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Trajectory Tracking Control Applied to an Electro-Hydraulic Actuator With Uncertain Parameters

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Abstract: Electro-hydraulic servo-systems are widely employed in industrial applications such as robotic manipulators, active suspensions, precision machine tools, and aerospace systems. They provide many advantages over electric motors, including high force to weight ratio, fast response time, and compact size. One major difficult in that precise control of electro-hydraulic systems cannot be easily obtained with conventional linear controllers, because of intrinsic nonlinearities. This work describes the development of an output feedback controller for an electro-hydraulic system that compensates the effects of nonlinearities and uncertain parameters. Numerical results are presented in order to demonstrate the control system performance.

Keywords: Electro-hydraulic actuator, Uncertain systems, Robust control, ADRC method.

INTRODUCTION

Electro-hydraulic actuators play an important role in many industrial applications. They are often considered as the most appropriate choice of actuation when large loads and high speeds are required. From the operation point of view, one of the main advantages of this type of actuator is the ability to maintain the load indefinitely, which can hardly be obtained by using electric actuators, without achieving overheating. Because they present a highly non-linear dynamic behavior, the efficient control of electro-hydraulic devices can not be easily obtained by means of conventional linear control techniques. In addition to the common nonlinearities generated by the compressibility of the hydraulic fluid, flow properties, and valve pressure, many of the electro-hydraulic systems are also subjected to large nonlinearities such as dead-zone, which occurs when the valve spool overlaps the fluid through-hole, preventing its flow even for a small displacement of the spool. In this context, the increase in the number of papers, proposing new control strategies for this class of systems, demonstrates the great interest of the industrial sector and the academic community for the theme. The most common approaches are the adaptive (Ahn *et al.*, 2014) and variable structure methodologies (Bessa *et al.*, 2010), but nonlinear controllers based on optimal tuning PID (Ye *et al.*, 2017), adaptive neural network and adaptive fuzzy system (Liem *et al.*, 2016) were also presented recently.

An emerging interest from both academia and industry in the *Active Disturbance Rejection Control* (ADRC) (Han, 1998) has been observed in recent years as discussed in (Madoński *et al.*, 2015). The potential of the ADRC method as a viable solution to industrial control has become increasingly evident after the recent adoptions by major industrial concerns, mainly because of its ability of disturbance rejection and its lack of requirement on the detailed mathematical model of the plant (Madoński *et al.*, 2015). Recently, a modified ADRC strategy was proposed in (Zachi *et al.*, 2017) to control electro-hydraulic actuator systems with uncertain control gain. In this work, the authors have discussed only the regulation problem in which the reference for the piston position was a constant set-point distance. In the present work, an improvement of the strategy proposed in (Zachi *et al.*, 2017) is developed. The main idea is to make the proposed method applicable to a more general class of movements, that is, in which the reference is a time-varying position.

1 PROBLEM STATEMENT

This paper considers the problem of controlling the piston displacement of an electro-hydraulic actuator for tracking a desired linear position trajectory, given with respect to a fixed coordinate system. As will be seen later in this paper, the actuator dynamical system, which is also denoted here by *plant*, possesses nonlinearities and parametric uncertainties that can degrade the system performance if they are not regarded in the controller design. Figure 1 shows a schematic diagram of the trajectory tracking control problem considered in this work.

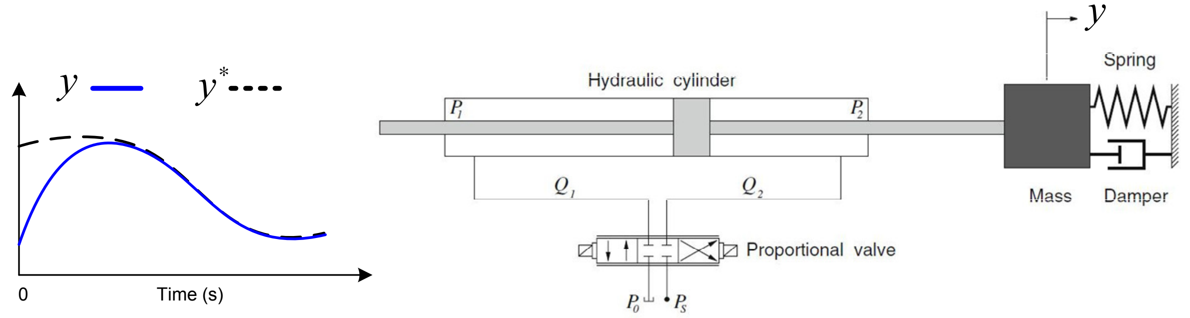

Figure 1 – Schematic block diagram of the electro-hydraulic servo-system.

2 METHODOLOGY

To solve the problem of controlling the actuator, in the presence of nonlinearities and parametric uncertainties, we developed a modified version of the Active Disturbance Rejection Method (ADRC) (Han, 1998, 2009; Madoński *et al.*, 2015). It consists basically of an extended observer that estimates the system states and non-measurable signals that act together with a state feedback control law. The key idea of the proposed method is to perform slight modifications both on the input/output dynamics of the plant and on the observer equations that brings some mathematics advantages to the closed loop control design. In contrast with the ADRC approaches reported recently in (Garran and Garcia, 2017; Xia *et al.*, 2018), the real contributions that are achieved in this work after applying such modifications are: (i) the exact knowledge of system parameters are not previously required in the control design; (ii) the control synthesis is simplified due to the reduced set of design constants; (iii) complexity reduction due to the use of linear components both on the observer and on the control law design.

3 SYSTEM MODEL

The electro-hydraulic system considered in this work consists of a four-way proportional valve, a hydraulic cylinder and variable load force. The variable load force is represented by a mass-spring-damper system. The schematic block diagram of the system under study is presented in Fig. 1. The behavior of the electro-hydraulic system, i.e. the piston linear displacement $y(t) \in \mathbb{R}$, is governed by the following third-order differential equation (Bessa *et al.*, 2010):

$$\dddot{y} = -a^T Y + b(Y,u)u - b(Y,u)d(u) \tag{1}$$

where $Y = [y, \dot{y}, \ddot{y}]^T$ is the state vector and $\mathbf{a} = [a_0, a_1, a_2]^T$ is the coefficient vector which values are defined by the combination of the parameters of the physical components of the mechanism. In Eq. (1), the term $b(Y,u) \in \mathbb{R}$ denotes the control coefficient of the system or simply the *control gain*, and the term $d(u) \in \mathbb{R}$ represents a dead-zone effect due to the overlap on the valve spool displacement. With respect to the dynamical system presented in Eq. (1), we assume that:
Assumption 1 the coefficients a_0, a_1 and a_2 are real, uncertain and constants.
Assumption 2 $b(Y,u)$ is not measurable but it is uniformly positive and bounded away from zero $\forall t$.

The next Section presents the modified ADRC.

4 THE PROPOSED ADRC WITH MODIFIED FRAMEWORK

Consider the class of dynamical systems defined by

$$\begin{aligned} y^{(n)} &= f\big(Y, d(t), h(t)\big) + bu(t)\,, \\ Y(t) &= [y,\ \dot{y},\ \cdots,\ y^{(n-1)}]^T\,. \end{aligned} \tag{2}$$

in which $y(t) \in \mathbb{R}$ is the output variable, $u(t) \in \mathbb{R}$ is the input variable, $d(t) \in \mathbb{R}$ is an external disturbance, $Y \in \mathbb{R}^n$ is the system state vector, $b \in \mathbb{R}$ is a bounded variable that will be denoted by the input *control gain*, and $h(t) \in \mathbb{R}$ represents nonlinear function of the system. In this work, we use the notation $y^{(n)}$ to represent the n-th order time derivative of $y(t)$. The function $f(Y, d(t), h(t))$ in (2) is usually denoted in the literature by the plant *generalized disturbance* term (Han, 2009; Madoński *et al.*, 2015). In order to simplify notation, we will henceforth represent the function $f(Y, d(t), h(t))$ by $f(t)$.

For analysis purpose, let us detach the system linear part from $f(t)$ so that it can appear explicitly in the plant representation, as follows

$$y^{(n)} \quad = \quad \underbrace{a^T Y + g(t)}_{f(t)} + bu(t)\,, \tag{3}$$

in which $a = [-a_0,\ -a_1,\ \cdots,\ -a_{(n-1)}]^T$, $Y(t) = [y,\ \dot{y},\ \cdots,\ y^{(n-1)}]^T$ and $g(t) = d(t) + h(t)$. In (3), $a \in \mathbb{R}^n$ denotes the system constant parameters, whose elements can either positive, negative or null.

In (Han, 1998), it is proposed the ADRC, witch is a control strategy robust to external disturbance and unmodeled dynamics. This robustness is obtained using an Extended State Observer(ESO)(Miklosovic and Gao, 2004) to estimate $f(t)$. However, one has not the same robustness with respect to change in the control gain b. Here, it is presented the modified ADRC, witch is an extension, to plants with unknown control gain, of the standard ADRC strategy proposed in (Han, 1998). Besides that, the proposed extension approaches the trajectory tracking control problem. Before proceeding with the design procedures, some assumptions are assumed:

Assumption 1 *The disturbance signal $d(t)$ and the nonlinear function $h(t)$ are both bounded and have uniformly bounded first order derivatives $\forall t$:*

$$H > |h(t)|\,,\ \ D > |d(t)|\,,\ \ \bar{H} > |\dot{h}(t)|\,,\ \ \bar{D} > |\dot{d}(t)|\,, \tag{4}$$

in which $H,\ D,\ \bar{H},\ \bar{D}$ are known positive real constants.

Assumption 2 *The plant parameters $a_0, \cdots, a_{(n-1)}$ are uncertain and upper bounded by a constant $a_M > 0 \in \mathbb{R}$:*

$$a_M > |a_i|\,, \quad i = 0, \cdots, n-1\,, \tag{5}$$

Assumption 3 *The high-frequency gain b is uncertain, has known sign and is lower bounded by:*

$$b_m < |b|\,, \quad b_m > 0 \in \mathbb{R}\,. \tag{6}$$

Assumption 1 is assumed to guarantee limits to the signals $d(t)$ and $h(t)$ and also to their rates of change. Since these signals will be estimated by an Extended State Observer(ESO)(Miklosovic and Gao, 2004), such a priori knowledge is fundamental for the choice of the bandwidth of estimator of them. By assuming Assumptions 2 and 3, the present work aims to consider a more general class of plants that possess a full set of uncertain parameters. In fact, considering such reduced knowledge about the system is a important step toward performing less conservative statements in the design of the proposed controller.

4.1 Proposed methodology

The main idea proposed in this work is to perform a structural transformation on the original system (3), in particular concerning the input/output behavior, in order to obtain a new dynamical system with advantageous format. For this end, we introduce an adjustable constant gain $\beta = K_0 sign(b)$, with $K_0 > 0$, in series with the plant output error and a linear stable filter Q_0 in parallel with them, as shown in Fig. 2.

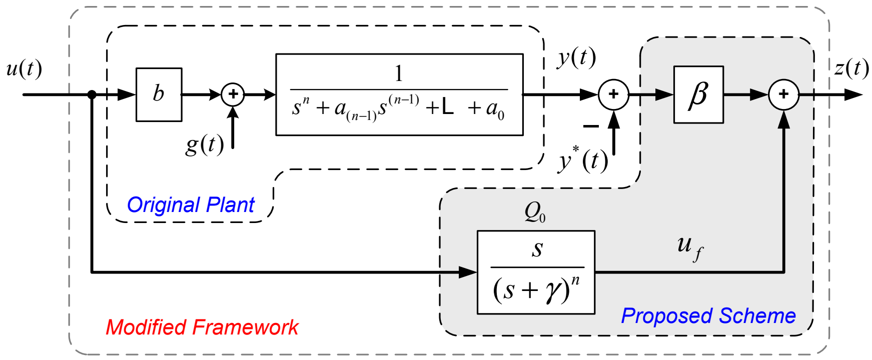

Figure 2 – Block diagram of the proposed solution.

In the present proposal, the positive design constant $\gamma \in \mathbb{R}$ is chosen such that $(s+\gamma)^n = s^n + \alpha_{(n-1)} s^{(n-1)} + \cdots + \alpha_0$ results in a stable polynomial. Based on the configuration of Fig. 2, the new output error can be written as:

$$z(t) = \beta e(t) + u_f(t)\,, \tag{7}$$

$$e(t) = y(t) - y^*(t)\,, \tag{8}$$

$$u_f^{(n)} = -\alpha^T \sigma_u + \dot{u}\,. \tag{9}$$

$$\alpha = [\alpha_0,\ \alpha_1,\ \cdots,\ \alpha_{(n-1)}]^T\,, \tag{10}$$

$$\sigma_u = [u_f,\ \dot{u}_f,\ \cdots,\ u_f^{(n-1)}]^T\,. \tag{11}$$

where y^* is a reference trajectory infinitely differentiable. By differentiating (7) n times, the dynamics of the new output error variable $z(t)$, with $b_p = \beta b$, will be given by:

$$z^{(n)} \;=\; \beta\,\underbrace{[a^T Y + g(t) + bu(t) - y^{(n)*}]}_{e^{(n)}} + u_f^{(n)} \;=\; \beta[a^T Y + g(t) - y^{(n)*}] - \alpha^T \sigma_u + b_p u(t) + \dot{u}(t)\,, \tag{12}$$

Detaching u_f from Eq. (7), we have that $u_f^{(i)} = z^{(i)} - \beta e^{(i)}$ where $i = 1, \cdots, n$. Then, using (10), we obtain:

$$z^{(n)} + \alpha^T Z(t) = \beta[a^T Y + g(t) - y^{(n)*}] + \beta\alpha^T e_p + \; b_p u(t) + \dot{u}(t)\,, \tag{13}$$

where $Z(t) = [z, \dot{z}, \cdots, z^{(n-1)}]^T$ and $e_p = [e, \dot{e}, \cdots, e^{(n)}]^T$.

In order to write the new description of the plant using the ADRC formalism, it is necessary to describe (13) as done in (3). Then, a new generalized perturbation function $\Omega(t)$ is defined as

$$\Omega(t) = \beta[a^T Y + g(t) - y^{(n)*}] + \beta\alpha^T e_p + b_p u(t)\,, \tag{14}$$

which reduces (12) to:

$$Z^{(n)} + \alpha^T Z(t) = \; \Omega(t) + \dot{u}(t)\,. \tag{15}$$

Note that the tracking problem $e(t) \to 0$ is now redefined in terms of the new output error $z(t)$. Besides that, the new control input $\dot{u}$ has unitary coefficient, meaning that the control law can be carried out without requiring the exact value of parameter b (3). Then, it is easy to conclude that a stabilizing control law is

$$\dot{u}(t) = -\Omega(t)\,.$$

However, since $\Omega(t)$ is not available, its estimative will be used, as it is shown in the next Section.

4.2 Extended State Observer (ESO) design

For (15), consider the following state variable definitions

$$\zeta(t) := \left[\zeta_1, \zeta_2, \cdots, \zeta_{(n+1)}\right]^T = [z(t), \dot{z}(t), \cdots, \Omega(t)]^T\,. \tag{16}$$

Assuming that $\Omega(t)$ is differentiable, the plant state-space representation of (15), in companion form, can be written as:

$$\dot{\zeta} = \underbrace{\begin{bmatrix} 0 & 1 & \cdots & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ -\alpha_0 & -\alpha_1 & \cdots & -\alpha_{(n-1)} & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}}_{A_m} \zeta \; + \underbrace{\begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \\ 0 \end{bmatrix}}_{B_\zeta} \dot{u} \; + \underbrace{\begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}}_{\Gamma} \dot{\Omega}(t), \tag{17}$$

$$z(t) = \underbrace{[1\; 0\; \cdots\; 0]}_{C}\,\zeta\,.$$

Since the pair (A_m, C) is always observable, a full-order ESO for (17) is then designed as follows:

$$\begin{cases} \dot{\hat{\zeta}} = A_m \hat{\zeta} + B_\zeta\; \dot{u} + L e_z\,, \\ \hat{z} = C\hat{\zeta}\,, \end{cases} \tag{18}$$

where $\hat{\zeta} \in \mathbb{R}^{(n+1)}$ represents the estimated state vector, $e_z := (z - \hat{z})$ is defined as the output estimation error and $L = [L_1\; L_2\; \cdots\; L_{(n+1)}]^T \in \mathbb{R}^{(n+1)}$ is the vector of the observer gains defined by $det[sI - (A_m - LC)] = (s + w_0)^{(n+1)}$ with $w_0 > 0$. By defining the ESO state error as

$$e_\zeta = \zeta - \hat{\zeta}\,, \tag{19}$$

the estimation error can be compute from (17), (18) and (19), resulting in:

$$\begin{cases} \dot{e}_\zeta = \underbrace{(A_m - LC)}_{\bar{A}_m} e_\zeta + \Gamma\,\dot{\Omega}(t)\,, \\ e_z = C e_\zeta\,. \end{cases} \tag{20}$$

Then, in order to compensate the disturbance term $\Omega(t)$, forcing the new error $z(t)$ to tend to zero, we propose the following control law:

$$\dot{u} = -\;\hat{\zeta}_{(n+1)}\,. \tag{21}$$

In the next section, the stability and convergence properties are carried out for the closed loop system obtained with the *Modified ADRC Framework*.

5 STABILITY ANALYSIS

By replacing the control law expression of (21) into (15), the closed loop dynamics for the error $z(t)$ becomes:

$$z^{(n)} + \alpha^T Z(t) \;=\; \Omega(t) - \hat{\zeta}_{(n+1)} \;=\; e_{\zeta(n+1)}\,. \tag{22}$$

Since the left-hand side of (22) corresponds to a linear, time-invariant and stable ODE, then a bounded $e_{\zeta(n+1)}$ will result in a bounded $z(t)$. Provided that the boundedness and convergence properties of the closed loop signals in (22) are dependent on the ESO (18) estimation error $e_{\zeta(n+1)}$, an investigation about it is needed. To verify the influence of the generalized disturbance term $\Omega(t)$ in the amplitude of the observer estimation error in (20), let us compute the transfer function from $\Omega(t)$ to $e_{\zeta(n+1)}$ by using:

$$G(s) = C_\Omega (sI - \bar{A}_m)^{-1}\Gamma, \tag{23}$$

with $C_\Omega = [0, \cdots, 0, 1]$. For a $(n+1)$-th order ESO, the computation of (23) can be easily accomplished by using math softwares with symbolic resources. However, due to the particular matrix structure of (20), such task becomes quite simplified. Thus, it can be concluded that the transfer function from $\Omega(t)$ to $e_{\zeta(n+1)}(t)$ assumes the following format:

$$e_{\zeta(n+1)} = \left[1 - \left(\frac{w_0}{s + w_0}\right)^{(n+1)}\right] \Omega(t)\,. \tag{24}$$

Remark 1 *Here, it is important to mention that the adoption of the mixed representation for both time and frequency domain quantities, in (24), is only for analysis purpose. In the cases in which $\Omega(t)$ (14) involves general nonlinear functions of the plant states, the definition of $\Omega(s)$ in frequency domain may be not consistent. So, as it is intended to analyze the amplitude behavior of the error signal in (24), without losing generality, we believe it is the most suitable input/output mathematical formalism to be utilized.*

Note, from (24), that the ESO (18) precision can be arbitrarily improved by increasing the absolute value of the characteristic roots at $-w_0$, which is the same of making the input/output equivalent gain $G(s)$ to be very close to zero. Since $e_{\zeta(n+1)} = \Omega(t) - \hat{\zeta}_{(n+1)}$, an interesting property that derives from (24) is the dynamical relation between the total disturbance term $\Omega(t)$ and its estimate $\hat{\zeta}_{(n+1)}$:

$$\hat{\zeta}_{(n+1)} = \underbrace{\left[\left(\frac{w_0}{s + w_0}\right)^{(n+1)}\right]}_{M(s)} \Omega(t)\,. \tag{25}$$

Inspired by the Bode Diagram of $M(s)$ (25), we define the following scaled relation between $\hat{\zeta}_{(n+1)}$ and $\Omega(t)$:

$$\hat{\zeta}_{(n+1)} = c_0\, \Omega(t)\,, \tag{26}$$

where $c_0 \in \mathbb{R}$ is introduced only for analysis purpose to represent a variable scaling factor whose value belongs to the interval $c_0 \in\,]0;1[$. Then, by rewriting (21), we have that

$$\dot{u} \;=\; -\,\hat{\zeta}_{(n+1)} \;=\; -\,c_0\Omega(t)\,. \tag{27}$$

From (14) and recalling that $e^{(n)} = \beta(a^T y + g(t) + bu - y^{(n)*})$, we have

$$\dot{u} = -c_0\beta\left(e^{(n)} + \alpha^T e_p\right)\,. \tag{28}$$

For the sake of clarity, the dynamical equation in (28) is represented by the block diagram of Fig. 3. Then, by rearranging the blocks, the diagram of Fig. 4 arises as a result, where the polynomial $P(s)$ is given by:

$$P(s) = s\left(s^n + a_{(n-1)}s^{(n-1)} + \cdots + a_0\right) + c_0\beta b(s+\gamma)^n\,. \tag{29}$$

Then, by expanding (29), we have that:

$$P(s) = s^{(n+1)} + \left[c_0\beta b + a_{(n-1)}\right]s^n + \left[c_0\beta b\alpha_{(n-1)} + a_{(n-2)}\right]s^{(n-1)} + \cdots + c_0\beta b\alpha_0\,. \tag{30}$$

As can seen from (30), some of the coefficients of $P(s)$ are linear combinations of the uncertain parameters a_i of the original plant (3) and of the known filter coefficients α_i, for $i = 0, \cdots, (n-1)$. To ensure the stability of $P(s)$, in the

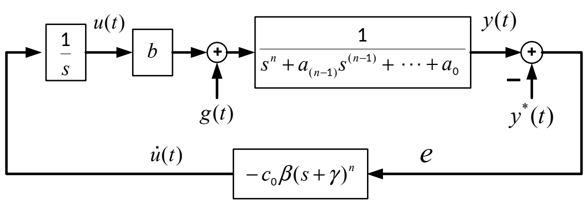

Figure 3 – Equivalent block diagram of Eq. (28).

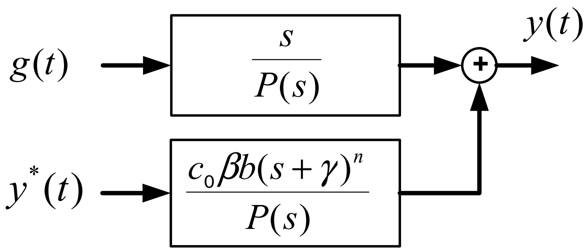

Figure 4 – Rearranged block diagram of Eq. (28).

presence of uncertain constants, we will choose β based on the constant upper and lower bounds a_M and b_m stated on Assumptions 2 and 3 respectively. Then,

$$\beta = K_0 sign(b) \;=\; \left(\frac{l_0 a_M}{b_m}\right) sign(b)\,, \tag{31}$$

in which $l_0 > 1$ is a free design constant. By replacing (31) in some of the coefficients of $P(s)$ in (30), we have that,

$$c_0\beta b + a_{(n-1)} = \left(\tfrac{c_0 l_0 a_M |b|}{b_m}\right) + a_{(n-1)}\,, \tag{32}$$

$$c_0\beta b\alpha_{(n-1)} + a_{(n-2)} = \left[\left(\tfrac{c_0 l_0 a_M |b|}{b_m}\right)\alpha_{(n-1)}\right] + a_{(n-2)}\,, \tag{33}$$

$$\vdots$$

$$c_0\beta b\alpha_0 = \left(\tfrac{c_0 l_0 a_M |b|}{b_m}\right)\alpha_0\,. \tag{34}$$

From (32)-(34), we can detach some important facts: (1) The term $|b|/b_m$ is greater than the unity; (2) The term $(a_M|b|)/b_m$ is greater than any a_i in (30); (3) By choosing l_0 (31) as a sufficiently large constant, the products $c_0\beta b\alpha_i$, $(i = 1,\cdots,n-1)$, can be made larger enough for dominating every uncertain a_i in (30). Then, by assuming l_0 is conveniently chosen, the polynomial $P(s)$ can be approximated by:

$$P(s) \;\approx\; s^{(n+1)} \;+\; c_0\beta b\,(s+\gamma)^n\,. \tag{35}$$

Moreover, if $c_0\beta b \;>>\; \gamma$ then (35) can be simplified even more to reach the form,

$$P(s) \;\approx\; (s + c_0\beta b)\,(s+\gamma)^n\,, \tag{36}$$

which is guaranteed to be stable. The approximation in (36) is addressed in Lemma 1 in the following, witch the proof is not shown by lack of space.

Lemma 1 *Consider a polynomial $(s+d_0)^n$ with $d_0 > 2\ \in \mathbb{R}$ and $n \in \mathbb{N}^*$. If $d_0 > n$, then there always exists a real constant*

$$B_0 > 10\left(\frac{d_0}{n}\right), \tag{37}$$

such that the polynomial

$$P_0(s) := s^{(n+1)} + B_0(s+d_0)^n\,, \tag{38}$$

can be approximated by

$$P_0(s) \approx (s+B_0)(s+d_0)^n\,. \tag{39}$$

Proof 1 *From (38), we know that*

$$(s+d_0)^n \;=\; S_q^T\alpha_q\,, \tag{40}$$

$$S_q \;=\; [s^n,\, s^{(n-1)},\cdots,\, 1]^T\,, \tag{41}$$

$$\alpha_q \;=\; [1,\, q_{(n-1)},\,\cdots,\, q_0]^T\,. \tag{42}$$

By expanding $P_0(s)$ from (39), based on (40), we obtain

$$(s+B_0)(s+d_0)^n = s(s+d_0)^n + B_0(s+d_0)^n = s^{(n+1)} + q_{(n-1)}s^n + \cdots + q_0 s + B_0 s^n + B_0 q_{(n-1)}s^{(n-1)} + \cdots + B_0 q_0 \tag{43}$$

$$(s+B_0)(s+d_0)^n = s^{(n+1)} + \left[B_0 + q_{(n-1)}\right] s^n + \left[B_0 q_{(n-1)} + q_{(n-2)}\right] s^{(n-1)} + \cdots + B_0 q_0 \,. \tag{44}$$

In (44), if B_0 is chosen sufficiently large, then dominant terms will arise inside the coefficients of $P_0(s)$ for $j = 1, \cdots, (n-1)$, i.e.

$$B_0 \gg q_{(n-1)} \,, \quad B_0 q_j \gg q_{(j-1)} \,. \tag{45}$$

Since (45) must hold for every j and n, let us analyze its viability. First of all, for the sake of simplicity, we assume that $d_0 > 2$ so that the values of α_q elements in (42) have an increasing pattern $1 < q_{(n-1)} < \cdots < q_0$. Thus, from (42) and according to the well-known Theorem of Binomial Coefficient Expansion, the three largest elements of α_q will be given by:

$$\underbrace{\left[\frac{n(n-1)}{2}\right] d_0^{(n-2)}}_{q_2} < \underbrace{n d_0^{(n-1)}}_{q_1} < \underbrace{d_0^n}_{q_0} \,. \tag{46}$$

So, by normalizing (46) by d_0^n, we have that:

$$\frac{n(n-1)}{2d_0^2} < \frac{n}{d_0} < 1 \,, \tag{47}$$

which reveals the additional sufficient condition $d_0 > n$ to be fullfilled. By expanding the normalization done in (47) to the other coefficients in (42), it is not difficult to verify that n/d_0 is the largest scaling factor that can occur between any two of them. By considering the worst case of (47), we obtain a more general sufficient condition than (45), namely :

$$B_0 \left(\frac{n}{d_0}\right) \gg 1 \,. \tag{48}$$

Then, without lost of generality, the choices of B_0 that can be suggested to satisfy (48) (and (45)) could be given by:

$$B_0 > 10 \left(\frac{d_0}{n}\right) . \tag{49}$$

Thus, once (48) is satisfied conveniently by (49), we can write from (44) ($j = 1, \cdots, (n-1)$), that

$$B_0 + q_{(n-1)} \approx B_0 \,, \quad \textit{and} \quad B_0 q_j + q_{(j-1)} \approx B_0 q_j \,, \tag{50}$$

and, then

$$(s+B_0)(s+d_0)^n \approx s^{(n+1)} + B_0 s^n + \left[B_0 q_{(n-1)}\right] s^{(n-1)} + \cdots + B_0 q_0 \approx s^{(n+1)} + B_0 (s+d_0)^n = (s+B_0)(s+d_0)^n \approx P_0(s). \tag{51}$$

Remark 2 *Note from the results of Lemma 1 applied to (32), that even without knowing exactly the values of the plant parameters (3), it is always possible to choose the design constant l_0 based on the upper and lower bounds assumed for those parameters, in order to guarantee the stability of the polynomial $P(s)$. Here, it is important to mention that, although the stability of $P(s)$ have been stated in the previous analysis by using conservative assumptions on the value of l_0, Lemma 1 has established an algorithm for choosing it in a less conservative manner.*

Remark 3 *The variable c_0 (26) introduced in the previous analysis is also an uncertain quantity. Although c_0 has influence in the choice of l_0, it does not affect the stability of $P(s)$ in (36) since it belongs to the interval $]0;1[$. Moreover, it is worthy remembering that, for accurate state estimation, it is always assumed that the speed of closed loop signals remain inside the observer bandwidth, far from its limit w_0. Thus, without lost of generality, one may assume $c_0 = 0.5$ in the design procedures, which is the lowest equivalent value for the mangitude of $M(s)$ (25) computed at w_0.*

5.1 Signals Boundedness analysis

From (36) and from the block diagram of Fig. 4, now we can state that:

$$y(t) = \underbrace{\left[\frac{c_0 K_0 |b|}{s + c_0 K_0 |b|}\right]}_{M_y(s)} y^*(t) + \left[\frac{s}{(s + c_0 \beta b)(s+\gamma)^n}\right] g(t) \,.$$

Note that $e^{(k)}(t)$, $k = 1, \cdots, n$, can be represented as

$$e^{(k)}(t) = [\bar{c}_0 - 1]\, y^{(k)*}(t) + \left[\frac{s^k}{(s + c_0 \beta b)(s+\gamma)^n}\right] g(t) \,, \tag{52}$$

in which $\bar{c}_0 \in]0;\ 1[$ is again an analysis variable that represents the possible equivalent magnitudes of $|M_y(s)|$ in the frequency interval $\omega \in [0,\infty[$. Notice that (52) can be written as:

$$e^{(k)}(t) \quad = \quad [\bar{c}_0 - 1]\, y^{(k)*}(t) + [1-\bar{c}_0] \left[1 - \frac{\gamma}{(s+\gamma)}\right]^{(k-1)} \times \left[\frac{1}{(s+\gamma)^{(n-k+1)}}\right] g(t)\,, \tag{53}$$

which, according to Assumption 1, implies in the following bound:

$$\|e^{(k)}(t)\|_\infty \quad < \quad \left|\frac{1-\bar{c}_0}{\gamma^{(n-k+1)}}\right| (H+D) + |1-\bar{c}_0|\ \|y^{(k)*}\|_\infty\,. \tag{54}$$

Thus, provided that y^* is an infinitely differentiable signal, we can conclude that $e^{(k)}(t)$ is bounded. However, since e_ζ is described by (20) and, rearranging (14), $\Omega(t) = \beta(e^{(n)} + \alpha^T e_p)$, then Ω, $\dot{\Omega}$ and the ESO estimate erros e_ζ are also bounded. In order to analyze the control signal $u(t)$, let us rearrange the blocks of Fig. 3 in order to detach the signal $\dot{u}$ as the diagram output and the signals y^*, $g(t)$ as the inputs. Then, after performing some block simplification, it is not difficult to find that:

$$u \quad = \quad \left(\frac{[-c_0\beta b](s+\gamma)^n}{P(s)}\right) \nu(t)\,, \quad \nu(t) = g(t) + \left[s^n + a_{(n-1)}s^{(n-1)} + \cdots + a_0\right] y^*\,, \tag{55}$$

which, by virtue of Lemma 1 and of Eq. (36), can be simplified to:

$$u(t) \;\approx\; \left(\frac{[-c_0\beta b]}{s + c_0\beta b}\right) \nu(t)\,. \tag{56}$$

As can be verified, the transfer function in (56) is BIBO stable. So, provided that $\nu(t)$ is formed by the uniformly bounded signals $g(t)$ (Assumptions 1) and $y^*, \dot{y}^*, \cdots,\ y^{(n)*}$, then we conclude that $u(t)$ is an uniformly bounded signal. Thus, we can conclude that the closed loop signals are all uniformly bounded $\forall t$. In the following, we present a list of steps for choosing the design parameters of the proposed ADRC controller. The main results of stability and convergence

Algorithm 1 Choosing design parameters

<u>Filter Q_0:</u>

1: In the filter Q_0 in (9) (Fig. 2), choose $\gamma > n$.

<u>Extended State Observer (ESO):</u>

1: From (4), compute the contants $H,\ D,\ \bar{H},\ \bar{D}$.
2: Compute the system largest speed $w_c = \max(\bar{H}, \bar{D}, 5\gamma)$.
3: Choose $w_0 \gg \omega_c$. We suggest $w_0 > 10w_c$.
4: Compute the ESO gain L by using $det[sI - (A_m - LC)] = (s + w_0)^{(n+1)}$.

<u>Output gain β:</u>

1: From (5) and (6), compute the constants a_M and b_m.
2: Define $c_0 = 0.5$, as suggested in Remark 3.
3: Based on (31), (35), (38) and (49), compute $K_0 \;>\; 20\left(\frac{\gamma\, a_M}{n\, b_m}\right)$.
4: Then, define β by using (31).

properties of the proposed modified ADRC scheme, are summarized in the following Theorem and Corollary.

Theorem 1 *(Output Tracking in General Uncertain Plants)*
Consider the output tracking control problem associated with the plant in (3), the extended state observer (ESO) designed in (18) and the control law defined in (21), in which $y^*(t)$ is the desired infinitely differentiable reference trajectory. Assume that Assumptions 1-3 are satisfied. If the design constants γ, K_0, w_0 are chosen based on Algorithm 1, then the following stability and convergence properties hold for the overall closed loop control system:

(i) All signals are uniformly bounded $\forall t$;

(ii) $\lim_{t\to\infty} |e^{(k)}(t)|$, for $k = 1\cdots n$, is upper bounded by:

$$\left|\frac{1-\bar{c}_0}{\gamma^{(n-k+1)}}\right| (H+D) + |1-\bar{c}_0|\ \|y^{(k)*}\|_\infty\,,$$

where $\bar{c}_0 \in]0;\ 1[$.

Corollary 1 *(Regulation in General Uncertain Plants)*
In Theorem 1, if the reference signal y^* is constant, then $\lim_{t\to\infty} |e^{(k)}(t)| \to 0$.

Proof 2 *Since a constant y^*, $\forall t \geq 0$, behaves like a step input, the equivalent amplitude of $|M_y(s)|$ (52) will be equal to its DC gain, which implies $\bar{c}_0 = 1$. So, the upper bounds for the system errors, stated in property (ii) of Theorem 1, become zero.*

SIMULATION RESULTS

In this section, we present and discuss simulation results of the proposed modified ADRC scheme applied to the electro-hydraulic actuator of Eq.(1), as depicted in Fig. 5. For the simulation, the dynamics of the overall control system

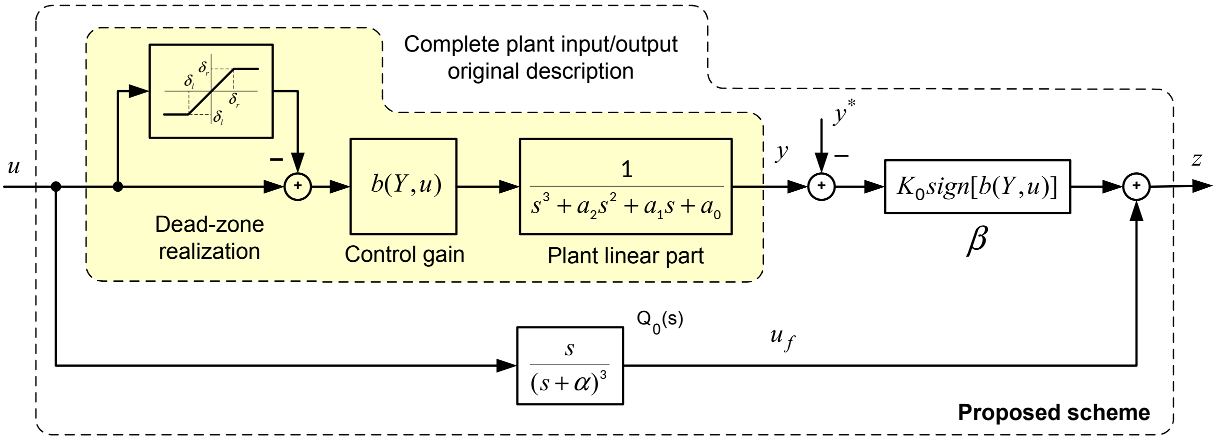

Figure 5 – Block diagram of the proposed (modified) ADRC scheme.

composed by: (i) the actuator in Eqs. (1), (ii) the extended observer (ESO) in Eq. (18), (iii) the filter Q_0 and the control law in Eq. (21), were all coded using Simulink^{TM} block programing. The parameters and design constants used in the simulation are described as follows. *For the plant* (Zachi *et al.*, 2017): $a_0 = 28$; $a_1 = 16838$; $a_2 = 93.73$; *For the observer*: $L_1 = 400$; $L_2 = 6\times10^4$; $L_3 = 4\times10^6$; and $L_4 = 10^8$; *For the control law*: $K_0 = \beta = 2.7$; *For the filter Q_0*: $\alpha_2 = 45$; $\alpha_1 = 675$ and $\alpha_0 = 3375$. The simulation sample time was 0.001 s. The curves in Fig. 6 illustrate the performance of the proposed ADRC strategy applied in tracking control problem in which the reference signal is given by Eq. (57). As predicted by the stability analysis (not presented in this extended abstract), the observer and the output errors both converge to a small residual region around zero.

$$y^*(t) = 0.005\ sin(0.2t)\ cos(0.5t)\ [\text{meters}]. \tag{57}$$

CONCLUSION

This work proposed a mathematical solution for the tracking control of an electro-hydraulic actuator with uncertain parameters, based on the Active Rejection Control method. The idea of the work was to develop an extension of the ADRC controller for uncertain systems in which the uncertainty in the control gain of the plant is also considered. To solve the problem of uncertainty over the original control gain of the plant, a modification in the original ADRC control scheme was proposed that consisted in the introduction of a constant gain in series with the system output error and a compensator in parallel with the plant. The central objective was to produce an input/output system equivalent to the original one but with a known control gain, without, however, changing the original control objective. An interesting feature of the developed strategy was the relaxation of the requirement of exact knowledge of the value of the original control gain of the plant. In this work, such requirement was reduced to the one of knowing its sign. The stability and convergence properties of the closed loop system were mathematically demonstrated and numerical simulation shown the good performance of the proposed control strategy.

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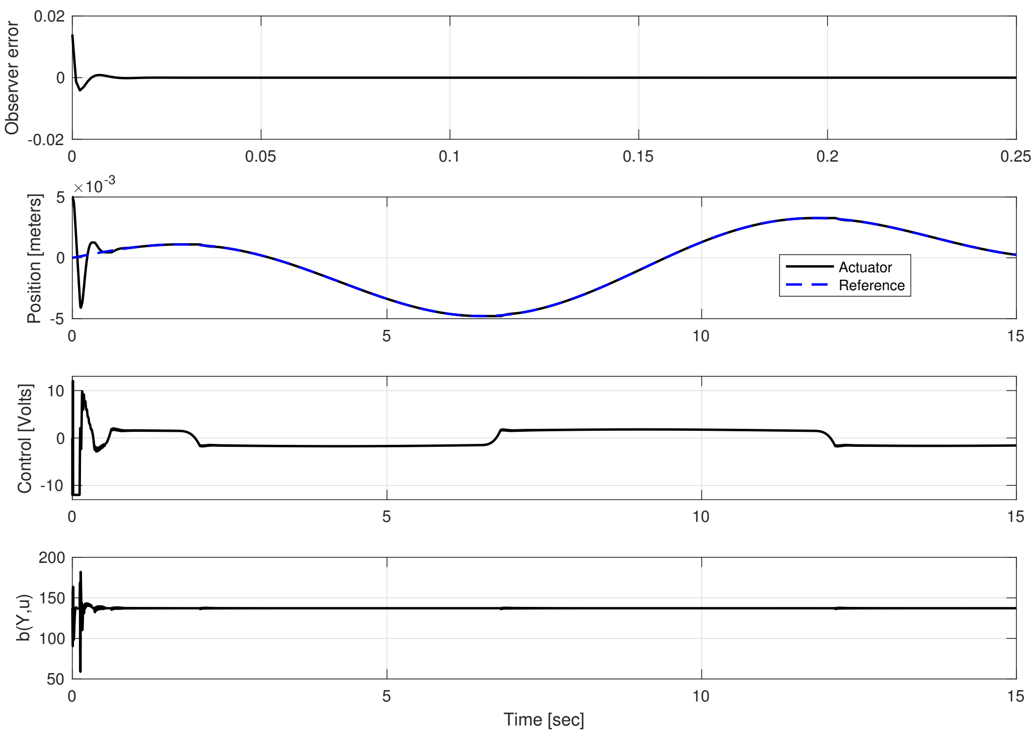

Figure 6 – Simulation results. Output tracking of the desired reference position y^*.

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