Non Linear Vibrations of Axially Loaded Viscoelastic Cylindrical Shells

Zenon del Prado [1], Nickollas Martins [2], and Paulo Gonçalves [1]

[1] Federal University of Goias, Schoool of Civil and Enviromental Engineering. , Av. Universitária, s/n, Setor Universitário, 74605-200, Goiânia, GO, Brazil

[2] Catholic University of Rio de Janeiro, Department of Civil Engineering, Rua Marquês de São Vicente, 225, Gávea, 22453-900, Rio de Janeiro, RJ, Brazil

Abstract: In this work the non linear vibrations of both axially and lateral loaded simply supported viscoelastic cylindrical shells are studied. For this the Donnell's non-linear shallow shell theory is used to model the shell, assumed to be made of a Kelvin-Voigt material type, and a modal solution with six degrees of freedom is used to describe the lateral displacements. The Galerkin method is applied to derive a set of coupled non-linear ordinary differential equations of motion. The influence of shell geometry, load and dissipation parameter are studied and special attention is given to parametric instability boundaries and bifurcation diagrams. Obtained results show that shell geometry together with the viscoelastic dissipation parameter have strong influence on the nonlinear dynamic behavior of the shells as displayed in parametric instability boundaries and bifurcation diagrams.

Keywords: Cylindrical shells, Viscoelastic Material, Nonlinear Vibrations

INTRODUCTION

Viscoelastic characteristics can be found in biological materials, elastomers and metals under high temperature. In literature, it can be found a large number of studies on elastic cylindrical shell dynamics but, just a small number of these works is related to the analysis of viscoelastic shells. Several mechanical models are used to describe viscoelastic material responses such as Maxwell, Kelvin-Voigt, Boltzman and Standard linear solid models.

Cheng and Zhang (2001) and Cederbaum and Touati (2002) studied the nonlinear dynamic behavior of viscoelastic cylindrical shells subjected to axial loads using the Von Kármán-Donnell non-linear shell theory. Later, Eshmatov and Khodjaev (2007a), Eshmatov (2007b), Eshmatov and Khodzhaev (2008) and Eshmatov (2009) studied vibrations and dynamic stability of viscoelastic cylindrical shells and cylindrical panels with and without concentrated masses using the Kirchhoff-Love hypothesis and Timoshenko theories by taking into account shear deformation and rotary inertia. The effect of lateral pressures on compressible non-linearly viscoelastic cylindrical and spherical shells under time-dependent pressures was studied by Antman and Lacarbonara (2009) and the thermal post-buckled characteristics of cylindrical composite shells with viscoelastic layers was studied by Shina et al (2009).

In this work, the influence of material properties and viscoelastic parameter on the non-linear vibrations of a perfect simply supported viscoelastic circular cylindrical shell subjected to an axial harmonic load was studied. Donnell's non-linear shallow shell theory is used to model the shell, which is assumed to be made of a Kelvin-Voigt material type, and a modal solution with six degrees of freedom which takes into account the essential modal couplings and interactions is used to describe the lateral displacements of the shell. The Galerkin method is applied to derive a set of coupled non-linear ordinary differential equations of motion that are, in turn, solved by the Runge-Kutta method.

MATHEMATICAL FORMULATION

Consider a thin-walled simply supported circular cylindrical shell of radius R, length L and thickness h. The axial, circumferential and radial coordinates are denoted by x, $y = R\theta$ and z and the corresponding displacements of the shell middle surface are denoted by u, v and w, as shown in Fig. 1. The shell is assumed to be made of a Kelvin-Voigt viscoelastic material with initial Young's modulus E, Poisson ratio ν, and density ρ.

The shell is subjected to an axial harmonic load:

$$\overline{N}_x = P_d \cos(\omega_f t) \tag{1}$$

where P_d is the coefficient of the amplitude of the load, ω_f is the frequency of the load and t the time.

The viscoelastic behavior of the material is modeled in the base of the Kelvin-Voigt viscoelastic theory. Considering the plane stress problem and the Kelvin-Voigt constitutive model of a viscoelastic material, the stress-strain relations can be written as:

$$\sigma_{xx} = E/\left(1-\nu^2\right)\left[\varepsilon_{xx} + \nu\,\varepsilon_{\theta\theta} + \eta\left(\varepsilon_{xx} + \nu\,\varepsilon_{\theta\theta}\right)_{,t}\right] \tag{2}$$

$$\sigma_{\theta\theta} = E/\left(1-\nu^2\right)\left[\varepsilon_{\theta\theta} + \nu\,\varepsilon_{xx} + \eta\left(\varepsilon_{\theta\theta} + \nu\,\varepsilon_{xx}\right)_{,t}\right] \tag{3}$$

$$\sigma_{x\theta} = E/\left[2\left(1+\nu\right)\right]\left[\gamma_{x\theta} + \eta\left(\gamma_{x\theta}\right)_{,t}\right] \tag{4}$$

where ε_{xx}, $\varepsilon_{\theta\theta}$ and $\gamma_{x\theta}$ are the strain components at an arbitrary point of the shell and η is the coefficient of the viscoelastic dissipation parameter measured in seconds.

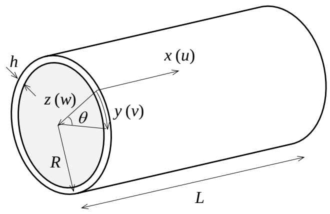

Figure 1 - Shell characteristics

The non-linear equation of motion, based on the Donnell shallow shell theory, in terms of a stress function *F*, the lateral displacement *w* is given by:

$$\frac{Eh^3}{12\left(1-\nu^2\right)}\left[\frac{\partial^4 w}{\partial x^4} + \frac{2}{R^2}\frac{\partial^4 w}{\partial x^2\,\partial\theta^2} + \frac{1}{R^4}\frac{\partial^4 w}{\partial\theta^4}\right] + \frac{Eh^3}{12\left(1-\nu^2\right)}\eta\frac{\partial}{\partial t}\left[\frac{\partial^4 w}{\partial x^4} + \frac{2}{R^2}\frac{\partial^4 w}{\partial x^2\,\partial\theta^2} + \frac{1}{R^4}\frac{\partial^4 w}{\partial\theta^4}\right] + \rho h\frac{\partial^2 w}{\partial t^2} =$$
$$\frac{1}{R^2}\frac{\partial^2 F}{\partial\theta^2}\left(\frac{\partial^2 w}{\partial x^2}\right) - 2\frac{1}{R^2}\frac{\partial^2 F}{\partial x\partial\theta}\left(\frac{\partial^2 w}{\partial x\partial\theta}\right) + \frac{\partial^2 F}{\partial x^2}\left(\frac{\partial^2 w}{R^2\partial\theta^2} + \frac{1}{R}\right) \tag{5}$$

The compatibility equation is given by

$$\frac{\partial^4 F}{\partial x^4} + \frac{2}{R^2}\frac{\partial^4 F}{\partial x^2\,\partial\theta^2} + \frac{1}{R^4}\frac{\partial^4 F}{\partial\theta^4} = E\,h\Bigg[-\frac{1}{R}\frac{\partial^2 w}{\partial x^2} + \frac{1}{R^2}\left(\frac{\partial^2 w}{\partial x\partial\theta}\right)^2 - \frac{1}{R^2}\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial\theta^2}$$
$$+\eta\left(-\frac{1}{R^2}\frac{\partial^3 w}{\partial x^2\partial t}\frac{\partial^2 w}{\partial\theta^2} - -\frac{1}{R^2}\frac{\partial^2 w}{\partial x^2}\frac{\partial^3 w}{\partial\theta^2\partial t} - \frac{1}{R}\frac{\partial^3 w}{\partial x^2\partial t} + \frac{2}{R^2}\frac{\partial^3 w}{\partial x\partial\theta\partial t}\frac{\partial^2 w}{\partial x\partial\theta}\right)\Bigg] \tag{6}$$

Where the stress function *F* is composed by a homogeneous and particular terms given by:

$$F = f_h + f_p \tag{7}$$

The homogenous component is then

$$f_h = \frac{1}{2}\overline{N}_x R^2\theta^2 \tag{8}$$

The lateral expansion of displacements *w*, satisfying the out-of-plane boundary conditions in terms of the circumferential and axial variables [15], is given by:

$$w = \xi_1(t)h\sin\left(m\pi x/L\right)\cos\left(n\theta\right) + \xi_2(t)h\sin\left(m\pi x/L\right)\sin\left(n\theta\right) + \xi_3(t)h\sin\left(m\pi x/L\right)$$
$$+\xi_4(t)h\sin\left(3m\pi x/L\right) + \xi_5(t)h\sin\left(5m\pi x/L\right) + \xi_6(t)h\sin\left(7m\pi x/L\right) \tag{9}$$

where $\xi_1(t)$, $\xi_2(t)$, $\xi_3(t)$, $\xi_4(t)$, $\xi_5(t)$ and $\xi_6(t)$ are the time-dependent non-dimensional modal amplitudes.

NUMERICAL RESULTS

Consider a simply supported viscoelastic cylindrical shell with the following physical properties: ρ = 1340.0 kg/m^3, ν = 0.195, E = 45.5e9 N/m^2. For this shell, three different geometries were selected looking to understand the effect of the viscoelastic parameter on the dynamic nonlinear response. Table 1 displays the selected shells geometries and their natural frequencies.

Table 1 – Shell selected geometries, natural frequencies, circumferential waves and longitudinal halfwaves

	L (m)	*R* (m)	*h* (m)	*L/R*	*R/h*	ω_o (rad/sec)	*m*	*n*
Case 1	0.4	0.2	0.002	100	2	3165.0	1	5
Case 2	0.4	0.4	0.002	200	1	2252.0	1	8
Case 3	0.4	0.2	0.0005	400	2	1597.0	1	7

The viscoelastic dissipation parameter η was assumed to have three different values1.0e-4 s, 5e-5s and 1.0e-5 s and, for each shell the parametric instability boundaries and bifurcation diagrams were plotted. The bifurcation diagrams were obtained using the force brute method, considering the excitation amplitude as control parameter.

Now, Figure 2 displays the parametric instability boundaries for the three adopted Cases and considering the assumed dissipation parameters, in all boundaries, the frequency parameter Ω was normalized considering the natural frequency of the shell for each Case and the load parameter Γ was normalized considering the axial critical load of each Case.

As can be observed, it is possible to see the principal and secondary instability regions, a small change on the value of the dissipation parameter strongly affects the instability parameter boundary and, when this value is reduced the boundary is shifted down. This behavior means that the instability loads of the shell are also changed and, depending on the values of the dissipation parameter, the instability may occur earlier. Then, the dissipation parameter affects mainly the principal and secondary wells been deeper for smaller values of η also, it can be seen that the peak which divides the principal and secondary wells, is not highly affected by the dissipation parameter and, only in Fig. 2b considering $\eta = 1e\text{-}5s$ and when $\Omega = 1.2$, the boundary becomes fractal with very different boundary if compared to other boundaries obtained with higher dissipation parameter.

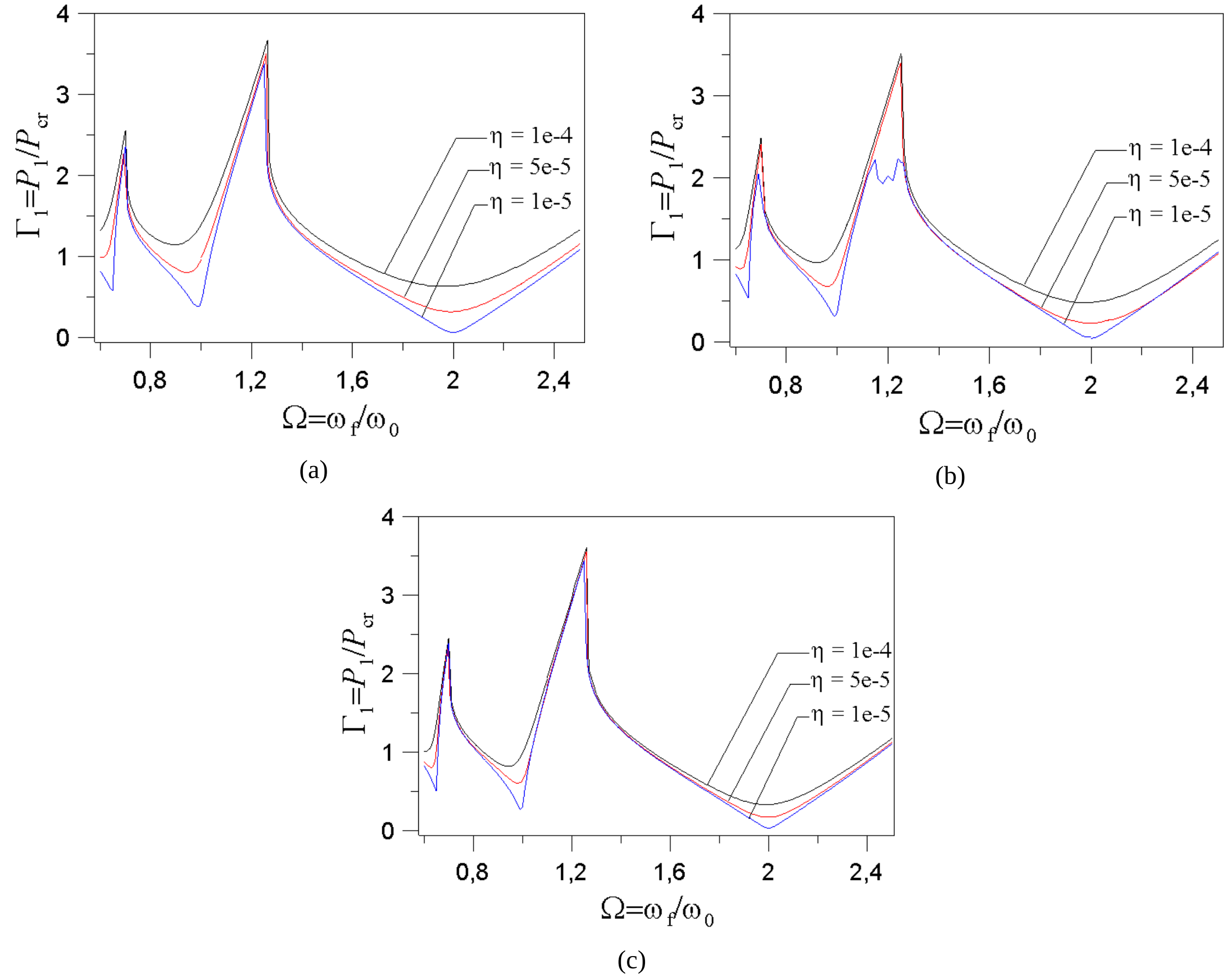

Figure 2 – Parametric instability boundaries. a) Case 1, b) Case 2 and c) Case 3

Figure 3 shows the bifurcation diagrams in the principal region of parametric instability (Ω = 2.0) of Fig. 2a. It is possible to observe that for low values of the load parameter Γ, the shell displays trivial solutions; as the load parameter is increased, there is a critical point where the shell jumps to small amplitude oscillations and there is the co-existence of small and large amplitude oscillations. If the load parameter is further increased, small amplitude oscillations disappear and only large amplitude oscillations will remain. In Fig. 3a, plotted for η = 1e-4, the region where small and large amplitude oscillations coexist is smaller than observed in Figs. 3b and 3c, that were obtained for smaller values of η. Then, the dissipation parameter has strong influence on the bifurcation point and , mainly, on the nonlinear vibrations of the shell.

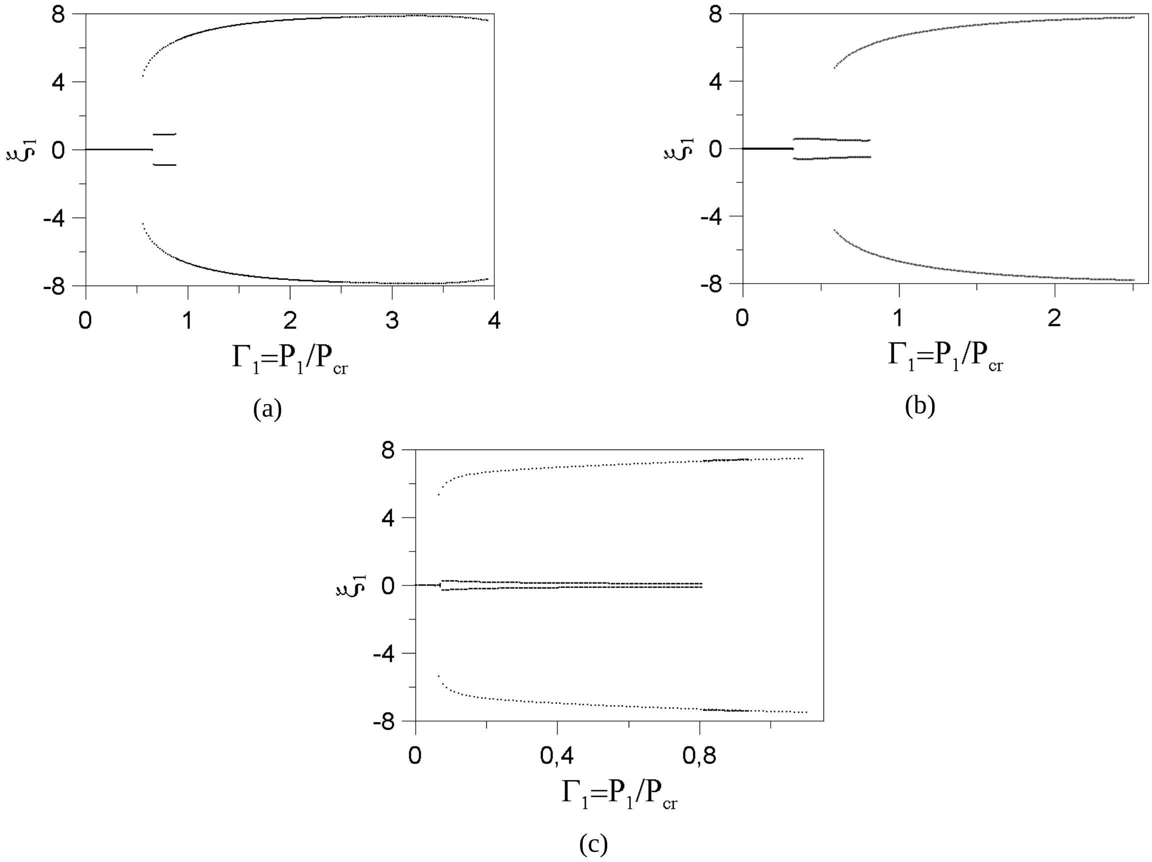

Figure 3 – Bifurcation diagrams Case 1, Ω = 2.0. a) η = 1e-4; b) η = 5e-5 and c) η = 1e-5

Figure 4 displays the bifurcation diagrams in the principal region of parametric instability (Ω = 2.0) of Fig. 2b. As can be observed, for low values of the load parameter Γ, the shell displays trivial solutions; as the load parameter is increased, there is a critical point where the shell shows super-critical bifurcations with small amplitude 2T oscillations as well as the coexistence with large 2T oscillations. After a critical value of the load parameter Γ, only large amplitude vibrations remain in the shell and, in Fig. 4b, for large values of the load parameter, it is possible to observe a region with chaotic large amplitude vibrations. It is possible to see, the influence of the dissipation parameter because, the region of coexistence of small and large amplitude oscillations depends on the value of η then, for higher values of η, the coexistence region will be smaller than for low values of η (Fig. 4b and Fig. 4c).

Finally, Fig. 5 shows the bifurcation diagrams in the principal region of parametric instability (Ω = 2.0) of Fig. 2c. As can be observed, after trivial solution, the shells displays super-critical bifurcation with small amplitude 2T oscillations and a window of very large 2T oscillations. The bifurcation diagram of Fig. 5b, obtained for η = 5e-5, displays chaotic large amplitude vibrations for high values of the load parameter; in similar way, the bifurcation diagram of 5c, obtained for η = 1e-5, the shell displays a period doubling window and after, large amplitude chaotic oscillations.

In all cases, the jumps, small or large amplitude vibrations as well as chaotic vibrations depend on the values of dissipation parameter as on the geometry relations affecting directly the non-linear dynamic behavior of the shell.

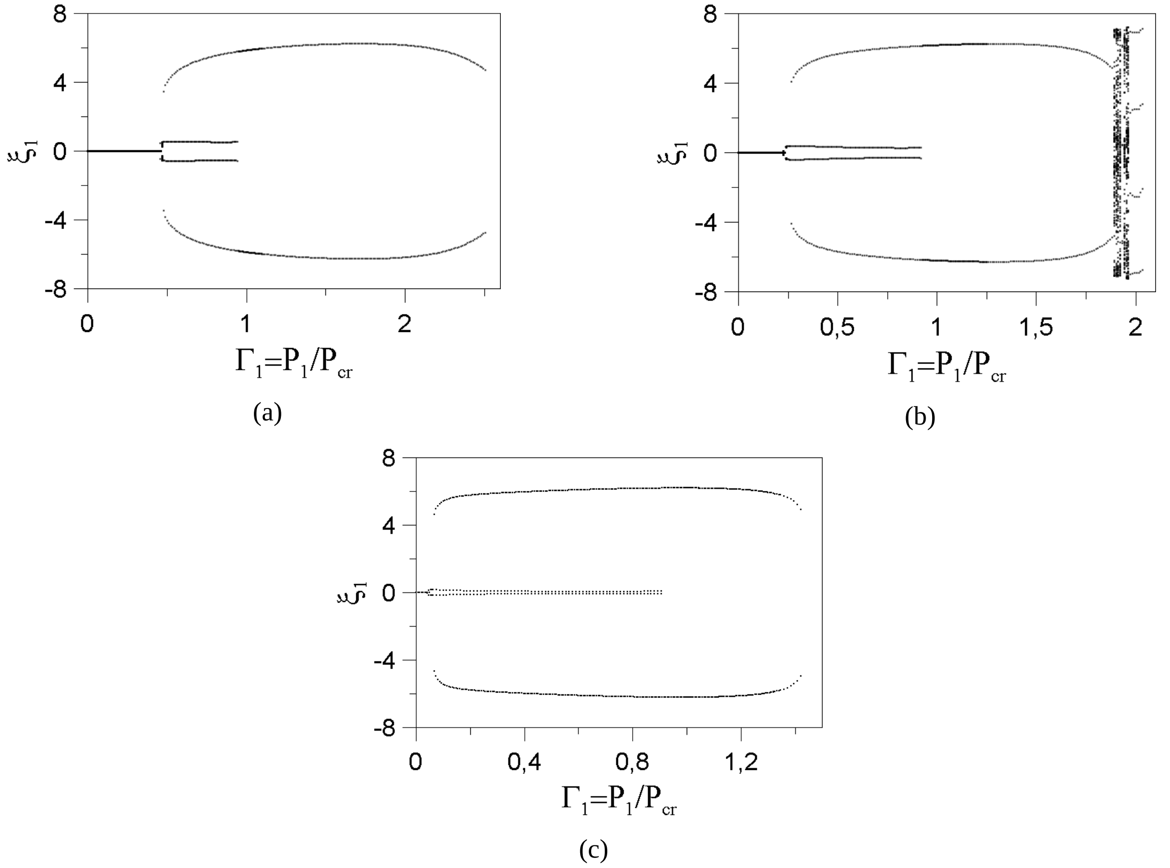

Figure 3 – Bifurcation diagrams Case 2, Ω = 2.0. a) η = 1e-4; b) η = 5e-5 and c) η = 1e-5

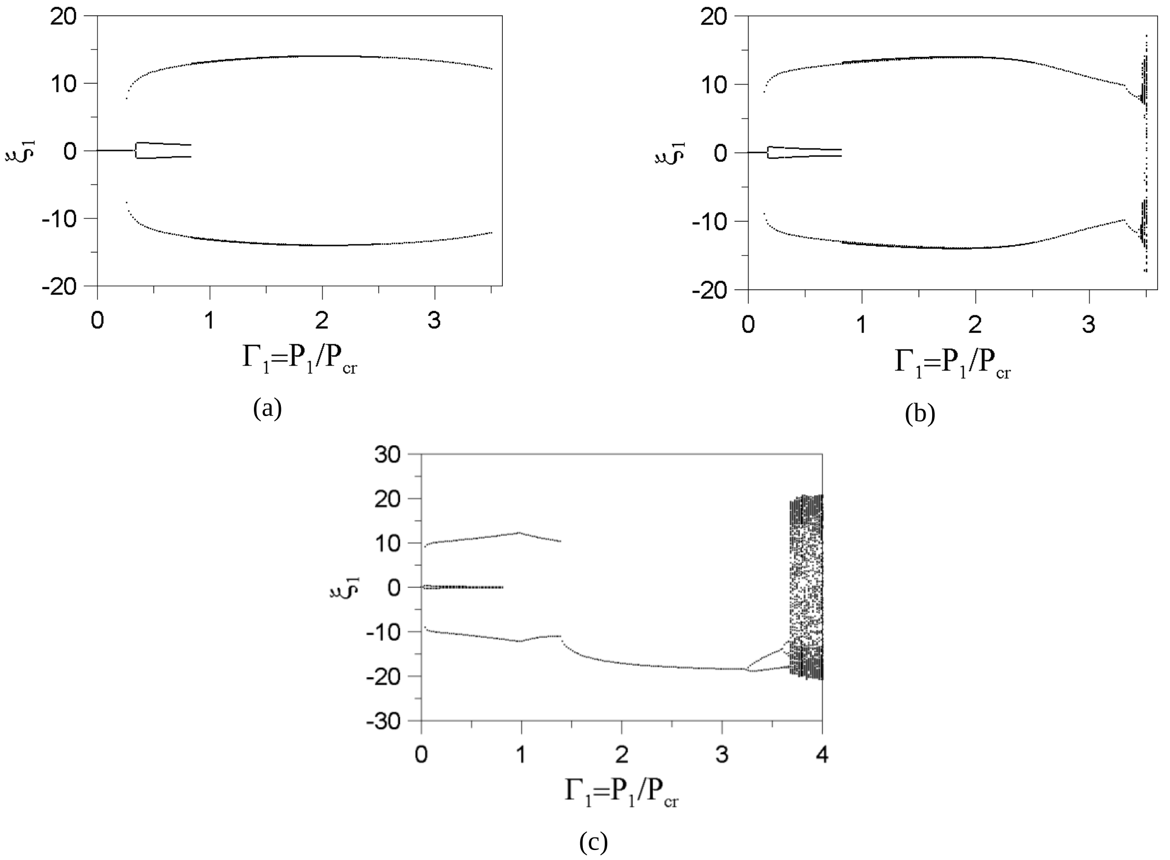

Figure 4 – Bifurcation diagrams Case 3, Ω = 2.0. a) η = 1e-4; b) η = 5e-5 and c) η = 1e-5

CONCLUSIONS

In this work, the non-linear vibrations analysis of a perfect viscoelastic Kelvin-Voigt simply supported cylindrical shell subjected to axial harmonic axial loads is analyzed. To model the shell, the Donnell's non-linear shallow shell theory is applied and an expansion with six degrees of freedom is used to describe the lateral displacements.

Parametric instability boundaries and bifurcation diagrams were obtained and, as observed. results show that a small change on the value of the viscoelastic dissipation parameter η of the Kelvin-Voigt material, strongly affects the parametric instability boundaries, bifurcation loads and type of post-critical paths. It is possible to observe that the complexity of the non-linear response, the non-linear paths and coexisting solutions depend on the external load, geometry relations and on the value of the viscoelastic dissipation parameter.

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