Chaos Control of a Smart Structure Using SMA Elements and Constrained Actuation

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Abstract: Shape memory alloys (SMAs) have been widely used in the smart structures creating adaptive behaviour that is interesting for several engineering applications. SMA thermomechanical behaviour can provide vibration attenuation or maintain the structure on a desired dynamical response. The two bar truss is an archetypal model to represent smart structures being investigated in this paper. Delayed feedback control method is employed providing low power consumption control that is able either to stabilize unstable periodic orbits or to promote bifurcation control. Thermal constraints are investigated, showing the real possibilities of this kind of control. Ideal and constrained thermal actuators are implemented via the SMA elements on the structure. Numerical results show situations related to controller constraints, defining its range of applicability.

Keywords: chaos control, smart structures, shape memory alloys.

INTRODUCTION

Shape memory alloys (SMAs) have been used as sensor and actuators on the so-called smart structures. The main purpose of these structures is to exploit adaptive behaviour that can be applied in different situations. An archetypal model of SMA smart structure is the two bar truss, known as the von Mises truss. It is usually employed to investigate stability of frame structures, flat arches and other structures that can be associated with bifurcation buckling. Its dynamics can represent a huge range of behaviours (Bazant and Cedolin, 1991).

The use of shape memory alloy (SMA) elements on structural systems can provide some desired results as vibration dissipation and adaptability. Nevertheless, constitutive nonlinearities introduce equilibrium points and make the dynamical response much more richer, leading to complex behaviours that include chaotic responses. The case of two-bar truss is of special complexity due to a combination of geometrical and constitutive nonlinearities. Therefore, the use of SMA elements needs to be preceded by a deep nonlinear dynamics analysis and control strategies can be useful in order to exploit its adaptive behaviour. Nonlinear dynamics of SMA two-bar truss was previously analysed by Savi *et al.* (2002) and Savi and Nogueira (2010).

Chaos control methods have the main objective to stabilise unstable periodic orbits (UPOs) embedded in the chaotic attractor, with a low energy cost. The extended time delay feedback control (ETDF) (Socolar *et al.*, 1994) is one of these methods that has been implemented on mechanical systems (Costa and Savi, 2018; de Paula and Savi, 2009). Configuration changes can be achieved by altering the targeted orbit, providing flexibility to the system.

Literature presents some articles that investigate the control of these smart structures, in particular the two-bar truss (Bessa *et al.*, 2009; de Paula *et al.*, 2014). Nevertheless, actuation is always considered as an ideal one, meaning that heat transfer constraints are not contemplated. This work deals with the application of the extended time delay feedback control applied to a two-bar shape memory alloy truss. The control strategy is used either to supress the chaotic behaviour or to avoid bifurcations. Thermal actuation is of concern and heat transfer constraints are incorporated on the controller. Based on that, two different scenarios are treated: an ideal one, where thermal actuation has not constraints; and a constrained one, where heat transfer limits SMA temperatures.

MATHEMATICAL MODEL

The two-bar truss is composed by two connected identical bars, free to rotate along their connection and their joint as can be seen on Figure 1. The bars have an angle ϕ with the horizontal plane and their bases are separated by a distance $2B$. The mass of the system is considered to be a lumped mass m at the connection between the two bars. A concentrated force $F(\mathrm{t})$ is applied to the connection. The equation of motion for the distance between the tip of the bars and the horizontal plane of their bases X is given by:

$$m\ddot{X} = -2P\sin(\phi) + F(t) \tag{1}$$

where P is the force given by the reaction of the two bars and depends on their constitutive model.

The two bars are considered to be composed of SMA and provide the thermal actuation needed to control the system. The SMA thermomechanical behaviour is described by a polynomial model that considers a Helmholtz free energy proposed by Falk (1980), resulting in the following a uniaxial stress σ:

$$\sigma = a_1\,[T - T_M]\epsilon - a_2\epsilon^3 + a_3\,\epsilon^5 \tag{2}$$

where $a_1, a_2,$ and a_3 are material parameters, T_M is the temperature below which only martensite is stable on a free stress configuration and ϵ is the strain. If T_A is defined as the temperature where the polynomial potential has only one equilibrium point it can be expressed as:

$$T_A = T_M + \frac{1}{4}\frac{a_2^2}{a_1 a_3} \tag{3}$$

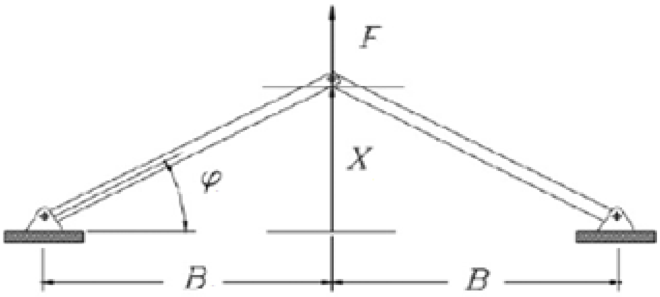

Figure 1 – Schematics of the two bar truss.

The strain of the two bars of length L with respect to a reference length L_0 can be given by:

$$\epsilon = \frac{L}{L_0} - 1 = \frac{\cos(\phi_0)}{\cos(\phi)} - 1 \tag{4}$$

where ϕ_0 is a reference angle defined by the reference length L_0.

Taking into consideration Eq. (2), Eq. (3) and Eq. (4) on Eq. (1), a cross-sectional area A, and introducing a harmonic external forcing $F = \Gamma\sin(\Omega t + \beta)$, where Γ is its amplitude, Ω its frequency and β is an initial phase of the bars. The system equation of motion can be expressed in non-dimensional units as:

$$\begin{aligned} x'' = \gamma\sin(\omega t + \beta^*) - \zeta x' + x\{&-[\theta - 1 - 3\alpha_2 + 5\alpha_3] \\ &+[\theta - 1 - \alpha_2 + \alpha_3](x^2 + b^2)^{-\frac{1}{2}} - [3\alpha_2 - 10\alpha_3](x^2 + b^2)^{\frac{1}{2}} \\ &+[\alpha_2 - 10\alpha_3](x^2 + b^2) + 5\alpha_3(x^2 + b^2)^{\frac{3}{2}} - \alpha_3(x^2 + b^2)^2\} \end{aligned} \tag{5}$$

where γ is a non-dimensional amplitude, ζ is an non-dimensional viscous dissipation, ω is an non-dimensional angular velocity $\omega = \frac{\Omega}{\Omega_0}$, $\beta^* = \frac{\beta}{\Omega_0}$, $x = \frac{X}{L_0}$, $t^* = t\Omega_0$, $\Omega_0 = \frac{2A\,a_1\,T_M}{mL_0}$, $b = \frac{B}{L_0}$, $\alpha_2 = \frac{a_2}{a_1 T_M}$, $\alpha_3 = \frac{a_3}{a_1 T_M}$, $\theta = \frac{T}{T_M}$ and $(\ )' = \frac{\mathrm{d}}{\mathrm{d}t^*}$.

The controller considers a thermal actuation force provided by the SMA elements with the following form:

$$F_{act} = x\left\{[\theta - \theta_0]\left([x^2 + b^2]^{-\frac{1}{2}} - 1\right)\right\} \tag{6}$$

where θ_0 is a reference temperature and it is assumed a homogeneous temperature of the bar.

Two kinds of controllers are treated. The ideal controller has the temperature as its accessible variable and can provide any force needed by the control strategy; and the constrained controller that changes SMA temperature with an accessible current via Joule effect and has its temperature variations constrained by the energy equation and current range. Therefore, the ideal actuator can provide any temperature and control force, while the constrained actuator has limitations defined by heat transfer.

Based on that, the constrained controller has a convective dissipation and a source of heat due to the Joule effect, associated with a current I. Hence, the non-dimensional energy equation for the bar can be defined as:

$$\theta' = h^*(\theta_{amb} - \theta) + R_r I^2 \tag{7}$$

where h^* is non-dimensional convective coefficient, R_r is a resistance of the bars and θ_{amb} is the non-dimensional ambient temperature.

The constrained controller has I as its accessible parameter. Hence, Eq. (7) limits the accessible temperatures and the possible actuation force. The first part of Eq. (7) governs the cooling processes as h^* defines how fast it can happen and cannot be accessed by the controller. The second part of the equation, on the other hand, controls heating and can be accessible by the current I that is limited to a maximum value of 10. Both controllers deal with the geometric restrictions using the same strategy: when x approaches these values the controller is turned off and after x leaves the vicinity of these values the controller is turned on again.

The controller is based on the extended time delayed feedback control (ETDF) (Socolar *et al.*, 1994) assuming that velocity x' is the observable variable, as well as the only accessible state to apply the control. Under this assumption, the control signal $g \in \mathbb{R}$ becomes:

$$g = K\left[[1-R]\sum_{j=1}^{\infty} R^{j-1}x'(t-j\tau) - x'(t)\right] \tag{8}$$

where K is a proportional gain, τ is a delay equal to the targeted UPO period and R is a parameter that balances the influence of the delayed states. Hence, for the ideal controller the actuation force is equal to the control signal ($F_{act} = g$), while the constrained controller can have a difference between its control force and the proposed control signal $e = (F_{act} - g)$.

NUMERICAL SIMULATIONS

Numerical simulations are carried out employing the forth order Runge-Kutta method with a time step of $\Delta t^* = \pi/1000\omega$, which ensures an relative error of the state space modulus lesser than 10^{-6}. The employed parameters are: $\alpha_2 = 1.2410^2$, $\alpha_3 = 1{,}4510^4$, $T_{tamb} = 293.15$ K, $b = 0.866$, $R_r = 20$ A^{-1}, $T_M = 288$ K, $\theta = 1.1$, $\omega = 0.5$, $h^* = 0.7$ and $\zeta_t = 0.05$. All other parameters are defined in each one of the different results.

The two-bar truss presents a chaotic behaviour considering uncontrolled response and $\gamma = 0.02$. It is confirmed by the Lyapunov exponents: $\lambda_1 = 0.1172$ bit/s and $\lambda_2 = -0.1854$ bit/s. The Poincaré section of this behaviour is presented in Fig. 2 for two values of the phase β^*. This result indicates the symmetry of the system with respect to the transformation $x \rightarrow -x$.

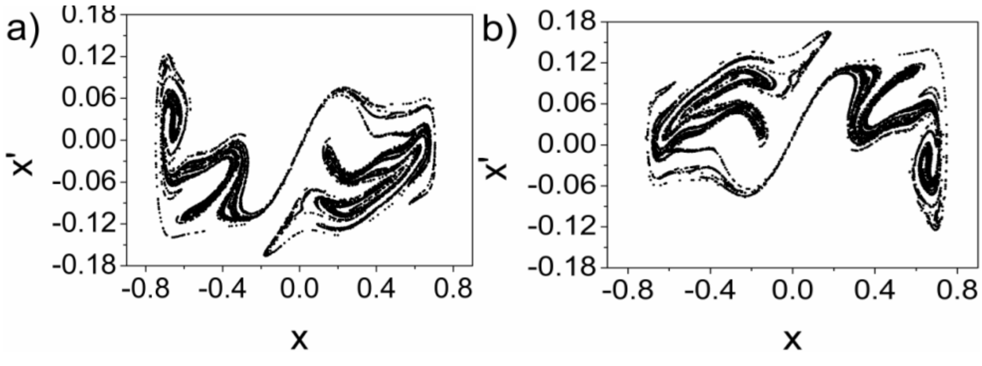

Figure 2 – Poincaré section for the chaotic behaviour for $\gamma = 0.02$. a) $\beta^* = 0$. b) $\beta^* = \pi$.

The analysis of unstable periodic orbits embedded in the chaotic attractor is now developed considering the recurrent points method (Auercach *et al.*, 1987). Among the UPOs, it is possible to show two different period-1 symmetric orbits. The ideal controller can stabilize these UPOs and the constrained controller is able to achieve stabilization depending on heat convection coefficient, presenting values that it works similar to the ideal one.

ETDF can be employed as a bifurcation control to avoid changes on the structure dynamical behaviour. Uncontrolled bifurcation diagram is compared with the two controllers diagrams, Fig. 3. The initial conditions of the system are $x_0 = 0.5$, $x'_0 = 0$, $\theta = 1.1$ and the parameter $\gamma = 0$. The slow quasi-static increase of γ is promoted in order to evaluate the system dynamics. The controllers approximate the infinite sum of Eq. (8) for its ten first terms; hence they are turned on after $t > 10\tau$ and stay on for all the rest of the simulation. The controller parameters $K = 0.2$ and $R = 0.2$ are set by the calculation of the Floquet exponents of the period-1 UPO following the procedure by Costa and Savi (2018).

Figure 3 compares the bifurcation diagrams for uncontrolled, ideal controller and constrained controller. Initially, the uncontrolled system has a periodic response and after some increase in γ it changes to another period-1 orbit. Further increases in γ push the uncontrolled system to a chaotic behaviour restricted to positive values of x. Before the parameter reaches $\gamma = 0.02$, the chaotic region of the uncontrolled system expands to negative values of the position x. Finally, at $\gamma = 0.0325$, the chaotic behaviour is replaced to a high amplitude periodic orbit. The controlled responses are different. Both constrained and ideal controller stays on a periodic response until they change to another period-1 response around $\gamma \sim 0.012$, and, afterward, stay on that orbit for the remaining values. Constrained and ideal controllers behaviours are the same, showing that the control constraints do not significantly influence the controller actuation under these conditions. However, for lower values of the parameter h^*, the constraints may influence the UPO stability.

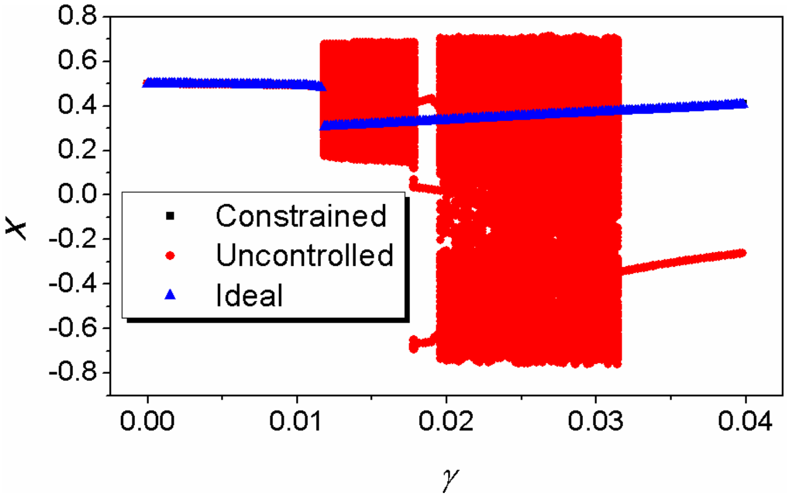

Figure 3 – Bifurcation diagram varying γ for both controlled systems and uncontrolled.

CONCLUSIONS

Chaos control of SMA two-bar truss is investigated using the ETDF approach. Constrained and ideal controllers are investigated. Results show that thermal actuation can be successfully employed to stabilize UPOs embedded in chaotic attractor. Besides, this approach can be employed for bifurcation control. The constrained and ideal controllers show similar results depending on the convection coefficient.

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