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Dynamic Response Computation in Phononic Crystal Cylindrical Shells by Wave Methods

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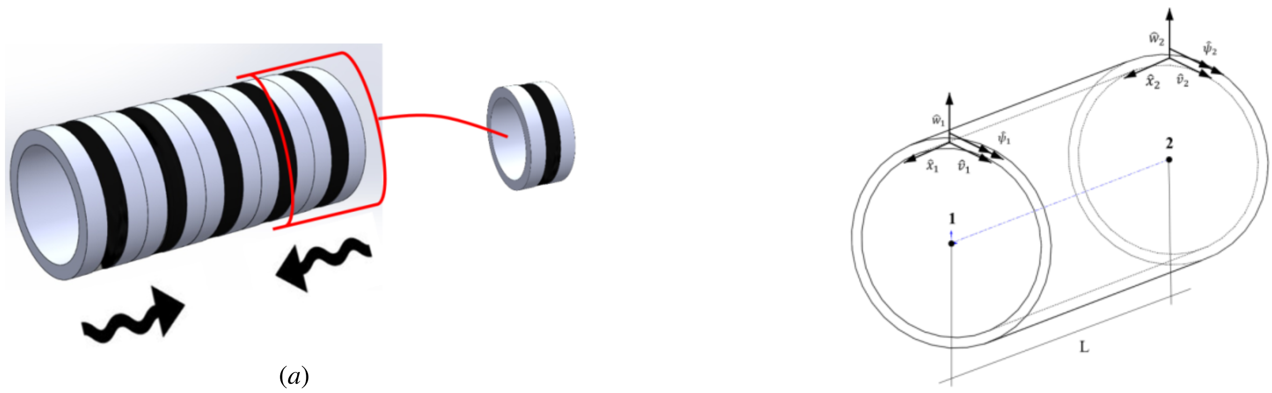
Abstract: Cylindrical shells are often used as structural elements in engineering. They are used for the construction of reservoirs, pressure vessels, fuselage, submarines, ducts and so on. Recently they have been used also as a new periodic composite material known as phononic crystals (PCs). PCs can be seen as structures having different elastic properties and/or different geometry which are periodically distributed along their space. Periodicity generates band gaps, i.e., stop bands or forbidden bands over which waves do not propagate. This feature enables one to propose new and efficient solutions for the vibration of structures. The aim of the paper is to investigate the wave propagation and properties of periodicity on the frequency band structure of PCs made with cylindrical shells. Band gaps generated by Bragg scattering effect are calculated for different test cases. Two methods are proposed for this investigation the Wave Spectral Element (WSE) method and the Wave Finite Element (WFE) method. Simulated results are presented in frequency domain as dispersion diagrams. Free and forced responses are under development and the results of band gaps generated will be presented as frequency response functions (FRF) in the final paper. Results calculated by WSE, WFE and conventional finite element (FE) method are compared among themselves.

Keywords: periodic structures, phononic crystals, band gaps, wave finite element method, spectral element method

INTRODUCTION

Shells have been used frequently as structural element in mechanical, civil, aerospace and naval engineering. It comes from the fact that shells can be used for large span structures with good stiffness to weight ratio. Shells are subjected to various complex loading and boundary conditions, that can lead to structural failure, therefore, a good understanding of the dynamic behaviour of shell elements is very important to guarantee a safe and inexpensive design.

Phononic crystals are artificial materials with discontinuities (inclusions, abrupt change of dimension or elastic property) which are periodically distributed along their dimensions. Figure 1a shows an example of phononic crystal cylindrical pipe made with two elastic properties distributed periodically along its length and a detail of the periodicity pattern (unit-cell). As a consequence of periodicity, those structures may exhibit frequency band gaps (stop or forbidden bands) where waves do not propagate. Based on this feature phononic crystals can be proposed as an efficient solution for vibration and noise control.



(*a*) (*b*)

Figure 1 – Phononic crystal: (a) cylindrical shell scheme made with two material properties; (b) two side cylindrical shell spectral element.

Wave propagation in periodic structures has been studied by many authors with origins that can be traced back to Newton and Rayleigh. The engineering study of periodic structures starts in the 70's with Mead, who wrote a good review about this subject (Mead, 1996). The band gap phenomena in structural dynamics started to be analyzed in the 90's (Sigalas and Economou, 1992). Since that, new approaches have appearing using periodicity together with analytical and numerical modeling to calculate complex structures at a low computational cost. Some of these are wave spectral element (WSE) method and wave finite element (WFE) method. Both methods model the periodic unit-cell using the dynamic stiffness matrix approach. The main difference between them is that the first uses the spectral element (SE) method (Doyle, 1997), while the second uses the finite element (FE) method (Mencik, 2014). The dynamic stiffness matrix formulation is expressed as a transfer matrix relation. Then, applying Floquet-Bloch theorem an eigenvalue vector transfer matrix problem is obtained, whose solution are the wavenumbers and corresponding wave modes which travel towards the right and left directions along the periodic structure. The WSE and WFE method has been applied to some phononic crystals made with rods, beams and plates (Nobrega et al., 2016; Silva and Arruda, 2012; Mencik, 2014).

Many researches in phononic crystals and metamaterials have been done in physics, but in engineering application these field remains an open challenge (Hussein et al, 2014). Phononic crystals produce band gaps, induced by Bragg scattering effect (destructive interference of waves) generated by periodicity. For a 1D periodic structure with a unit-cell of length d, band gaps would appear around frequencies governed by the Bragg condition, $d = n(\lambda/2), (n = 1, 2, \ldots)$ where λ is the wavelength. Phononic crystals studies have been focused on bulk waves and more recently it have been extended to elastic waves in structures.

In this paper, the WSE and WFE methods are applied to compute band gaps in phononic crystal cylindrical shells whose elastic properties change periodically as shown in Fig. 2. The motivation is to demonstrate that WSE and WFE method are accurate and, also evaluate their efficiencies (computational time) for modeling and simulating phononic crystal cylindrical shell. A comparative study between both methods (WSE and WFE) and the analytical solution of a cylindrical shell (Leissa, 1973) is presented here in terms of dispersion diagrams, and the results are in good agreement. Both methods are been extended to calculate free and forced responses for the phononic crystal cylindrical shell model and will be presented in the final paper. The dynamic stiffness matrix of the circular cylindrical shell spectral element is formulated. In the governing equations time derivatives are transformed by using the spectral decomposition, while the circumferential coordinate is eliminated by applying the solution in the form of Fourier series (Kolarević et al., 2016). A two-side cylindrical shell spectral element with four degrees-of-freedom/side is formulated as shown in Fig. 1b. A similar procedure using numerical approaches, wave finite element (WFE) method (Mencik, 2014), will be applied to a cylindrical shell model and presented in the final paper.

Computational Procedure

The WSE and WFE method uses the concept of transfer matrix methods to compute the wave modes along periodic structures. The periodic structures which are considered here are assumed to be 1D periodic in the sense that they are composed of identical unit-cells along a certain straight direction. By considering the wave modes, the forced response of phononic crystal cylindrical shell can be computed in an efficient way. The computational tasks involved in the WSE and WFE method can be summarized as follows.

(1) SE or FE model of a unit-cell: Let's consider the unit-cell (materia$_1$ +material$_2$ +material$_1$) of the periodic pipe shown in Fig. 2, made with two material properties. It consists in obtaining the dynamic stiffness matrix of the unit-cell, which can be modeled by SE or FE. By using SE only three cylindrical shell spectral elements will be enough, one for each elastic property changes. While by FE it consists in obtaining the mass, stiffness and damping matrices of the unit-cell, which can be done using a commercial or ?home-made? FE software. Attention should to be paid about the size of the FE mesh, i.e., it has to be fine enough to accurately capture the wavelengths of the wave motion. Then, the condensation of the dynamic stiffness matrix is prepared, which might be cumbersome for unit-cells having many internal DOFs.

(2) Transform the dynamic stiffness matrix in the transfer matrix formulation and applying Floquet-Bloch theorem: Partitioning the dynamic stiffness matrix in left and right boundary state vectors, the transfer matrix of the unit-cell can be easily derived. Then, applying Floquet-Bloch theorem it has the wave eigenproblem given by,

$$\mathbf{T}\mathbf{q} = e^{\mu}\mathbf{q} \tag{1}$$

where $\mathbf{T}$ is the transfer matrix, $\mathbf{q}$ is the state vector (eigenvectors) and e^{μ} is the eigenvector with $e^{\mu} = ikd$, is the attenuation constant and k is the wavenumber. A more detailed formulation about WSE and WFE can be found in references (Mencik, 2014; Nobrega at al., 2016; Sousa et al., 2017; Pereira and Dos Santos, 2017).

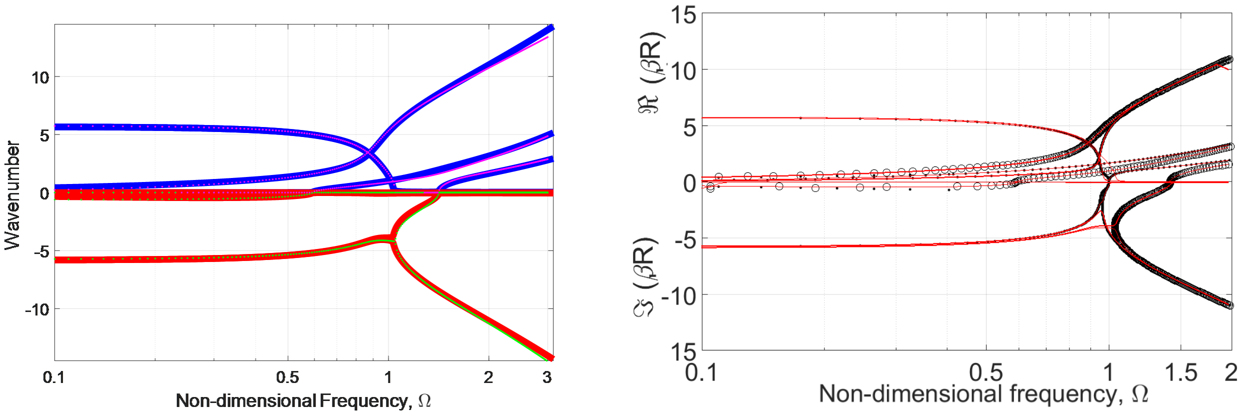
(3) Computation of the wave modes: It involves a well-posed eigenproblem based on skew symmetric matrices to compute the eigensolutions of the unit-cell transfer matrix. The order of eigenvalues/vectors are important to obtain the correct answers.

4) Computation of the forced response is under development and will be presented in the full paper. By considering the wave modes, well-conditioned matrix formulations can be proposed to compute the dynamic response of finite-length

periodic structures whose left and right ends can be subject to arbitrary kinds of boundary conditions or coupling conditions.

RESULTS AND DISCUSSION

Two examples are presented, one to verify the accuracy of WSE and WFE to calculate an homogeneous cylindrical shell (Fig. 2) and the other to show its capacity to calculate band gaps in a phononic crystal (Fig. 3). The first test consists of a free-free homogeneous cylindrical shell made of inox steel; the other test concerns a free-free phononic crystal (Figure 1) made of periodic unit-cells with two materials (inox steel and polyacetal). Figure 2 shows the dispersion diagrams for the longitudinal wave mode in the homogeneous cylindrical shell calculated by the Analytical Solution and compared with both proposed methods WSE (Fig. 2a) and WFE (Fig. 2b). It can be seen very good agreement between the Analytical and the methods (WSE and SFE), corroborating others results in literature Pereira and Dos Santos (2017).



(*a*) (*b*)

Figure 2 – Dispersion diagrams for the homogeneous cylindrical shell: (a) Analytical (blue- ℜ) and (red- ℑ) and WSE method (green-ℜ) and (magenta-ℑ); (b) Analytical (black dots and circles) and WFE (red solid lines).

Figure 3 shows the dispersion diagrams for the longitudinal wave mode in the phononic crystal cylindrical shell calculated by the WSE and WFE for the first harmonic modes ($m = 0$). As expected, it can be seen very good agreement between the methods (WSE and SFE) at low frequency bands, but as the frequency band increases some divergences appears. However, both methods are still able to identify clearly the locations and widths of band gaps, which occur at the locations where the imaginary part of the wavenumber becomes negative (evanescent wave) in the dispersion diagram.

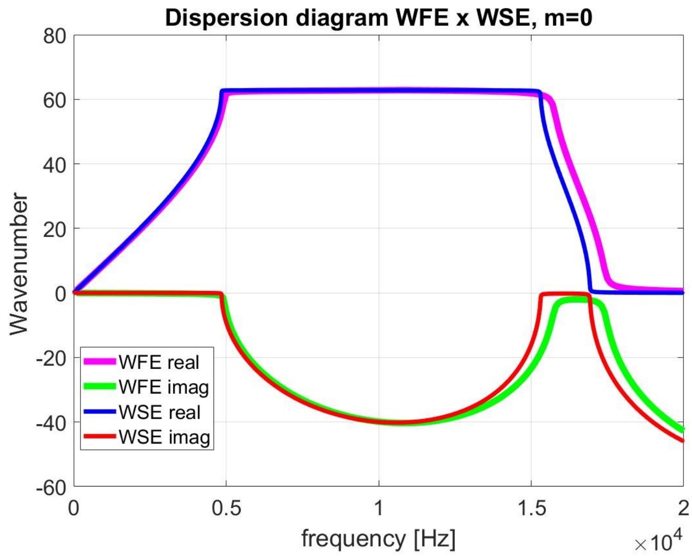


Figure 3 – Figure 3: Dispersion diagrams for the phononic crystal cylindrical shell by WSE and WFE methods for the harmonic mode m = 0.

Final Remarks

The WSE and WFE methods were considered and implemented to compute the wave modes in free-free homogeneous and phononic crystal cylindrical shells. The accuracy of WSE and WFE to calculate the homogeneous cylindrical shell as compares with the Analytical Solution was clearly demonstrated. Also, the WSE and WFE approaches constitute relevant numerical tools which are capable of predicting band gaps locations and widths in the dispersion diagrams of phononic crystals. Now, the efficiency of WSE and WFE will be analyzed regarding the computation of band gaps and the dynamic responses and the results of band gaps generated will be evaluated. Simulated results are under way and will be presented as dispersion diagrams and frequency response functions (FRF) in the final paper.

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