Aeroelastic stability analysis of a rectangular wing with PID control

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Abstract: Research in aeroelasticity composes a significant part on aircraft design. Aiming both safety and efficiency, vibration control mechanisms should be implemented for achieving better results. In this context, this work deals with active aeroelastic control using PID control theory. A tridimensional model of a uniform and rectangular wing is assumed to provide computational results. Strip Theory is used to model the aerodynamical behaviour, and the pk frequency match method provides the frequency and damping data in each flight condition. From the results, it is possible to observe that the gain of the controllers increases the flight envelope, and the increase of derivative gain has a higher influence when compared to proportional gain.

Keywords: Aeroelastic Stability, PID control, flutter

INTRODUCTION

Aeroelasticity can be defined as field of knowledge that deals with the study of the interaction between inertial, elastics and aerodynamics forces (Bisplinghoff et al., 1996). This interaction may result in problems like flutter, divergence, reversal control, buffeting and others that could culminate in catastrophic disasters (Dowell et al., 2005). The flutter phenomena must be highlighted, which is an important dynamical instability present since the beginning of aviation (Hodges and Pierce, 2011) that results in self-oscillations and eventual failure. In general terms, the flutter speed depends on properties of both the fluid and the structure, which together form a coupled aeroelastic system (McEver et al., 2007).

Thereby, aeroelastic control mechanisms could be implemented. An approach that has shown reasonable results is the inclusion of vibration control mechanisms, which can be passive or active. The first one makes use of dissipative materials insertions to achieve aeroelastic control. The passive approach keeps the stability of the system, the consequent reduction of failure and low energy is required (Leo, 2007), but passive modification may have weight penalty and cost constraints (Singh, 2015).

The active aeroelastic control case is based on the use of sensors and actuators that can be triggered according with the structural behaviour. However, strategies including active control, in most of cases, commits the requirements of structural flexibility. Also, there are the desired closed loop behaviour, where the modification of the system matrices complicates the real time implementation during an entire flight operation in several flight conditions (Mukhopadhyay, 2003). Moreover, active aeroelastic control gives rise to time delay which can deteriorates the control performance and lead to instability condition (Pendleton et al., 2000).

In this work, an active control is addressed, involving a PID controller, aiming to suppress the instability of a rectangular wing. Several controller gains values are assumed to establish a relationship between these parameters and the increase of the flight envelope.

MATHEMATICAL MODELING

The baseline model is equivalent to a tridimensional, uniform rectangular wing, with chord c, spanwise s, and distributed mass m. Two degrees of freedom are considered on this proposal, which represent the bending and torsion movements, respectively represented by the variables κ and θ, as illustrated on Fig. 1.

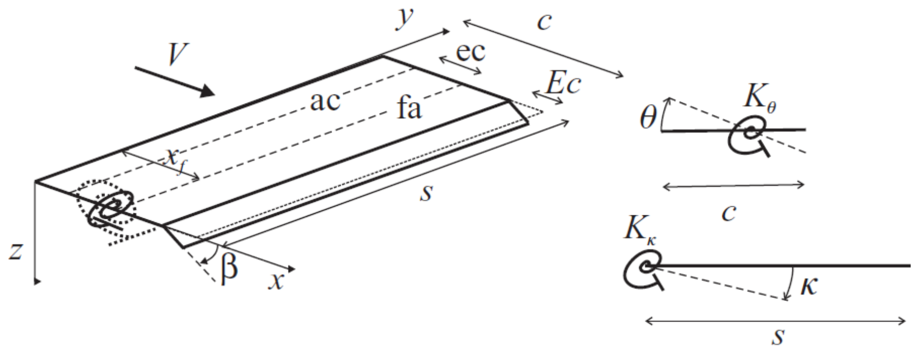

Figure 1 – Physical Model for a tridimensional rectangular wing. (Wright and Cooper, 2014)

It's possible to see on Fig. 1 a control surface, which is distributed along all spanwise. This structure has an infinite stiffness attachment to the wing but can be moved to any angle β that is demanded. Inertia effects of the control surface can be ignored without significant differences in the results. Thus, the control surface is not involved in the basic dynamics of the wing, but simply acts as an excitation device (Cooper, 2015). The position of the elastic axis is identified by x_f .The rotational and translational rigidities of the elastic suspension are considered on the variables K_θ e K_κ .

Strip Theory is used to model the aerodynamic forces distributed along the wing which is subjected to an airflow with velocity V and density ρ , as Eq. To consider the effects of a control surface, the lift and pitch moment are modified including $M_{\dot\theta}$ (to unsteady aerodynamic effects) and terms proportional to the displacement and velocity of control surface (Fung, 1959). This assumption is the basis of PID control theory and these terms are called feedback gains, represented by K_v and K_d .

$$\begin{aligned} dL &= \frac{1}{2}\rho V^2 c\left[a_w\left(\frac{y}{s}\theta + \frac{y^2}{s^2 V}\dot\gamma\right) + a_c\beta\right]dy \\ dM &= \frac{1}{2}\rho V^2 c^2\left[a_w e\left(\frac{y}{s}\theta + \frac{y^2}{s^2 V}\dot\gamma\right) + M_{\dot\theta}\frac{cy}{4sV}\dot\theta + b_c\beta\right]dy \end{aligned} \tag{1}$$

Using Lagrange formulation, assuming the prescript movement as Eq.(2), it is possible to obtain the structural and aerodynamic matrices for the rectangular wing, as showed on the Eq. (4).

$$z(x,y) \approx \left(\frac{y}{s}\right)^2 \kappa + \left(\frac{y}{s}\right)\left(x - x_f\right)\theta \tag{2}$$

$$\begin{gathered} \begin{bmatrix} \dfrac{mcs}{5} & \dfrac{ms}{4}\left(\dfrac{c^2}{2} - cx_f\right) \\ \dfrac{ms}{4}\left(\dfrac{c^2}{2} - cx_f\right) & \dfrac{ms}{3}\left(\dfrac{c^3}{3} - c^2 x_f + x_f^2 c\right) \end{bmatrix} \begin{Bmatrix} \ddot\kappa \\ \ddot\theta \end{Bmatrix} + \rho V \begin{bmatrix} \dfrac{c a_w s}{10} & 0 \\ -\dfrac{c^2 e a_w s}{8} & \dfrac{-c^3 M_{\dot\theta} s}{24} \end{bmatrix} \begin{Bmatrix} \dot\kappa \\ \dot\theta \end{Bmatrix} + \\ \left(\rho V^2 \begin{bmatrix} 0 & \dfrac{c a_w s}{8} \\ 0 & \dfrac{-c^2 e a_w s}{6} \end{bmatrix} + \begin{bmatrix} k_\gamma & 0 \\ 0 & k_\gamma \end{bmatrix}\right) \begin{Bmatrix} \kappa \\ \theta \end{Bmatrix} = \rho V^2 \begin{Bmatrix} \dfrac{-c a_c s}{6} \\ \dfrac{c^2 b_c s}{4} \end{Bmatrix} \beta \end{gathered} \tag{3}$$

Assuming the representation of a vector g as a combination of proportional and derivative matrices (PID theory) , as Eq.(4),

$$\{g\} = \rho V^2 \begin{Bmatrix} \dfrac{-ca_c s}{6} \\ \dfrac{c^2 b_c s}{4} \end{Bmatrix} = K_v \begin{bmatrix} g_1 & -g_1 x_f \\ g_2 & -g_2 x_f \end{bmatrix} \begin{Bmatrix} \dot{\kappa} \\ \dot{\theta} \end{Bmatrix} + K_d \begin{bmatrix} g_1 & -g_1 x_f \\ g_2 & -g_2 x_f \end{bmatrix} \begin{Bmatrix} \kappa \\ \theta \end{Bmatrix} = [F] \begin{Bmatrix} \dot{\kappa} \\ \dot{\theta} \end{Bmatrix} + [G] \begin{Bmatrix} \kappa \\ \theta \end{Bmatrix} \quad (4)$$

and including the Eq.(4) in Eq.(3) and rearranging the matrixes, the aeroservoelastic problem is mathematically defined as in Eq.(5).

$$[A]\{\ddot{q}\} + \left(\rho V[B] + [D] - [F]\right)\{\dot{q}\} + \left(\rho V^2 [C] + [E] - [G]\right)\{q\} = 0 \quad (5)$$

NUMERICAL SIMULATIONS

Based on the formulation presented in the previous sections, numerical simulations have been performed aiming at evaluating the influence of the PID control gains on the flutter speeds. For this purpose, the values of the parameters provided in Tab. 1 are used in a MATLAB® routine.

Table 1 – Baseline parameters for computational simulations.

Properties	Value
Chord (c)	2m
Semispam (s)	7.5m
Elastic Axis position (x_f)	0,48c
Mass Axis position (x_m)	0,5c
Mass per unit area (m)	200kg/m²
Flapping frequency (f_κ)	5Hz
Pitch Frequency (f_θ)	10Hz
Air density (ρ)	1,225kg/m³
Lift Curve Slope (a_w)	2π
Nondimensional pitch damping $(M_{\dot\theta})$	-1,2

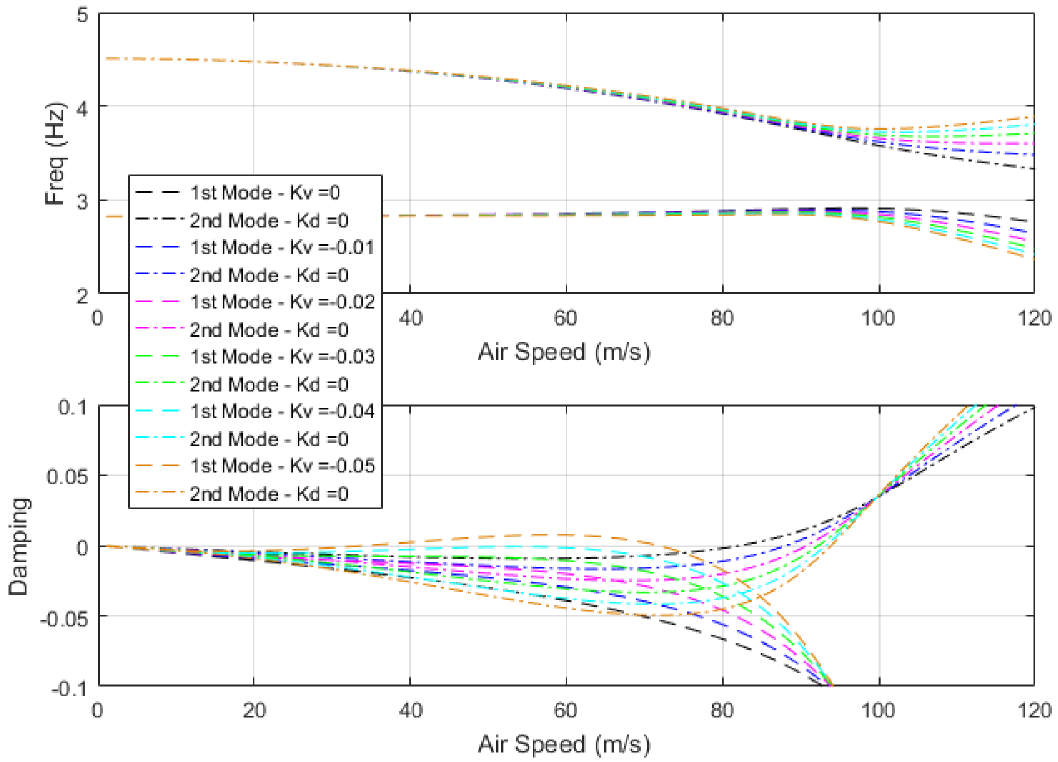

Figure 2 – Vgf diagrams for flutter analysis with Derivative feedback Gain Variation.

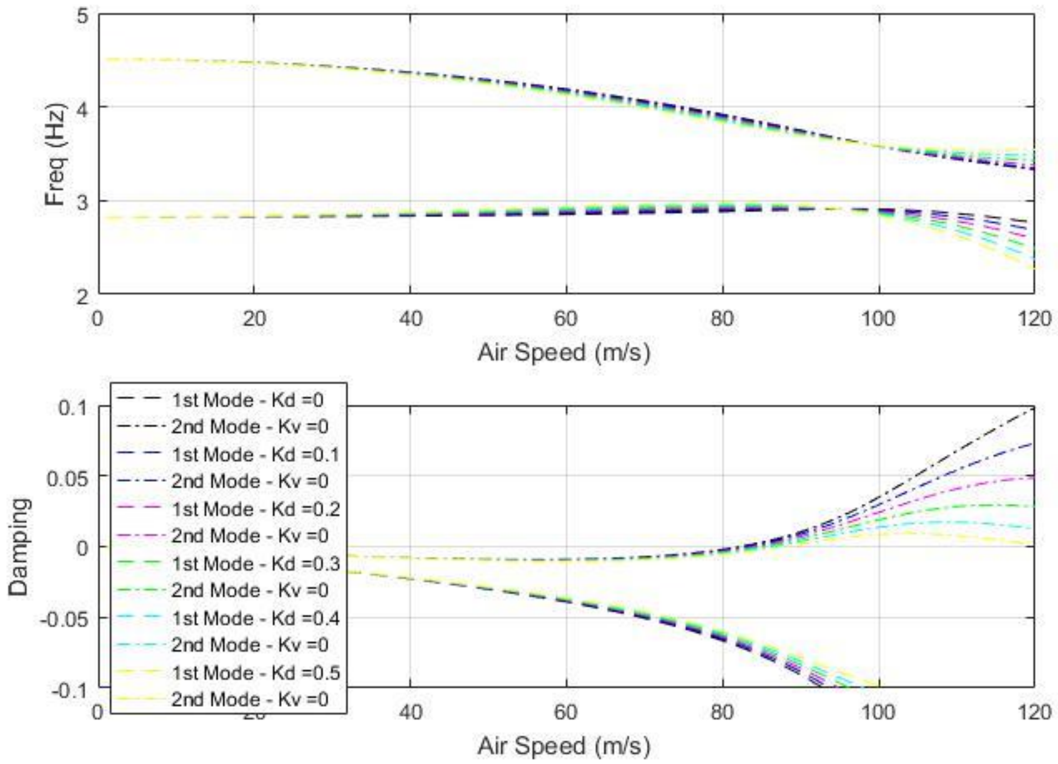

Figure 3 – Vgf diagrams for flutter analysis with Proportional feedback Gain Variation.

The stability analysis can be performed based on the examination of the complex eigenvalues associated with the equations of motion presented on the section above. It is also possible to build a Vgf (Velocity, Damping and Frequency) diagram and observe the flutter velocity, which occurs when the damping term becomes unstable (positive). Frequency Matching '*pk*' Method is used in each flight condition of interest.

Table 2 – Instability condition associated with feedback gains.

Feedback Gain (Kv)	Critical Velocity(m/s)	Feedback Gain (Kd)	Critical Velocity(m/s)
0,0	82,3	0,0	82,3
-0,01	88,2	0,1	83,1
-0,02	90,8	0,2	84,0
-0,03	92,3	0,3	85,0
-0,04	93,2	0,4	86,2
-0,05	93,9	0,5	87,7

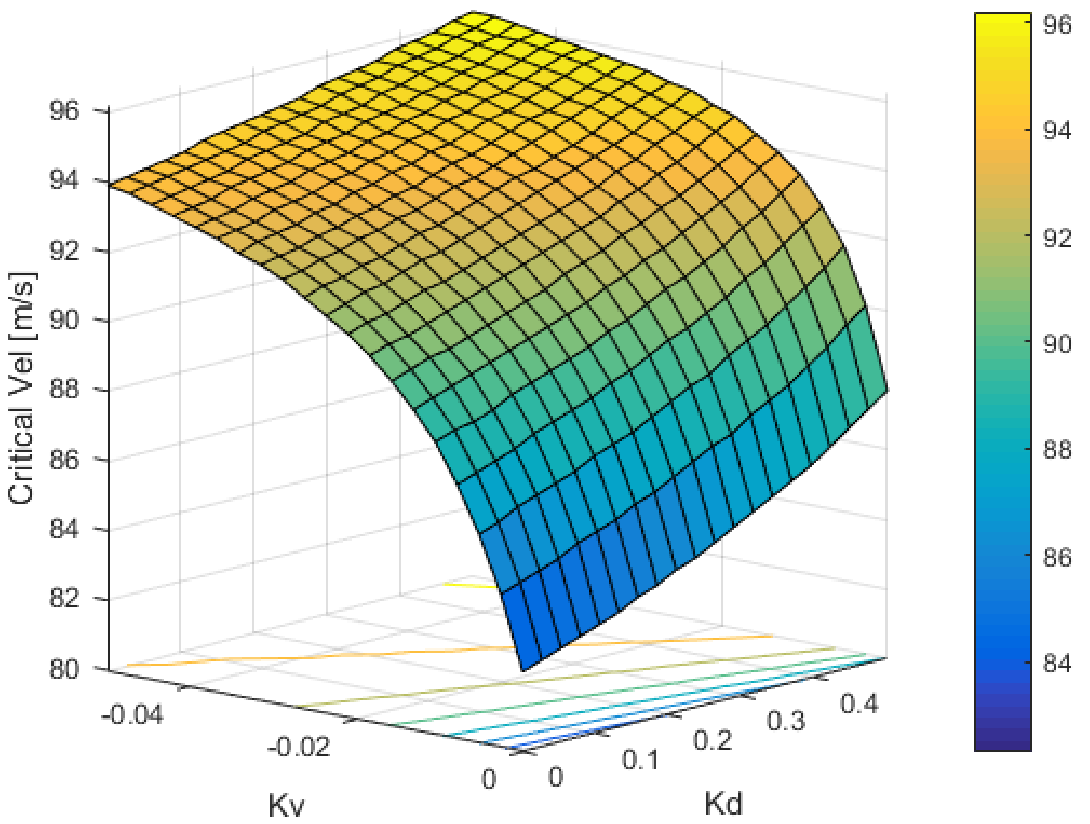

Figure 4 – Behavior of critical velocity of flutter according with combination of Derivative and Proportional Gain Feedback variations.

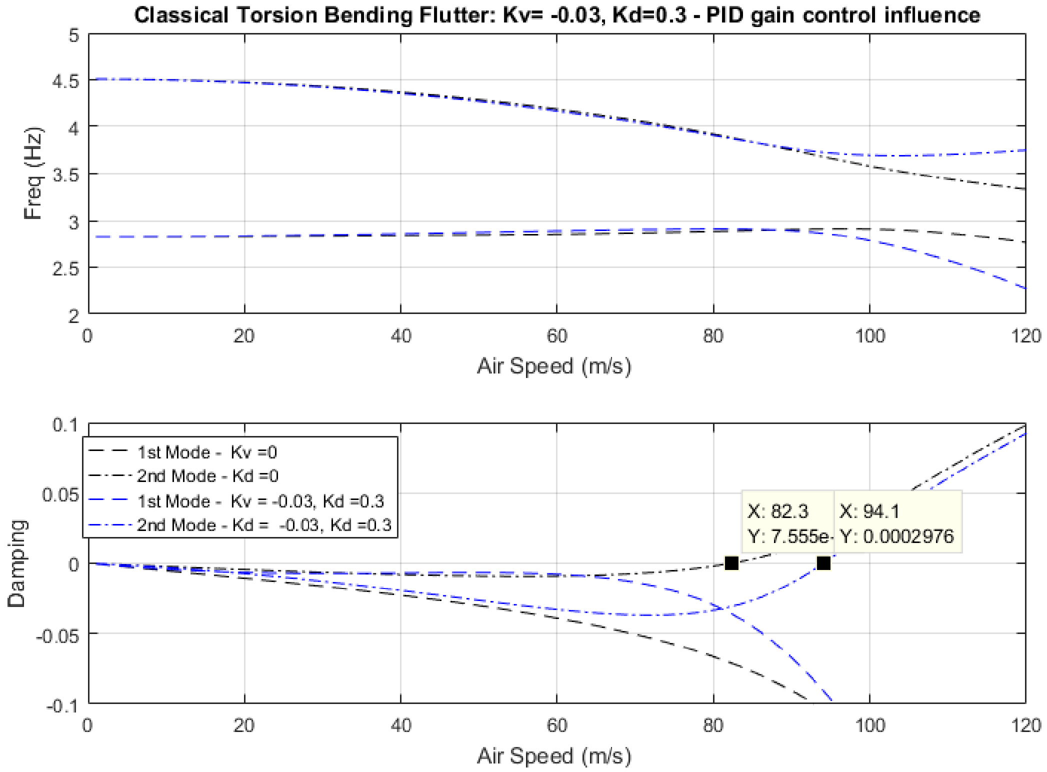

Figure 5 - Vgf diagrams for flutter analysis with Proportional and Derivative feedback Gain consideration.

It is possible to observe in Fig.2 that, as the gain of the derivative controller increases, the critical flutter velocity follows in the same direction. However, it is possible to observe that the system experiences, in the case of greater gains, a jump of instability in one of the vibration modes of vibration. This behavior is characterized as soft flutter (Cooper, 2015) or hump mode, and in most of the cases, the structural damping (which are not considered here) suppress this phenomenon.

Similarly, it is verified that the critical flutter speed increases with the proportional controller, but at a much lower rate (as shown Tab.2 and Fig.3), when compared to the previous case. However, it is not observed a biased behavior to instability of other modes of vibration of the system.

The surface illustrated in the Fig.4, it is also evident that the behavior of the aeroelastic system is much more influenced by the derivative gain than by the proportional gain, leading to the conclusion that the application of the first one is much more advantageous than the second one, in the case of active aeroelastic control.

In order to observe the both proportional and derivative gains and the influence of those combination, the best value of critical speed (with no stability behavior in second mode) for the set of combinations performed to generate the surface in Fig.4 is showed in Vgf diagram on the Fig.5. The value of critical speed is approximately 91m/s when the proportional gain is 0,3 and the derivative gain is -0,03. In other words, the inclusion of proportional gain does not improve effectively the results than those generated only with derivative gain.

CONCLUSIONS

This paper presents an implementation of a model based on a rectangular uniform wing subject to an airflow, including active control technics based on the PID theory. The feedback gains were varied and the flight envelope for each case was analyzed. According with results, it’s possible to visualize the increase of flutter velocity with the increase of the feedback gains, in general terms. However, better results were achieved for the derivative feedback gains, since small variations of this parameter causes major modifications on the system stability.

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