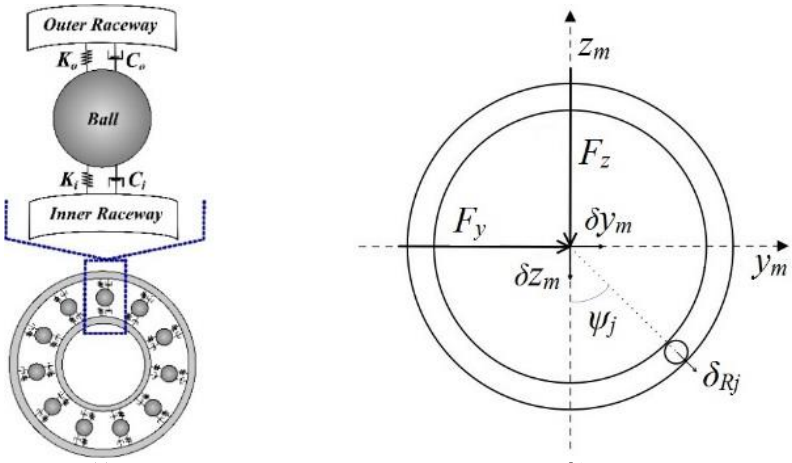
METHODOLOGY

Contact Reduced Model and Bearing Equilibrium

Using the Newton-Raphson iterative method, Radaelli (2013) presents a solution of the dynamic equilibrium of the spheres of a rolling bearing with angular contact. Hertz contact is the model used, in which, for each sphere, the forces equilibrium at high speed is made.

Using loading and velocity data of each sphere, Moes dimensionless parameters are introduced in the EHD lubrication model by Nonato (2010), based on Verner (2000), characterizing each ball/raceway contact, applying the Reynolds equation, oil film thickness, viscosity-pressure and density-pressure relations and forces equilibrium. The solution is achieved by a multilevel finite difference method, resulting in the oil film pressure and thickness distribution, reaction forces, displacements, and velocities at the internal and external contact of each sphere.

The oil film is represented as a spring-damper system, as schematized in Fig. 1. With the static force variating of the EHD simulation results the restorative force is encountered and with the transient solutions is the resultant dissipative force is calculated.



(a) (b)

Figure 1. (a) Equivalent model of the contact forces between inner and outer raceway (Bizarre, 2014). (b) Representation of the radial displacement of the j-th sphere of a ball bearing, Carrer, Bizarre and Cavalca (2018).

The static balance in the bearing is given by the sum of the reaction forces of the balls in x, y and z directions. Since Z is the total number of balls of each bearing and ψ_j is the Azimuth angle of each ball element, Eq. (1), (2) and (3) are obtained, considering the bearings degrees of freedom δy_m and δz_m, in Fig. 1.(b), as y_m and z_m, respectively, resulting in the restorative forces shown in the Eqs. (1), (2) and (3).

$$F_y = \sum_{j=1}^{Z} \left[\left(K_R \left(z_m \cdot cos\left(\psi_j\right) + y_m \cdot sin\left(\psi_j\right)\right)^{d_R} + \Delta F_R \right) \cdot cos\left(\psi_j\right) \right] \quad (1)$$

$$F_z = \sum_{j=1}^{Z} \left[\left(K_R \left(z_m \cdot cos\left(\psi_j\right) + y_m \cdot sin\left(\psi_j\right)\right)^{d_R} + \Delta F_R \right) \cdot sin\left(\psi_j\right) \right] \quad (2)$$

$$F_x = \left(K_A \cdot x_m^{d_A} + \Delta F_A \right) \cdot Z \quad (3)$$

The dissipative bearing force is described by Eq. (4), (5) and (6), multiplying the sphere contact equivalent damping, C, by the position time derivative, where ω_c stands for bearing cage velocity.

$$F_{\dot{y}} = \sum_{j=1}^{Z} \left[\left(C\left(\dot{z}_m \cdot cos\left(\psi_j\right) + \dot{y}_m \cdot sin\left(\psi_j\right)\right) + \omega_c \left(z_m \cdot sin\left(\psi_j\right) + y_m \cdot cos\left(\psi_j\right)\right)\right) \cdot cos\left(\psi_j\right) \right] \quad (4)$$

$$F_{\dot{z}} = \sum_{j=1}^{Z} \left[\left(C\left(\dot{z}_m \cdot cos\left(\psi_j\right) + \dot{y}_m \cdot sin\left(\psi_j\right)\right) + \omega_c \left(z_m \cdot sin\left(\psi_j\right) + y_m \cdot cos\left(\psi_j\right)\right)\right) \cdot sin\left(\psi_j\right) \right] \quad (5)$$

$$F_{\dot{x}} = (C \cdot \dot{x}_m) \cdot Z \quad (6)$$

Rotor-Bearing System Modeling

The rotor-bearing assembly was discretized in 15 shaft's elements, resulting in 16 knots, as it can be seen in the Fig. 2, were the disc has its inertia and applicable external forces applied in the point 7 and the bearings reaction forces are applied in the knots 2, bearing 1, and 15, bearing 2. Each knot has five degrees of freedom, x, y, z, θ_y e θ_z, and Ω is the shaft rotational speed.

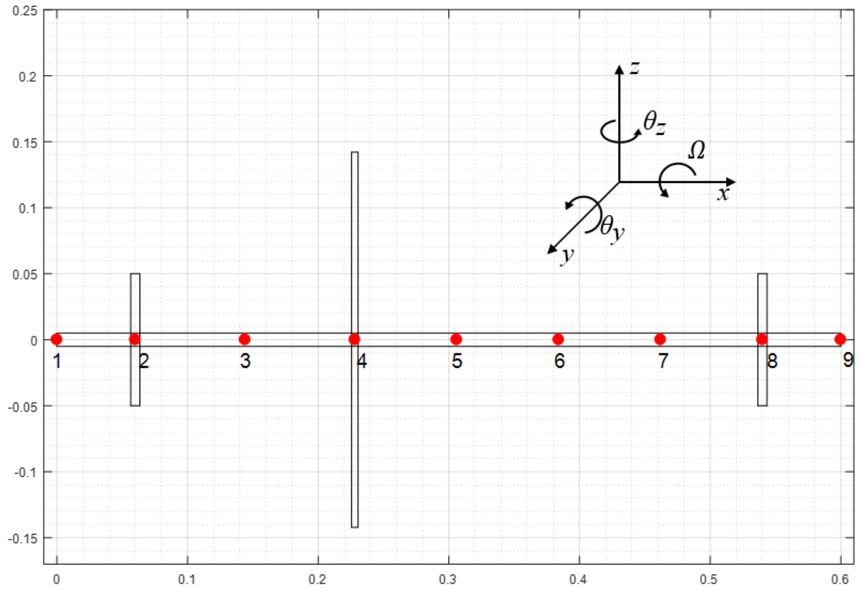


Figure 2. Finite element representation of the rotor-bearings system model in meters scale.

The complete system equation of motion is described in Eq. (7). M, G e K are, respectively, the mass, gyroscopic and stiffness matrixes. They are assembled using the element matrixes validated in Tuckmantel (2010), using Timoshenko bean theory:

$$M \cdot \ddot{u}(t) \; + \; (D \; + \; G) \cdot \dot{u}(t) \; + \; K \cdot u\,(t) \; = \; f(u, \dot{u}, t) \tag{7}$$

The D matrix represents the shaft dissipative forces and is written as proportional to the stiffness matrix by multiplying it by a damping coefficient, β. The f vector contains all external forces and bearings reaction forces in its respective knot degree of freedom.

RESULTS AND DISCUSSION

Geometrical and operational information about the simulated rotor model is shown in Table 1 and the bearings geometric characteristics are presented in Table 2.

Table 1.Rotor geometrical and operational characteristics.

	Description	Value
l	Distance between bearing (m)	0.6
m	Disc mass (kg)	1.7478
ρ	Shafts density (kg/m³)	7875
E	Shaft Modulus of elasticity (Pa)	$210{\cdot}10^{9}$
I	Shaft area moment of inertia (m^4)	$2.3235{\cdot}10^{-9}$
I_d	Disc diametral moment of inertia (m^4)	$6.2{\cdot}10^{-3}$
I_p	Disc polar moment of inertia (m^4)	$1.23{\cdot}10^{-2}$
A	Shaft cross-sectional area (m^2)	$6.8349{\cdot}10^{-4}$
β	Proportional Damping Coefficient	$2.0{\cdot}10^{-4}$
F_a	Applied Load (N), x-direction	500
F_{est}	Applied Load (N), z-direction	500
ε	Mass eccentricity (m)	$1.0{\cdot}10^{-5}$

Table 2. Bearings geometrical characteristics

	Description	Value
Z	Number of spheres in the bearing	11
D	Sphere diameter (m)	$7.0 \cdot 10^{-3}$
d_m	Bearing pitch diameter (m)	$42.5 \cdot 10^{-3}$
α	Nominal angular contact (°)	15

Considering a double pinned supported rotor (rigid bearings), the Campbell Diagram is on shown in Fig. 3 to facilitate the system critical speeds identification. It is possible to point out three ones: 27 Hz and 56 Hz (backward) and 29 Hz (forward).

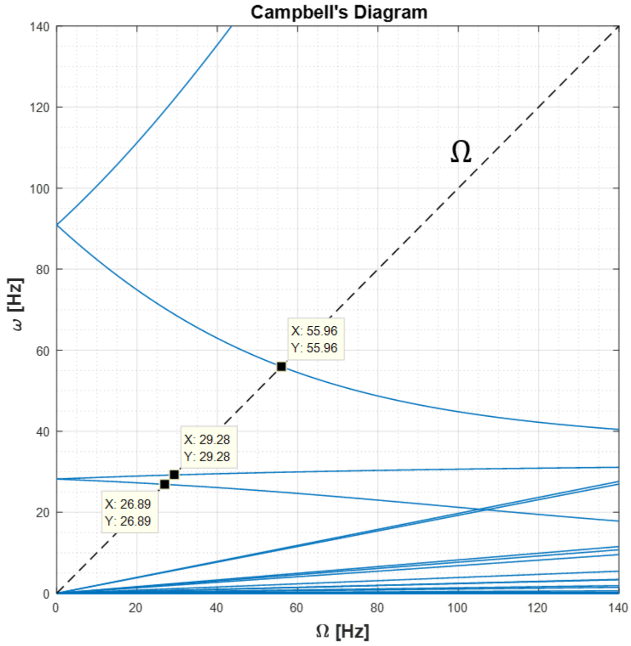


Figure 3. Campbell's Diagram with rigid bearings.

In Fig. 4, the shaft knots orbits are shown and the points of external forces application, the two bearing and the disc, are highlighted. The two bearings orbits are presented independently. Even though both orbits have similar formats, it is noticeable that the bearing 1 has a larger amplitude due to its disc proximity and lower bias, since it is under a greater vertical load.

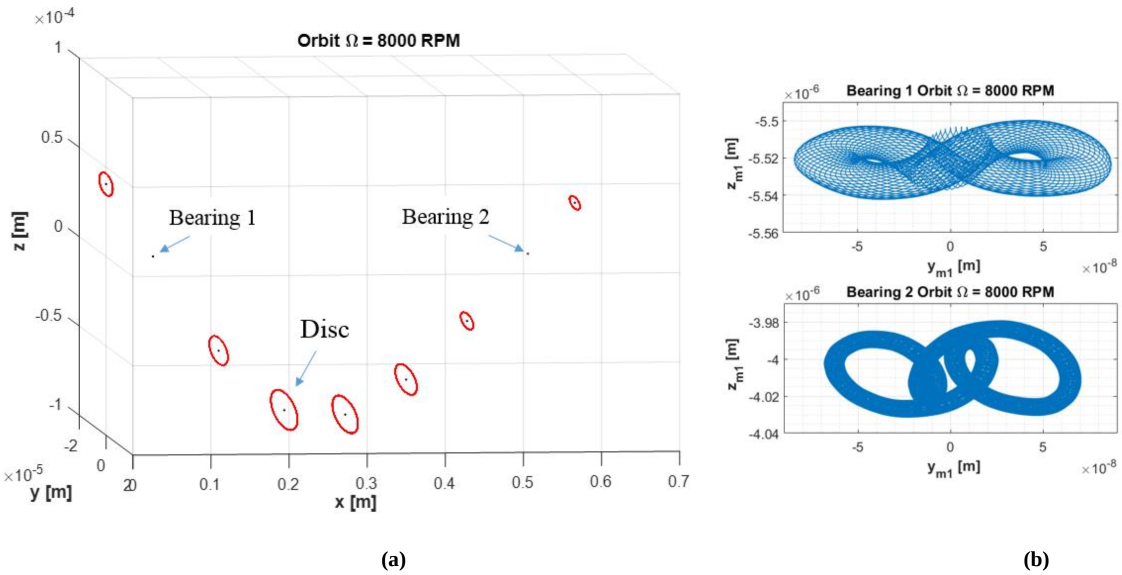


Figure 4. (a) Shaft points orbit. (b) Bearings 1 and 2 orbits.

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