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Comparison between 6x6 and 6x4 vehicle behavior in steady-state cornering

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Abstract: The model presented has the objective of studying the behavior of a 3-axle vehicle in single steady-state cornering, according to Newton-Euler rigid vehicle equations and Bakker-Pacejka tire model, comparing the 6x6 and 6x4 configurations. The vehicle has two front steering axles and a rear axle with no steering. To compare both models, it was considered that the torque is provided by a mathematical model of engine and equally distributed to the traction axles, and the vehicle weight is equally distributed to each tire. The vehicle is an extended rigid body with 3 degrees of freedom: x and y position and z rotation, and develops a planar motion. The model studied is the simple handling model, also known as bicycle planar model, that considers a half car moving in a plane. The vehicle develops a cornering in low velocities and low steering angle, as consequence the centrifugal effects, suspension deformation, camber variation, dynamic weight transference and air resistance are disconsidered. It was developed a model considering slip and rotational inertia of the tires, according to the Bakker-Pacejka tire model. In the 6x6 configuration, all the wheels are driver wheels, it means that all of them receive torque from the powertrain (that is assumed to be the same), and has longitudinal forces and lateral forces from tire deformation and rolling resistance. In the 6x4 configuration, the driven wheel, that is, the wheel that receives no torque from the powertrain, is the front wheel and has only lateral forces from tire deformation and rolling resistance, and the other wheels are driver wheels. This model works in SIMULINK/MATLAB platform. As conclusion, it is shown a mathematical model for the 6x6 and 6x4 configuration, that represents the longitudinal and lateral behavior of the vehicle, assuming the restrictions previously mentioned.

Keywords: vehicle dynamics, lateral dynamics, Bakker-Pacejka tire model, simple handling model, bicycle planar model.

INTRODUCTION

In the phase of research and development of an engineering project, the computational modelling is an important tool to reduce the time and costs on prototype fabrication and testing, increasing the effectiveness of this phase. A good computational model must be simple but must represent as much as possible the real conditions.

In the context of vehicle modelling dynamics, the cornering behavior in lateral dynamics occupies an important status. The lateral dynamics of a vehicle is important for what is called vehicle handling (Gilespie, 1992). The handling of a vehicle measures the responsiveness of a vehicle to driver input. Secondly, the cornering movement causes lateral centrifugal acceleration, implying rolling effects that limits maximum velocity.

A vehicle simpler computational model, disconsidering centrifugal effects, suspension deformation, camber variation, dynamic weight transference and air resistance is a good approximation of the reality when the vehicle develops a cornering in low velocities and low steering angle. In a steady-state cornering, and due to constant velocity and force modules actuating in the vehicle, a three degrees of freedom is a good approximation.

In this study, a steady-state cornering is studied, considering a 3-axle vehicle with 6x4 and 6x6 configurations. The vehicle has two front steering axles and a rear axle with no steering.

It was developed a model considering slip and rotational inertia of the tires, according to the Bakker-Pacejka tire model. In the 6x6 configuration, all the wheels are driver wheels, that is, all of them receive torque from the powertrain (that is assumed to be the same), and has longitudinal forces and lateral forces from tire deformation and rolling resistance. In the 6x4 configuration, the driven wheel that is the wheel that receives no torque from the powertrain is the front wheel and has only lateral forces from tire deformation and rolling resistance, and the other wheels are driver wheels.

TIRE COORDINATE FRAME AND TIRE FORCE SYSTEM – CONSIDERING SLIP AND TIRE ROTATION INERTIA

In the case of considering tire slip and tire rotational inertia, a tire model must be considered. To describe the tire model, firstly, we need to characterize the tire coordinate frame and tire force system.

To describe the tire-road interaction and force system, we attach a Cartesian coordinate frame at the center of the tireprint, as shown in Figure 1, assuming a flat and horizontal ground. The x-axis is along the intersection line of the tire-plane and the ground. Tire plane is the plane made by narrowing the tire to a flat disk. The z-axis is perpendicular to the ground, opposite to the gravitational acceleration **g**, and the y-axis makes the coordinate system a right-hand triad (Jazar, 2009)

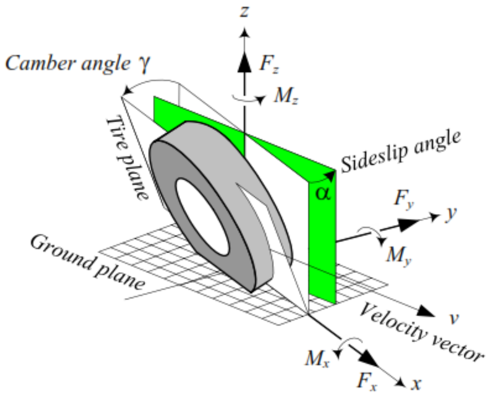

Figure 1 - Tire coordinate system (Jazar, 2009)

There are three forces on the tire acting from the ground. The tractive force or longitudinal force F_x is the force exerted on the tire by the road an let the vehicle move forward to the main direction. The lateral force F_y perpendicular of the F_x and the normal force F_z, act as a result of the ground reaction. Besides, there are three moments involved, the overturning moments M_x exerted on the tire by the road, rolling resistance moment M_y and the auto-aligning moment M_z.

The study focus on the two main forces involves in the lateral movement, the lateral force F_y and the longitudinal force F_x.

1. Lateral force F_y

The lateral force developed at the tire-ground contact patch is usually called the cornering force F_y. The lateral force depends on the vertical force on the tire, the sideslip angle α and the camber angle γ.

Sideslip angle α, or deviation angle, is the angle between the tire direction and the velocity vector of the tire, as figure 1 shows.

The sideslip angle can be calculated by the following formula:

$$\alpha = \arctan\left(\frac{v_y^C}{v_x^C}\right) \quad (1)$$

Where v_x^C and v_y^C are the *x* and *y* velocity of the tire in the tire referential system.

Camber angle γ is the angle between the vertical plane of the wheel and the vertical direction of the ground.

The lateral force is generated by the sideslip angle α, camber angle γ or its combination. In cars, camber angle has a smaller contribution on the lateral force than the sideslip angle, which does not occur in motorcycles. The relationship between the cornering force and the slip angle is important to the directional control and stability of road vehicle.

a. Lateral force developed by the sideslip angle $F_{y\alpha}$

When the tire is moving on the xy plane in a curve, the tire deformation causes internal lateral forces; these lateral forces tend to increase as the tire elements move the contact region to the point where the lateral forces outweight the friction with the ground. In these points, the elements begin to slip. The fact that the tire has a slip region does not mean that it loses its grip on the ground. As the sideslip angle presented by it increases, the area of the region of sliding increases until the slip region completely covers the contact region, and the tire really lose grip.

The integration of each lateral loading into each contact region results in the total lateral force. The asymmetric growth of the lateral force along the contact region causes the resulting lateral force to position behind the center of that region, from a distance called "pneumatic trail". The moment generated is called self-alignment torque or auto-alignment torque M_z .

b. Lateral force developed by the camber angle $F_{y\gamma}$

As explained before, camber is the inclination of the wheel plane from a plane perpendicular to the road surface when viewed from the fore and aft directions of the vehicle. Its main purpose is to achieve axial bearing pressure and to decrease the king-pin offset.

In this study the camber angle takes zero value to simplify the simulation, thus does not generates lateral force.

2. Rolling resistance of tire

The rolling resistance is a force generated by the hysteresis effects on tire compression and expansion while moves on road surface. Therefore, the energy lost on this effect of compression and expansion is generated by a force opposite to the direction of motion, and proportional to the normal force on the tireprint (Jazar, 2009).

$$F_r = \mu_r F_z \qquad (2)$$

Where μ_r is called rolling friction coefficient (depends on tire speed, inflation, pressure, sideslip and camber angle) and F_z is the normal force on the tire.

The following function (Jazar, 2009) is a good approximation for the rolling resistance for passenger car tires driving on concrete pavement, dependent on speed:

$$\mu_r = \mu_0 + \mu_1 v_x^2 \qquad (3)$$

Where μ_0 =0.015 and μ_1 =7.10^{-6} s²/m² and v_x is the vehicle velocity.

3. Longitudinal Force (Traction force)

To accelerate or brake a vehicle is necessary to developed forces between the tire and the ground. The mechanism for these longitudinal forces to be generated has a certain similarity with the mechanism of lateral forces generation already, which also has a region of elastic deformations and a region of slip. The longitudinal forces could be for traction or braking. For each of these two cases there are slightly different behavior, although the mechanism for forces are analogous.

4. Bakker-Pacejka tire model and "Magic Formula"

In the construction of the model of a vehicle for the study of its dynamics, a good representation of tire behavior is critical to consistent results. Some papers propose the use of linear models. These models show good results only for small deviation angles and / or low values of longitudinal sliding. Close to the conditions of the limits of adhesion, these models are very inefficient, requiring the use of models that can represent the non-linear behavior of the tire for any conditions. In the searching for the best mathematical representation of the nonlinear characteristics of a tire, several approaches have been presented in the literature.

Many studies took empirical results from experiments, which let achieve better quantitative result that represents the dynamics tire behavior, commonly represented by equations as Fourier's series, polynomials expressions, etc. or special equations. The use of series has the disadvantage to manage a lot of coefficients which let fit the goals with the experiments results. Besides, this method lose precision under each iterations and extrapolations, getting coefficients without physical meaning.

In 1987 an empirical method for characterizing tire behavior known as Magic Formula has been developed and used in vehicules handling simulations.(Wong, . By using some special functions, we obtain the lateral force, the longitudinal force and the self-alignment torque of the tire. This equation has different coefficients depending on each tire. Today, the "Magic formula" achieved great reputation and have been adopted for many simulations programs.

To understand about Bakker-Pacejka tire model and "Magic Formula", firstly it is important to introduce the following definitions:

a. Effective rolling radius R_e

Consider a vertically loaded wheel that is turning on a ground. The effective rolling radius of the wheel R_e, is defined by the following equation:

$$R_e = \frac{v_x}{\omega} \quad (4)$$

Where:

v_x: Vehicle velocity or longitudinal speed
ω: Angular speed of the wheel.

b. Longitudinal slip

When a driving torque is applied to a pneumatic tire during the acceleration or braking, due to the tire slip, generates a difference between the angular speed ω and the angular speed on pure rolling (without tire deformation) . The difference of the speed is determined at each moment prior to the application of the tractive or braking torque and calculated from the longitudinal velocity of the center of mass of the wheel (v) and the free-rolling radius of the tire R by the following equation:

$$\sigma = 1 - \frac{v_x}{\omega . R} \quad (5)$$

The longitudinal force F_x, of traction or braking generated in the tire/ground contact is usually shown as a function of longitudinal sliding σ. The longitudinal force is zero when σ is equal to zero (pure rolling condition).

c. Characteristics of the "Magic formula"

The "Magic formula" can be used to fit experimental tire data for characterizing the relationships between the cornering force and slip angle, self-aligning torque and slip angle, or braking effort and skid (Wong, 2001). It is expressed by the following equation:

$$y(x) = D \sin\{C \arctan[B\,x - E(B\,x - \arctan(Bx))]\} \quad (6)$$

$$Y(X) = y(x) + S_v \quad (7)$$

$$x = X + S_h \quad (8)$$

Where:

$Y(X)$: Represents cornering force, self-aligning torque or braking effort.
X: Denotes slip angle (for cornering force or self-aligning torque) or skid (for braking effort).
B: Stiffness factor.
C: Shape factor.
D: Peak factor.
E: Curvature factor.
S_h and S_v: Horizontal shift and vertical shift, respectively.

The product BCD is the angular coefficient of Y(X) at its origin. In case of calculating the lateral force, BCD represents the tire sideslip coefficient or cornering stiffness for one vertical force and camber angle. The C coefficient, define the extension of the sin function of the "Magic formula" and fit the Y(X) curve. Beside, this coefficient lets the curve assume the characteristic of the lateral force, longitudinal force or self-aligning torque. The B coefficient, also called as stiffness factor, does not have a meaning by itself, only it is important with the angular coefficient of the Y(X) at its origin. The coefficient E is the curvature factor and limited the function according it assume maximum values. The S_h and S_v represents the slip on the curve developed by the asymmetry of the cords of the belt of the tire, for the taper of the tire and for the rolling resistance and the camber angle.

The B, C, D, E, S_h and S_v coefficients are in function of the vertical force F_Z and the camber angle γ. They could be obtained by other mathematical expressions presented hereafter.

1) Lateral force

The lateral force is calculated by "Magic formula" and its coefficient is obtained by the following mathematical expressions:

$$D = \mu_{ym} F_Z \quad (9)$$

Where:

$$\mu_{ym} = a_1 F_Z + a_2 \quad (10)$$

μ_{ym}: Lateral friction coefficient.
a_1: Coefficient that represents the relationship between the lateral friction and vertical force.

a_2: Coefficients that represents the level of lateral friction.

$$BCD = a_3 \sin\left[2 \arctan\left(\frac{F_z}{a_4}\right)\right](1 - a_5\,|\gamma|) \quad \textbf{(11)}$$

Where:

a_3: Coefficient that represents the maximum cornering stiffness.
a_4: Coefficient that represents the relative vertical force at maximum cornering stiffness.
a_5: Coefficient that represents the sensibility of the cornering stiffness for the camber angle.

$$C = a_0 \quad \textbf{(12)}$$

The shape factor C assume values as 1.30.

$$B = \frac{BCD}{CD} \quad \textbf{(13)}$$

$$E = a_6 F_z + a_7 \quad \textbf{(14)}$$

$$S_h = a_8\,\gamma + a_9\,F_z + a_{10} \quad \textbf{(15)}$$

$$S_v = a_{11}\,F_z\,\gamma + a_{12}\,F_z + a_{13} \quad \textbf{(16)}$$

The values of coefficients a_0 to a_{13} for the same tire are tabulated as result of the experimental data mentioned.

2) Longitudinal force

The calculus of the longitudinal force involves coefficients that are obtained by the expression shown as following:

$$D = \mu_{xm} F_Z \quad \textbf{(17)}$$

Where:

$$\mu_{xm} = b_1 F_Z + b_2 \quad \textbf{(18)}$$

μ_{xm}: Coefficient of longitudinal friction.
b_1: Coefficient that represents the relationship between the longitudinal frictions with the vertical force.
b_2: Coefficient that represents the level of the longitudinal friction.

$$BCD = (b_3 {F_z}^2 + b_4\,F_z)\,e^{(-b_5 F_z)} \quad \textbf{(19)}$$

$$C = b_0 \quad \textbf{(20)}$$

Usually adopted $b_0 = 1.65$

$$B = \frac{BCD}{CD} \quad \textbf{(21)}$$

$$E = b_6 {F_z}^2 + b_7\,F_z + b_8 \quad \textbf{(22)}$$

$$S_h = b_9\,F_z + b_{10} \quad \textbf{(23)}$$

$$S_v = 0 \quad \textbf{(24)}$$

The values of coefficients b_0 to b_{10} for the same tire are tabulated as result of the experimental data mentioned.

3) Interaction between the lateral and longitudinal force.

The formulas of previous sections apply only in cases where lateral and longitudinal forces act separately. If the tire simultaneously produces forces in the lateral and longitudinal directions, the situation becomes different, since the force used in one direction limits the available force in the other direction. By applying a braking or tractive force to the tire which is subject to a particular slip angle, the lateral force is reduced when compared to the condition without braking or traction. The same applies to longitudinal force when the tire is subjected to lateral force.

One way of graphing the relationship between lateral force and longitudinal force, when applied simultaneously to a tire, is through several curves, drawn for different values of α, limited by a polar diagram representing the maximum force that the tire can generate for each condition. (Wong, 2001)

The Magic formula presented before is applicable when the tire is under the lateral dynamic or longitudinal dynamic studies. In order to obtain an equation with both forces involves, it is necessary developing a new methodology based on the first seen before.

From figure 2, it is possible to define a longitudinal sliding σ_x (not to be confused with longitudinal slip) and the lateral sliding σ_y as:

$$\sigma_x = \frac{V_{sx}}{V_r} \tag{26}$$

$$\sigma_y = \frac{V_{sy}}{V_r} \tag{27}$$

Where:

V_{sx}: Absolute value of the sliding speed component in X' direction;
V_{sy}: Absolute value of the sliding speed component in Y' direction;
V_r: Absolute value of the longitudinal speed.

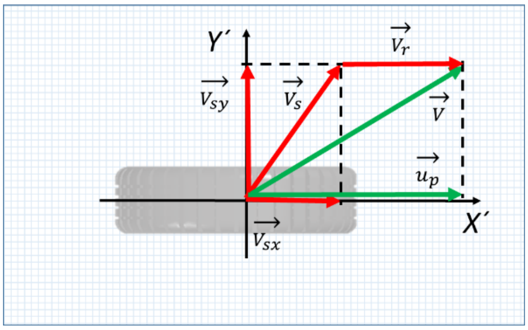

Figure 2 - Sliding speed and longitudinal speed

Figure 2 depict the way to obtain de longitudinal slip σ and sideslip angle α as following:

$$\sigma = \frac{V_{sx}}{v_x} \tag{28}$$

$$\tan\alpha = \frac{V_{sy}}{V_{sx} + V_r} \tag{29}$$

Where:

v_x: tire longitudinal speed

After a little algebra, and rewriting the equation of the sliding in both direction:

$$\sigma_x = \frac{\sigma}{1+\sigma} \tag{30}$$

$$\sigma_y = \frac{\tan\alpha}{1+\sigma} \tag{31}$$

At the beginning, the lateral force F_y exists when the sideslip angle α is equal to zero according to the coefficients S_h and S_v. For this case is defined de sideslip angle $\delta\alpha$ as the angle when the lateral force is zero. For low values of this angle, it could be expressed as following:

$$\delta\alpha = S_h + \frac{S_v}{BCD} \tag{32}$$

Consequently, the longitudinal slip σ for low values of α, without lateral force, it defined the $\delta\sigma$. In this study we are considering $S_v = 0$.

$$\delta\alpha = S_h \tag{33}$$

The relative sliding speed δV_{sx} and δV_{sy} according to $\delta\alpha$ and $\delta\sigma$ are:

$$\delta V_{sx} = V_r \frac{\delta\sigma}{1+\delta\sigma} \approx -V_r\,\delta\sigma \tag{34}$$

$$\delta V_{sy} = V_r \tan\delta\alpha \approx V_r\,\delta\alpha \tag{35}$$

The resultant sliding speed V_{sxtot} and V_{sytot} are:

$$V_{sxtot} = V_{sx} + \delta\, V_{sx} \quad \textbf{(36}$$

$$V_{sytot} = V_{sy} + \delta\, V_{sy} \tag{37}$$

Thus, the longitudinal and lateral sliding:

$$\sigma_{xtot} = \frac{\sigma}{1+\sigma} + \delta\sigma \tag{38}$$

$$\sigma_{ytot} = \frac{\tan\alpha}{1+\sigma} + \delta\alpha \tag{39}$$

Where:

σ_{xtot}: Total longitudinal sliding
σ_{ytot}: Total lateral sliding

The magnitude of the total theoretical sliding could be obtained as a sum of its components:

$$\sigma_{tot} = \sqrt{{\sigma_{xtot}}^2 + {\sigma_{ytot}}^2} \tag{40}$$

The curves obtained $F_x(\sigma_{tot})$ and $F_y(\sigma_{tot})$ respectably, when there is only lateral and longitudinal sliding are called "basic original curves".

The real values of the lateral and longitudinal are obtained multiplying the forces F_{x0} and F_{y0} , from the basic original curves by the relationship between the components of the theoretical sliding in X´ and Y´ directions and the Total theoretical sliding σ_{tot}:

$$F_x = \frac{\sigma_{xtot}}{\sigma_{tot}} F_{x0} \tag{41}$$

$$F_y = \frac{\sigma_{ytot}}{\sigma_{tot}} F_{y0} \tag{42}$$

A problem of physical nature arises when the values of longitudinal and relative lateral sliding to the peaks of F_{x0} and F_{y0} differ considerably. When the tire reaches a total theoretical sliding value, σ_{tot} between these two peaks, the situation occurs where there is total sliding in relation to a curve and partial slip in relation to another curve. The slip of the tire occurs in relation to a global condition, and not in relation to only one direction. In this case, it is defined a normalized total slip σ^* defined in relation to the maximum slip values in the X' and Y' directions:

$${\sigma_x}^* = \frac{\sigma_{xtot}}{\sigma_{xm}} \tag{43}$$

$${\sigma_y}^* = \frac{\sigma_{ytot}}{\sigma_{ym}} \tag{44}$$

$$\sigma^* = \sqrt{{\sigma_x}^{*2} + {\sigma_y}^{*2}} \tag{45}$$

Where:

σ_{xm}: Relative theoretical sliding for the peak of F_{x0}
σ_{ym}: Relative theoretical sliding for the peak of F_{y0}.

Consequently

$$F_{x0}{}^{*} = F_{x0} - \varepsilon\left(F_{x0} - F_{y0}\right)\left(\frac{\sigma_y{}^{*}}{\sigma^{*}}\right)^2 \quad (46)$$

$$F_{y0}{}^{*} = F_{y0} - \varepsilon\left(F_{y0} - F_{x0}\right)\left(\frac{\sigma_x{}^{*}}{\sigma^{*}}\right)^2 \quad (47)$$

Where:
$\varepsilon = \sigma^{*}$, for $\sigma^{*} \leq 1$;
$\varepsilon = 1$, for $\sigma^{*} > 1$;

FORCE FROM ENGINE TORQUE

The engine torque is transmitted from the engine to the tires by the vehicle transmission system. Therefore, the force generated on tires depends on engine performance and transmission characteristics (gear transmission ratios and differential transmission ratio). In this study, the transmission efficiency, considering losses in transmission, is not considered, although in many studies it is simply estimated by a constant efficiency coefficient.

At full throttle condition, the torque performance, in N·m, of an internal combustion engine can be estimated from a quadratic function that depends on engine angular velocity, respecting the interval of minimum and maximum rotation of the engine. For a spark ignition engine, the coefficients of the quadratic polynomial are calculated as follows (Jazar, 2009):

$$T = P_1 + P_2\omega_e + P_3\omega_e{}^2 \quad (48)$$

$$P_1 = \frac{P_M}{\omega_M} \quad (49)$$

$$P_2 = \frac{P_M}{\omega_M{}^2} \quad (50)$$

$$P_3 = -\frac{P_M}{\omega_M{}^3} \quad (51)$$

Where P_M is the maximum engine power in kW ω_M is the rotation of maximum engine power in rad/s and ω is the engine rotation in rad/s.

The engine torque is then multiplied by the vehicle gearbox and differential ratios. This multiplication increases transmitted torque and decreases transmitted rotation.

RIGID VEHICLE NEWTON-EULER DYNAMICS

In this study, we consider the car as a rigid body moving in the xy plane. We also consider only 3 degrees of freedom that are: translation on x and y axes, and rotation about the z axis. According to these considerations, we have the following Newton-Euler equations (Jazar, 2009):

$$F_x^C = m.\dot{v}_x^C - m\omega_z v_y^C \quad (52)$$

$$F_y^C = m.\dot{v}_y^C + m\omega_z v_x^C \quad (53)$$

$$M_z = \dot{\omega}_z I_z \quad (54)$$

Where all the velocities, forces and moments are calculated in the chassis referential.

SIMULINK BAKKER-PACEJKATIRE MODEL

The following parameters were considered to the simulation:

Table 1 – Coefficients and parameters for simulation

Lateral Coefficient		*Longitudinal Coefficient*		*Vehicle*		*Description*
$a1$	0	$b1$	0	M	$4.800\ kg$	*Vehicle total mass*
$a2$	1688	$b2$	1688	I_z	$6.000\ kg\ m^2$	*Moment of inertia of the vehicle on z − axis*
$a3$	4140	$b3$	0	Lf	$1.8\ m$	*Distance between front axis and cg*
$a4$	6.0260	$b4$	229	Lr	$1.8\ m$	*Distance between rear axis and cg*
$a5$	0	$b5$	0	w	$2.3\ m$	*Track*
$a6$	−0.3589	$b6$	0	R	$0.50\ m$	*Rolling radius of the free − rolling tire*
$a7$	1	$b7$	0	I_{wz}	$4.0\ kg\ m^2$	*Moment of inertia of the wheel on z − axis*
$a8$	0	$b8$	−10	P_{max}	$282\ kW$	*Maximum power of the engine*
$a9$	0	$b9$	0	ω_{Pmax}	$2100\ RPM$	*Angular velocity for maximum power of the engine*
$a10$	0	$b10$	0	i_d	4.35	*Diferential transmition ratio*
$a11$	0	− − −	− − −	i_1	5.6	*First gear transmition ratio*
$a12$	0	− − −	− − −	i_2	3.45	*Second gear transmition ratio*
$a13$	0	− − −	− − −	− − −	− − −	− − −
Other parameters: $g = 9.81\ m/s^2$ *Tire*: $P205/60\ R15$						

In the 6x6 configuration, all the wheels are driver wheels, that is, all of them receive torque from the powertrain (that is assumed to be the same), and has longitudinal forces and lateral forces from tire deformation and rolling resistance. In the 6x4 configuration, the driven wheel, that is, the wheel that receives no torque from the powertrain, is the front wheel and has only lateral forces from tire deformation and rolling resistance, and the other wheels are driver wheels.

It is assumed that the steering angle of the front tire in both configurations increases from zero to one degree linearly, and then decreases linearly with the same rate. The simulation starts with a straight movement of the vehicle, until it reaches a constant velocity and then the direction changes as explained.

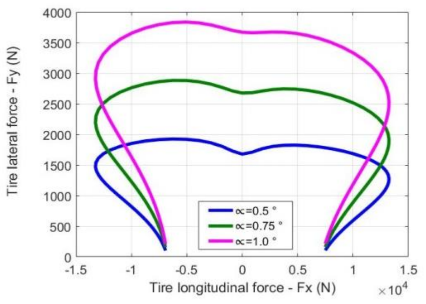

Figure 3 – Tire lateral force vs Tire longitudinal force

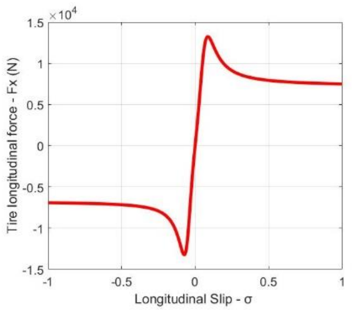

Figure 4 – Tire longitudinal force vs Longitudinal Slip

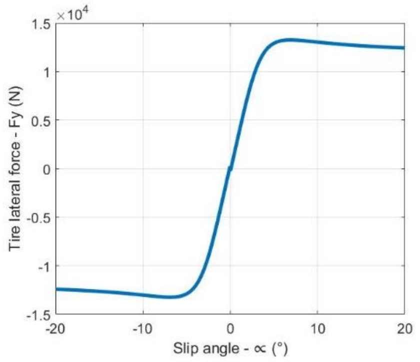

Figure 5 – Tire lateral force vs Slip angle

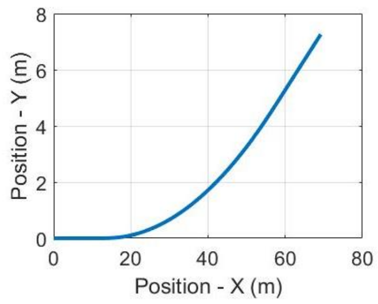

Figure 6 – Trajectory

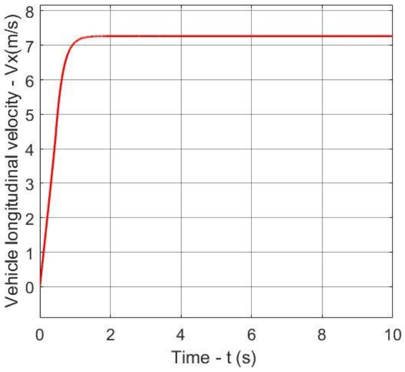

Figure 7– Vehicle longitudinal velocity vs Time

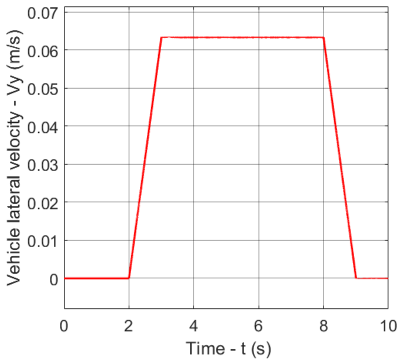

Figure 8 – Vehicle lateral velocity vs Time

In the case of 6x6 configuration, the tires develop longitudinal and lateral forces calculated from Bakker-Pacejka tire model. On the other hand, the 6x4 configuration the moved tire developed only the lateral force from "Magic formula". It is important to emphasize that, in this last configuration; no torque is transmitted from the motor to the front axle, but is equally divided between the intermediate and rear axles.

It was observed that the graphs obtained of the 6x6 and 6x4 configurations were slightly the same. Figure 3 shows the relation between the longitudinal and lateral forces of the tire, considering the method described in this paper. Figure 4 shows the behavior of the longitudinal force in function of the longitudinal slip. In the simulation, the longitudinal slip varies from 1, in the beginning of the movement, to approximately $2 \cdot 10^{-3}$, when the longitudinal slip is very small and the vehicle velocity approximates to the tire velocity.

The behavior of the lateral force in function of the slip angle is shown in figure 5, and the vehicle trajectory in figure 6. Figure 7 reaches the maximum velocity of approximately 7.3 m/s. The velocity is limited due to the maximum rotation of the engine and the reaction forces of the ground on the tire. In figure 8 is represented the vehicle lateral velocity at the simulated time.

CONCLUSION AND PERSPECTIVES

In this paper, it was studied the dynamic behavior of a 6x6 and 6x4 vehicle on curves developed on small velocities and small steering angles.

It was presented the main equations that govern this dynamic problem, and it was simulated at SIMULINK block diagram to analyze the dynamic behavior and estimate the parameters involved, as longitudinal velocity, lateral velocity and trajectory.

We conclude that, for small velocities and small steering angles, the vehicle dynamic behavior in the 6x6 and 6x4 configurations are slightly the same.

As a continuation of this work, we intent to improve the model to take into account bigger angles and velocities and then compare the 6x6 and 6x4 configurations in more detail.

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