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MODELING OF MUTUAL HEAT TRANSFER BY THERMAL RADIATION IN FINS

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Abstract. *The present work describes the thermal profile of double fins, where their surfaces interact thermally, generating mutual effects, perpendicular to their primary surface and in steady state, which dissipates heat by conduction, convection and thermal radiation. Neumann and Dirichlet boundary conditions are established, characterizing that heat dissipation occurs only on the fin faces, in addition to predicting that the ambient temperature is homogeneous. Heat transfer analysis is performed by computational simulations using appropriate numerical methods. The Finite Differences Method is applied to the problem formulation. For the real situation approximation, the thermal conductivity of the silicon is considered as a function of temperature at each point, which makes the equation that governs the nonlinear problem. Finally, the comparison of the results obtained with typical results proves that the assumptions of variable thermal conductivity, heat dissipation by thermal radiation and mutual interaction are crucial to obtain results that are closer to reality.*

Keywords: *Extended surface, Heat dissipation, Thermal radiation, Mutual effect*

1. Introduction

The present work presents the mathematical modeling that describes the thermal distribution along two longitudinal fins, with rectangular geometric profile attached to the same base, and therefore, to the same thermal source. Several considerations should be taken into account for the preliminary feasibility of this work.

The central objective of this analysis is to observe the thermal behavior of fins that dissipate heat from a given primary surface through the heat transfer processes of conduction, convection and thermal radiation.

The analysis of such temperature profile aims to present the importance of considering the effects of thermal radiation on heat dissipation and mutuality of effects between two fins, so that, compared to the thermal behavior in the absence of such effects, there is a considerable discrepancy in the results.

Since the majority of the works that involve the topic considered consider the parameter of thermal conduction constant, this study proposes to consider the variation of such parameter to bring even greater applicability and approximation of reality.

2. Mathematical approach

Some considerations should be made: The thermal transient is neglected; The fin is not a source of its own heat; and the double-fins have exactly the same geometric dimensions and are positioned symmetrically.

Based on such definitions, and according to Quarteroni *et al.* (2010), the thermal distribution in the directions of each Cartesian axis of the fin, is

$$\frac{\partial}{\partial x}\left(k(T)\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(k(T)\frac{\partial T}{\partial y}\right)+\frac{\partial}{\partial z}\left(k(T)\frac{\partial T}{\partial z}\right)=0. \quad (1)$$

By the definitions of fins, the width and height are much larger than the thickness, therafter it is concluded that only in the axis y the temperature differences between their points are considerable. This formulation suggests that the problem is analyzed in a one-dimensional approach.

2.1 Boundary conditions

The Fig.1 presents a *Single Fin* which has only one surface extended to the primary surface and *Double Fin* which has two extended surfaces to the primary surface, where the distance between them is very small.

Two mathematical boundary conditions (b.c.) were used. These conditions are very common in mathematical problems involving differential equations, as shown in Fig.1.

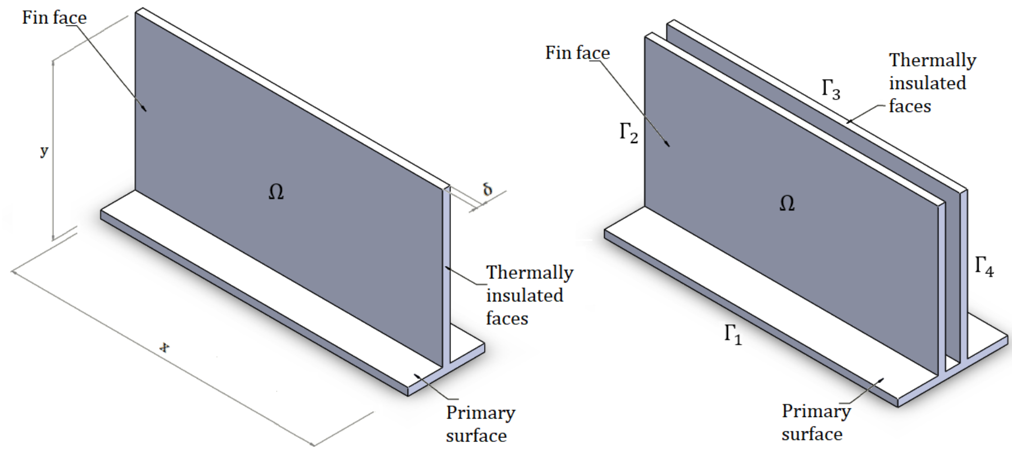

Figure 1. Boundary and domain of fins

- The face Γ_1 $(y = 0)$ is conditioned with a certain temperature imposed on it. (Dirichlet b.c.)
- The faces Γ_2 $(x = 0)$, Γ_3 $(x = L)$ and Γ_4 are thermally isolated. (Neumann b.c.)
- In the faces $z = 0$ e $z = \delta$, heat dissipation is considered by thermal convection and radiation, according to Li *et al.* (2016) and Dogonchi and Ganji (2016).

2.2 Analytical insertion of radiation effects

When considering the effects of thermal radiation, it is assumed that the boundary conditions on the fin faces will be altered, since it is precisely on both faces of the fin that the thermal dissipation occurs. Until here the analytical solution was exposed only with the effects of convection.

This insertion is given by adding the equation of the *Stefan-Boltzmann's Law*, which represents the heat flux transferred by thermal radiation.

In the situation involving thermal radiation dissipation and variable thermal conductivity, the thermal phenomenon presents itself as non-linear, considerably hindering an analytical solution.

For the total analysis of the problem, one has to:

$$z = 0 \Rightarrow k(T)\frac{dT}{dz} = h\left(T - T_\infty\right) + \varepsilon\sigma T^4 \tag{2a}$$

$$z = \delta \Rightarrow -k(T)\frac{dT}{dz} = h\left(T - T_\infty\right) + \varepsilon\sigma T^4 \tag{2b}$$

Equations (2) are similarly symmetrical, since the geometric conditions imposed on the problem produce a symmetrical heat dissipation on both faces of the fin.

When working with extended surfaces, one should consider an essential fin feature, which is its geometry as a very thin plate. Thus, by integrating the differential equations above, we conclude through *Average Value Theorem* (Apostol, 1969) that

$$\frac{d}{dz}\left(k(T)\frac{d\overline{T}}{dz}\right) = \frac{1}{\delta}\{-h[(\overline{T} - T_\infty) + \varepsilon\sigma T^4] - h[(\overline{T} - T_\infty) + \varepsilon\sigma T^4]\} = -\frac{2}{\delta}[h\left(\overline{T} - T_\infty\right) + \varepsilon\sigma T^4]. \tag{3}$$

Relating the Eq.(1) and Eq.(3), we have a new formulation for the temperature profile.

$$k(T)\frac{\partial^2 T}{\partial x^2} + k(T)\frac{\partial^2 T}{\partial y^2} + \left(\frac{\partial k(T)}{\partial y} \cdot \frac{\partial T}{\partial y}\right) - \frac{2}{\delta}[h(T - T_\infty) + \varepsilon\sigma T^4] = 0. \tag{4}$$

The equation (4), therefore, it is the equation that models the thermal distribution with radiance in the fin, still in a two-dimensional approach. As previously explained, the studied situation considers that the entire primary surface is at a prescribed homogeneous temperature, ie, along this surface, all points are at the same temperature, which leads to the conclusion that along the axis x of the fin , there is no variation of temperature, so the derivative of temperature in relation to x equals zero.

$$k(T)\frac{\partial^2 T}{\partial y^2} + \left(\frac{\partial k(T)}{\partial y} \cdot \frac{\partial T}{\partial y}\right) - \frac{2}{\delta}[h(T - T_\infty) + \varepsilon\sigma T^4] = 0, \tag{5}$$

where k is a function of T, which in turn is a function of y, so k will be treated as a variable in function of y.

The mathematical result of the thermal interaction between the fins in a radiation-only perspective is given by the integral expressed in Eq(6), developed by Sobral (2017), which represents the form factor of the thermal radiation between the surfaces.

$$E_{mut} = \int_0^b \sigma\overline{T}^4(\xi)\left(\frac{d^2}{2[(y-\xi)^2 + d^2]^{\frac{3}{2}}}\right) d\xi. \tag{6}$$

2.3 Numerical methods

For the characterization of the thermal conductivity profile, it is necessary to trace parameters that relate the thermal conductivity (k) and temperature (T).

The method used was the *Least Squares Method* (LSM), where the empirical data were stipulated Ho *et al.* (1972) and the approximation is illustrated in Fig.3 for silicon data.

Simplifying and adjusting the minimization approximation equations, it follows that

$$an + b\sum_{k=1}^{n} x_i = \sum_{k=1}^{n} y_i \quad \text{and} \tag{7a}$$

$$a\sum_{k=1}^{n} x_i + b\sum_{k=1}^{n} x_i^2 = \sum_{k=1}^{n} x_i y_i \tag{7b}$$

The solution of the system represented by the equation (7) is given by equations (8).

$$A = \frac{\sum(\ln y) - b\sum(\ln x)}{n} \quad \text{and} \tag{8a}$$

$$B = \frac{n\sum(\ln x \ln y) - \sum(\ln x)\sum(\ln y)}{n\sum[(\ln x)^2] - (\sum(\ln x)^2)} \tag{8b}$$

Where $a = exp(A)$ e $b = B$.

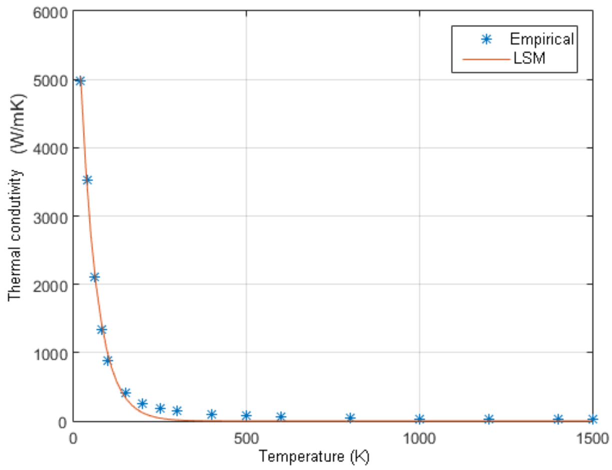

Figure 2. Curve fitting by LSM in exponential type

In order to solve numerically the partial derivatives present in the problem studied, is used the Finite Differences Method(FDM). Holman (2010)

Given these relations, after algebraic adjustments takes the following form.

$$T_j = \frac{\delta}{2k_j + 2hl^2}\left[T_{j+1}\left(\frac{k_j - k_{j-1}}{2} + k_j\right) - T_{j-1}\left(\frac{k_j - k_{j-1}}{2} - k_j\right)\right] + \frac{2hl^2 T_\infty}{2k_j + 2hl^2} - \frac{2\varepsilon\sigma T_{rad}}{2k_j + 2hl^2}. \tag{9}$$

3. RESULTS AND CONCLUSIONS

The analysis and processing of all the data, methods and processes exposed in this work have resulted in some extremely relevant conclusions.

Since this work argues that certain phenomena can not be neglected, all simulation procedures occur contemplating the most varied situations so that it is possible to make comparisons of the results and determine the relevance of the study.

The simulation environment was maintained in all situations, except for the parameters that characterize the preponderant differences that will be compared.

It was used to carry out this work the commercial software *Matlab*, which allowed the creation of an algorithm that contains all the mathematical data and process the required information and calculations.

$T_\infty \rightarrow$ Environment temperature $(= 300K)$
$T_{base} \rightarrow$ Primary surface temperature $(= 500K)$
$j_{max} \rightarrow$ Number of vertically oriented nodes $(= 50)$
$L_y \rightarrow$ Fin height $(= 10mm)$
$\delta \rightarrow$ Fin thickness $(= 1mm)$
$tol_T \rightarrow$ Specified accuracy$(= 10^{-6})$

It is worth mentioning that the parameters stipulated above can be handled in a convenient manner in order to investigate results in other circumstances.

In order to allow better visualization and understanding of the obtained results, some graphs were generated.

To evaluate the behavior of thermal conductivity at different temperature values, the data were processed in six different ways from the combinations that will be exposed in the following subsections.

3.1 Single fin, without radiation and constant k for any T

The first results studied were evaluating single fins, following concepts proposed by Murray (1938) and Gardner (1945), who developed a list of assumptions known as *Murray-Gardner Hypotheses*. These statements seek to simplify as much as possible the differential calculations governing the effects studied. Among these hypotheses, one of them is the evaluation of the heat dissipation in extended surfaces considering its constant thermal conductivity in all the points and despising the effects of radiation. The results obtained in this evaluation follow in Fig.3.

Temperature profile	
Node	Single, without radiation, constant k
1	500
2	450,0089
3	412,5134
4	384,3901
5	363,2963
. . .	
46	300,0005
47	300,0004
48	300,0004
49	300,0003
50	300,0003
Color legend	
500	T 300,0003

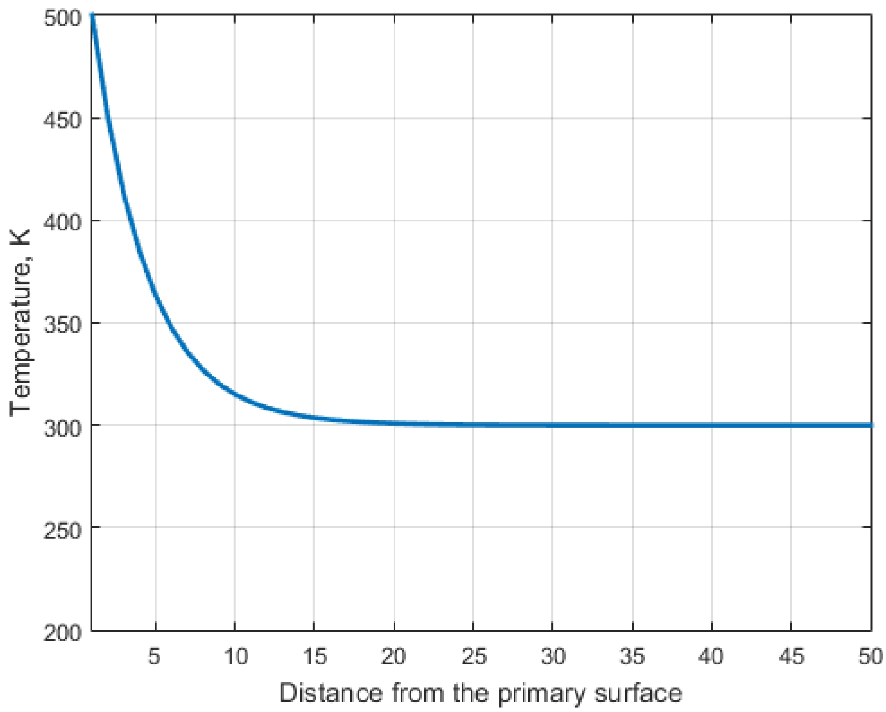

Figure 3. Thermal profile of single fin, without radiation and constant k

3.2 Single fin, without radiation and variable k in function of T

One of the parameters proposed in this work is the insertion of the effects generated by the phenomenon of variation of thermal conductivity, as a function of temperature. Such an analysis has been described in the subsection 2.3 Kraus *et al.* (2002) have suggested that the Murray-Gardner hypothesis that evaluates the constant thermal conductivity in all directions, when disregarded, proposes results that are closer to a real model. Fig.4 shows the results in the evaluation of single fins, including only the effects of variation in thermal conductivity.

Temperature profile	
Node	Single, without radiation, variable k
1	500
2	494,1872
3	488,6525
4	483,3804
5	478,3563
. . .	
46	394,0088
47	393,81
48	393,6777
49	393,6116
50	393,6116
Color legend	
500	T 393,6116

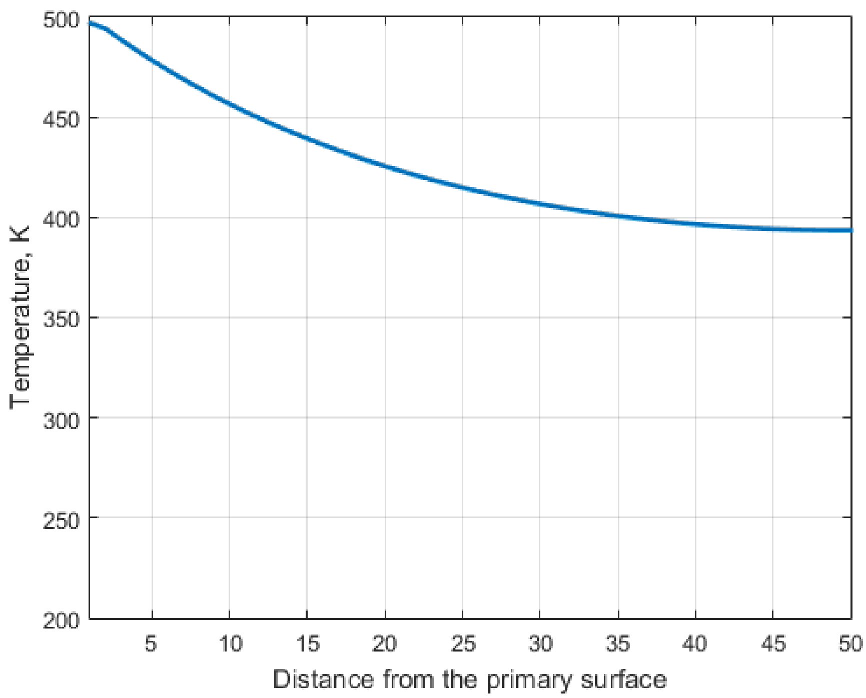

Figure 4. Thermal profile of single fin, without radiation and variable k

3.3 Single fin, with radiation and constant k for any T

Many authors have already proposed studies of extended surfaces considering the effects of thermal radiation on heat dissipation. Callinan and Berggren (1959) have brought one of the earliest publications on such considerations with fins. The relevance of this consideration can be proven by several situations, since thermal radiation does not depend on a material medium, so applications in the vacuum or rarefied atmosphere can be analyzed. Fig.5 presents the results of insertion of these effects in the exposed situations.

Temperature profile	
Node	Single, with radiation, constant k
1	500
2	354,3908
3	287,8075
4	252,6131
5	232,7033
. . .	
46	203,2476
47	203,2476
48	203,2476
49	203,2476
50	203,2476
Color legend	
500 — T — 203,2476	

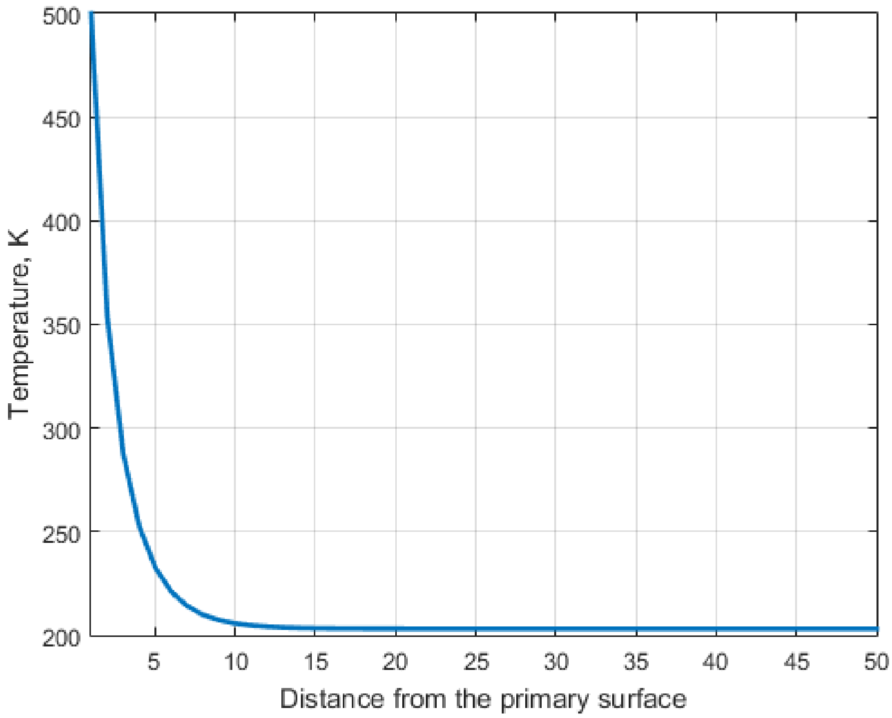

Figure 5. Thermal profile of single fin, with radiation and constant k

It is worth mentioning an important part of this analysis where it has been identified that, once the thermal radiation dissipation effects have been inserted, the last term of the thermal profile equation ensures that such effects do not depend on the ambient temperature.

$$\frac{dQ}{dt} = \varepsilon A \sigma T^4 \tag{??}$$

This premise is guaranteed by Stefan-Boltzman's Law, where heat transfer is not by exchange with the fluid, but by electromagnetic emission. This leads to the conclusion that this consideration allows the fin temperature profile to reach values below room temperature.

3.4 Single fin, with radiation and variable k in function of T

The combination of the effects generated by the heat dissipation by thermal radiation, besides the evaluation of the thermal conductivity variation as a function of the temperature at each point was proposed by Cohen (1969) and brings a closer approximation to real results. To the nonlinear differential equation (as a function of the radiation term) the approximation calculation of thermal conductivity values is added by means of appropriate numerical methods. Fig.6 shows the results obtained in this situation.

Temperature profile	
Node	Single, with radiation, variable k
1	500
2	472,3525
3	449,3305
4	429,8068
5	413,0058
. . .	
46	255,3705
47	255,143
48	254,9917
49	254,9162
50	254,9162
Color legend	
500	T 254,9162

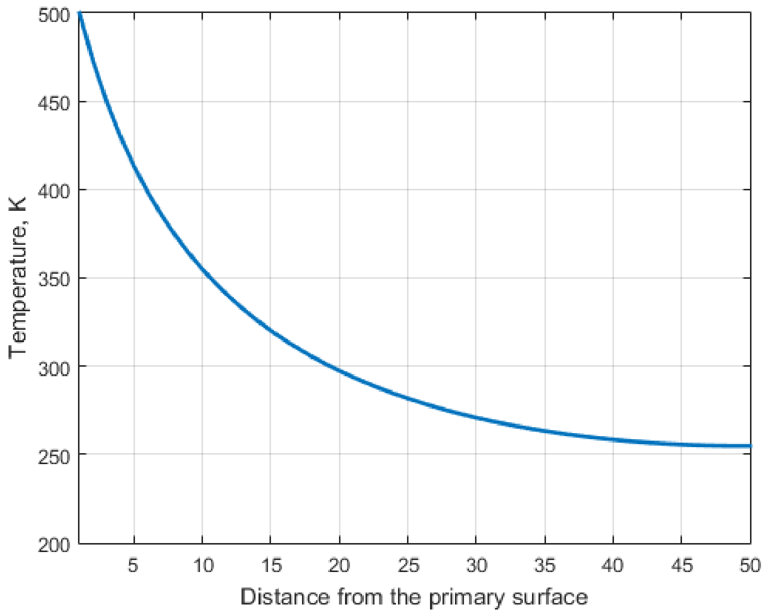

Figure 6. Thermal profile of single fin, with radiation and variable k

3.5 Double fin, with radiation and constant k for any T

This situation provides an interesting approximation of a real model, since the consideration in two fins brings with it the concept of thermal interaction, that is, a fin, besides dissipating heat, as already said, also receives heat from the neighboring fin, this being a mutual relationship. Eckert *et al.* (1960) proposed the analysis of the effects generated by the thermal interaction between fins. Fig.7 gives the results of such an analysis.

Temperature profile	
Node	Double, with radiation, constant k
1	500
2	444,2809
3	382,2041
4	347,5135
5	327,5989
. . .	
46	263,9203
47	263,8854
48	263,8242
49	263,7282
50	263,7282
Color legend	
500	T 263,7282

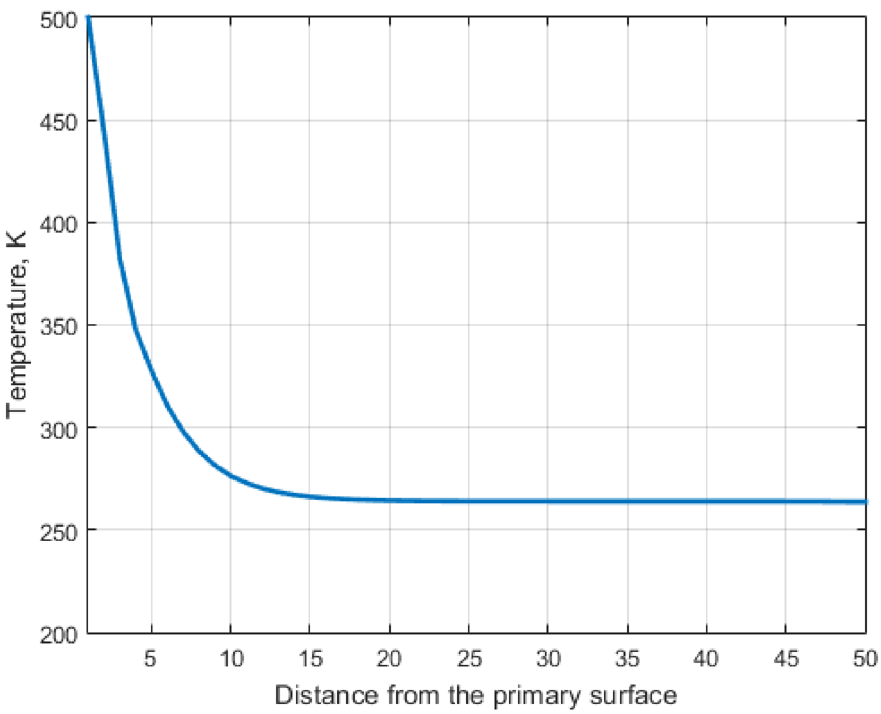

Figure 7. Thermal profile of double fin, with radiation and constant k

3.6 Double fin, with radiation and variable k in function of T

Finally, the main results which are expected from this work are the profile of temperatures in a situation contemplating a double fin, which interact with one another with heat dissipation by thermal radiation, which is absorbed by the neighboring fin, whose thermal conductivity varies depending on the temperature at each point.

Since the real purpose of this work is to present a model that approximates reality, the result of the processing is presented in Fig.8 by means of graphic plotting.

Temperature profile		
Node	Double, with radiation, variable k	
1	500	
2	482,6424	
3	468,8099	
4	456,4912	
5	445,4575	
.	.	.
46	326,8627	
47	326,6801	
48	326,5574	
49	326,4948	
50	326,4948	
Color legend		
500	T	326,4948

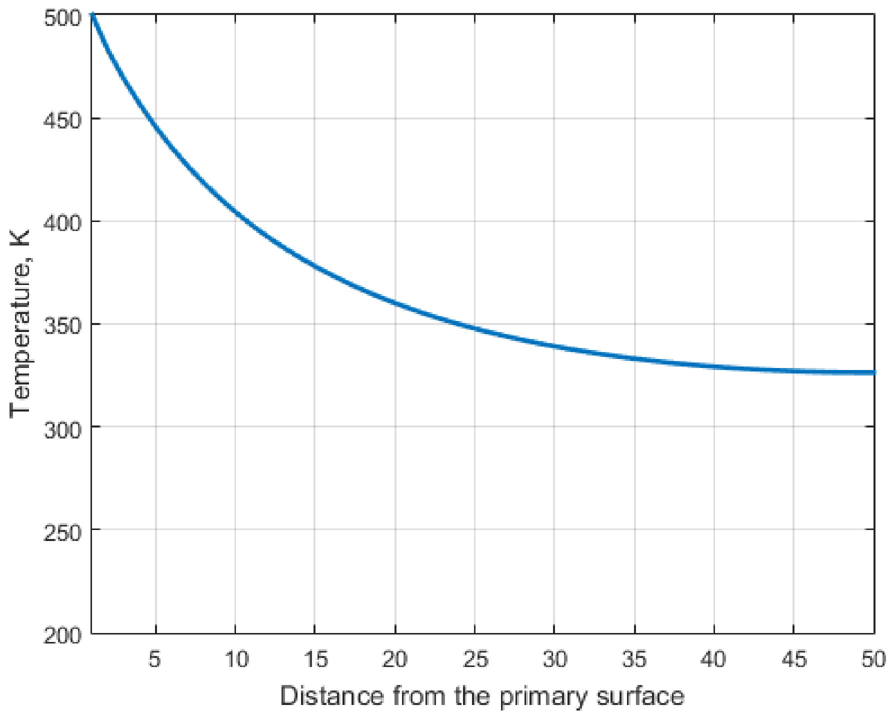

Figure 8. Thermal profile of double fin, with radiation and variable k

In order to demonstrate the importance of this study and the analyzes and proposed situations, the numerical results of the first ten nodes and the last ten nodes were organized in Table 1 so that it was possible to understand the thermal variation in all situations contemplated in this study for some nodes processing.

Table 1. Results

Node	Single, without radiation, constant k	Single, without radiation,	Single, with radiation, constant k	Single, with radiation, variable k	Double, with radiation, constant k	Double, with radiation, variable k
1	500	500	500	500	500	500
2	450,0089251	494,1872126	354,3908378	472,3524942	444,280935	482,6424284
3	412,5133876	488,6525468	287,8074834	449,3305075	382,2040758	468,8098877
4	384,390061	483,3803863	252,6130777	429,8067921	347,513475	456,4911812
5	363,2963107	478,3563131	232,7032644	413,0057567	327,5989135	445,4575289
6	347,4750564	473,566998	221,0363878	398,3737754	310,4276272	435,5274798
7	335,6084095	469,0001041	214,0653018	385,5035749	297,7868752	426,5530895
8	326,7078945	464,6442	209,8530357	374,0879182	288,336919	418,4118607
9	320,032111	460,4886814	207,2908814	363,8900482	281,4249096	411,0014401
10	315,0249753	456,5237016	205,7262299	354,7241598	276,4148008	404,2356503
.	.	.	.	.	.	.
.	.	.	.	.	.	.
.	.	.	.	.	.	.
41	300,0020204	396,0121361	203,2475953	257,6853864	263,9614506	328,7001292
42	300,0015216	395,4751017	203,247595	257,0609146	263,9596154	328,2060183
43	300,0011503	395,0068734	203,2475949	256,5188023	263,9563464	327,7764943
44	300,0008758	394,6067778	203,2475948	256,0572996	263,9504378	327,4101654
45	300,0006751	394,2742409	203,2475947	255,6749303	263,9397131	327,1058838
46	300,0005316	394,0087856	203,2475947	255,3704793	263,9202786	326,8627397
47	300,0004333	393,8100308	203,2475947	255,1429835	263,8853516	326,6800596
48	300,000372	393,6776898	203,2475946	254,9917232	263,8241691	326,5574259
49	300,0003425	393,6115694	203,2475946	254,9162167	263,7282069	326,4948319
50	300,0003416	393,6115684	203,2475939	254,9162157	263,728206	326,4948309

Color legend		
500	T	200

In addition, a graph in Fig.9 was generated that overlapped all the results analyzed by this work in order to demonstrate the discrepancy between the results in all situations analyzed in this work.

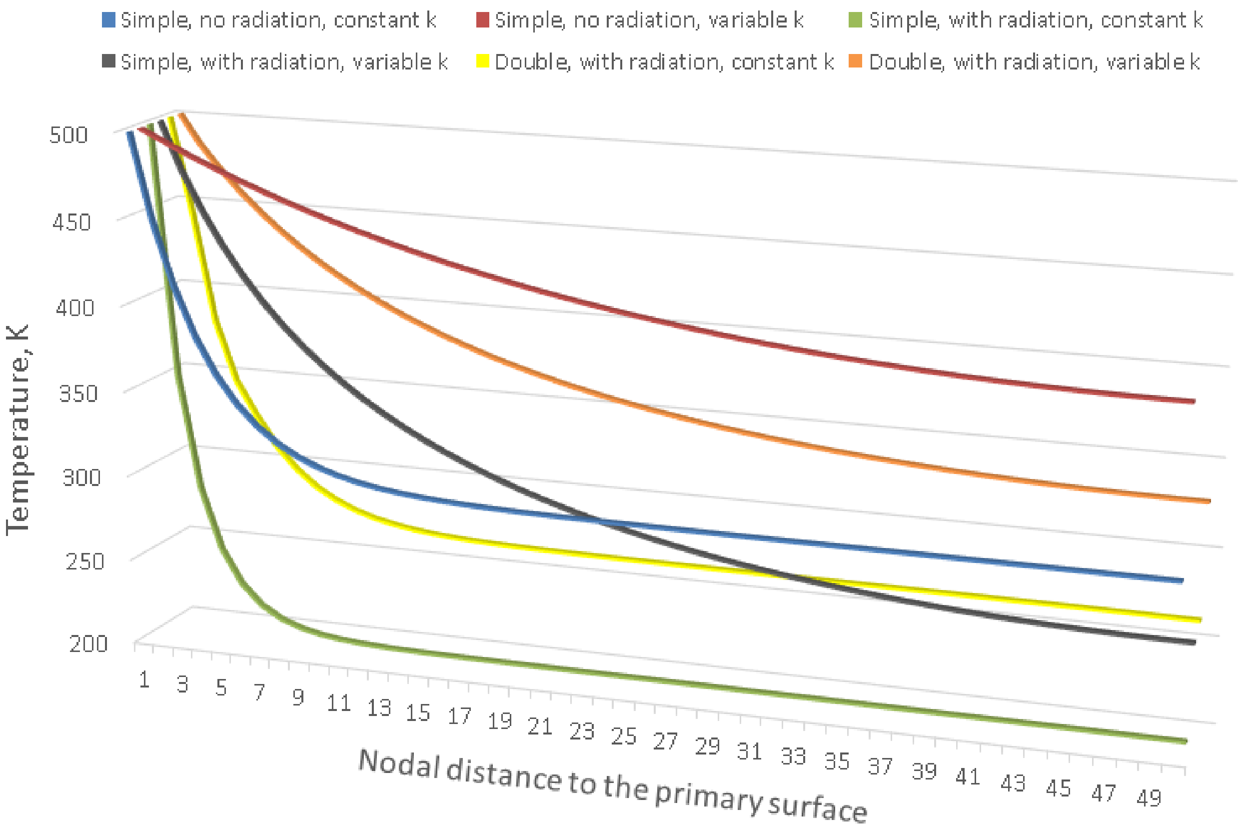

Figure 9. Comparison of thermal profiles

Since the final objective of this analysis is based on the results in the double-fin situation, with thermal radiation dissipation and variable thermal conductivity, the numerical results of the first ten nodes and the last ten nodes were compared to that, where such comparisons are expressed in the percentage difference to the conclusive result of this work.

It is worth mentioning that variations in values of k are much more evident in a context in which low and medium temperatures are employed, since the higher the values of T, the more the thermal conductivity profile approaches a straight line, in asymptotic way.

In this study, it was noted the enormous importance of evaluating commonly overlooked phenomena, which generate very considerable discrepancies to the final results in several applications.The errors related to the expected result reached maximum values of 49.84%, in the case of single fins with constant thermal conductivity.

It can be concluded, therefore, that thermal dissipation analysis, in order to approach a real model, should never neglect the variation of thermal conductivity, the effects of thermal radiation and mutual interaction.

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